

# VCE Specialist Mathematics Units 3&4

**Suggested Solutions** 

2024 Trial Examination 1

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Note: Deduct 1 mark for a single error.

## Question 1 (4 marks)

Applying implicit differentiation gives:

$$1 \times \tan^{-1} y + x \times \frac{1}{1+y^2} \frac{dy}{dx} + 2\frac{dy}{dx} = 2x$$
 A2

Substituting point (0, 1) gives:

$$\frac{\pi}{4} + 0 + 2\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = -\frac{\pi}{8}$$
$$m_{\perp} = \frac{8}{\pi}$$
A1

$$\therefore y = \frac{8}{\pi}x + 1$$
 A1

### Question 2 (3 marks)

u = x	du = dx	
$dv = \cos(x)dx$	$v = \sin(x)$	
		M1

$$\int_{0}^{\pi} x \cos(x) dx = [x \sin(x)]_{0}^{\pi} - \int_{0}^{\pi} \sin(x) dx$$

$$= 0 + [\cos(x)]_{0}^{\pi}$$
A1

#### Question 3 (5 marks)

b. 
$$\operatorname{Arg}\left(\left(2-\frac{2}{\sqrt{3}}i\right)^n\right) = \pi k, \ k \in \mathbb{Z}$$
  
 $-\frac{n\pi}{6} = \pi k$   
 $n = 6k \ \mathbf{OR} - 6k, \ k \in \mathbb{Z}$   
Note: Consequential on answer to Question 3a.

#### Question 4 (4 marks)

**a.** The vector resolute of  $\underline{a}$  in the direction of  $\underline{b}$  is given by  $(\underline{a} \cdot \underline{b})\underline{b}$ .

$$\hat{b} = \frac{1}{\sqrt{1+9+25}} \left( -\underline{i} + 3\underline{j} - 5\underline{k} \right)$$

$$= \frac{1}{\sqrt{35}} \left( -\underline{i} + 3\underline{j} - 5\underline{k} \right)$$
A1
$$\underline{a} \cdot \hat{b} = \frac{1}{\sqrt{35}} \left( (2 \times -1) + (-4 \times 3) + (1 \times -5) \right)$$

$$= \frac{1}{\sqrt{35}} (-2 - 12 - 5)$$

$$= -\frac{19}{\sqrt{35}}$$

$$\left( \underline{a} \cdot \underline{b} \right) \underline{b} = -\frac{19}{35} \left( -\underline{i} + 3\underline{j} - 5\underline{k} \right)$$

$$= \frac{19}{35} \underline{i} - \frac{57}{35} \underline{j} + \frac{19}{7} \underline{k}$$
A1

**b.** The Cartesian equation is given by  $\underline{n} \cdot \overline{P_0P}$ , where  $\underline{n} = \underline{a} \times \underline{b}$ ,  $P_0 = (3, 1, -1)$  and P = (x, y, z).

$$\begin{split} \mathbf{a} \times \mathbf{b} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -4 & 1 \\ -1 & 3 & -5 \end{vmatrix} \\ &= (20 - 3)\mathbf{i} - (-10 + 1)\mathbf{j} + (6 - 4)\mathbf{k} \\ &= 17\mathbf{i} + 9\mathbf{j} + 2\mathbf{k} \\ 17(x - 3) + 9(y - 1) + 2(z + 1) = 0 \\ & 17x + 9y + 2z = 58 \end{split}$$
 M1

Question 5 (3 marks)	
$W \sim N(25, 1.8^2)$	
$\overline{W} \sim N\left(25, \left(\frac{1.8}{\sqrt{36}}\right)^2\right)$	A1
$\Pr(\overline{W} < a) = 0.84$	
$\Pr(Z < 1) = 0.84$	A1
$1 = \frac{a - 25}{\frac{1.8}{2}}$	
a = 25.3  kg	A1
Question 6 (4 marks)	
$f'(x) = 4x^3e^{2x} + 2x^4e^{2x}$	
$=2e^{2x}\left(2x^3+x^4\right)$	A1
(1/2) $(0.3, .4)$ $(0.2)$ $(0.2, .3)$	

$$f''(x) = 4e^{2x} (2x^3 + x^4) + 2e^{2x} (6x^2 + 4x^3)$$
  
=  $4x^2 e^{2x} (x^2 + 4x + 3)$   
 $f''(x) = 0$  A1

$$4x^{2}e^{2x}(x+1)(x+3) = 0$$

$$x = -3, -1, 0$$

$$f''(-3^{-}) > 0 \Rightarrow \text{ concave up for } x < -3$$

$$f''(-3^{+}) < 0$$

$$f''(-1^{+}) > 0 \Rightarrow \text{ concave up for } x > -1, x \neq 0 \text{ (stationary point of inflection at 0)}$$

$$x \in (-\infty, -3) \cup (-1, \infty) \setminus \{0\}$$

Question 7 (4 marks)

$$S = 2\pi \int_{0}^{1} x^{3} \sqrt{1 + (3x^{2})^{2}} dx$$
  
=  $2\pi \int_{0}^{1} x^{3} \sqrt{1 + 9x^{4}} dx$  M1  
 $u = 1 + 9x^{4} \rightarrow du = 36x^{3} dx$  M1

$$S = 2\pi \int_{0}^{10} \frac{1}{36} u^{\frac{1}{2}} du$$
A1

$$= \frac{\pi}{18} \left[ \frac{2}{3} u^{\frac{3}{2}} \right]_{1}^{10}$$
$$= \frac{\pi}{27} (10\sqrt{10} - 1)$$
A1

#### Question 8 (6 marks)

a. 
$$\dot{t}(t) = -e^{t}\dot{t} + (2e^{2t} - 2e^{t})\dot{t}$$
 A1  
 $\dot{t}(0) = -e^{0}\dot{t} + (2e^{0} - 2e^{0})\dot{t}$   
 $= -\dot{t}$   
 $|\dot{t}(0)| = \sqrt{(-1)^{2} + 0^{2}}$   
 $= 1$   
b.  $x = -e^{t}$   
 $x^{2} = (-e^{t})^{2}$ 

$$x^{2} = e^{2t}$$

$$y = e^{2t} - 2e^{t} - 1$$

$$= x^{2} + 2x - 1$$
A1

Given that  $t \ge 0$ ,  $e^t \ge 1 \Rightarrow -e^t \le -1 \Rightarrow x \le -1$ . A1

**c.** Let *P* be a point on the curve  $y = x^2 + 2x - 1$  (from **part b.**). The coordinates of *P* are given by  $(x, x^2 + 2x - 1)$ . Finding the distance from *P* to the origin gives:

$$d_{OP} = \sqrt{x^{2} + (x^{2} + 2x - 1)^{2}}$$
$$\frac{d}{dx}(d_{OP}) = \frac{1}{2}(x^{2} + (x^{2} + 2x - 1)^{2})^{-\frac{1}{2}} \times (2x + 2(x^{2} + 2x - 1)(2x + 2))$$
M1

Minimising the distance gives:

$$\frac{d}{dx}(d_{OP})=0$$

Considering the numerator only gives:

$$(2x + 2(x^{2} + 2x - 1)(2x + 2)) = 0$$
  

$$2x + 4x^{3} + 4x^{2} + 8x^{2} + 8x - 4x - 4 = 0$$
  

$$4x^{3} + 12x^{2} + 6x - 4 = 0$$
  

$$2x^{3} + 6x^{2} + 3x - 2 = 0$$
  
When  $x = a$ :  

$$2a^{3} + 6a^{2} + 3a - 2 = 0$$

M1 Note: Consequential on answer to **Question 8b.** 

Question 9 (4 marks)	
$-2x^2 + 13x - 1$ A $Bx + C$	N / 4
$\frac{1}{(x+5)(x^2+4)} = \frac{1}{x+5} + \frac{1}{x^2+4}$	
$-2x^{2} + 13x - 1 = A(x^{2} + 4) + (Bx + C)(x + 5)$	
Substituting $x = -5$ into the equation gives:	
$A((-5)^{2}+4)+(B(-5)+C)(-5+5) = -2 \times (-5)^{2}+13 \times -5-1$	
29 <i>A</i> = -116	
A = -4	
Substituting $x = 0$ and then $A = -4$ gives:	
$A((0)^{2}+4)+(B(0)+C)(0+5) = -2 \times (0)^{2}+13 \times 0 - 1$	
$4 \times -4 + 5C = -1$	
C = 3	
Substituting $x = 1$ and then $A = -4$ , $C = 3$ gives:	
$A((1)^{2} + 4) + (B(1) + C)(1 + 5) = -2 \times (1)^{2} + 13 \times 1 - 1$	
$5 \times -4 + 6B + 6 \times 3 = 10$	
<i>B</i> = 2	A1
$\int \left(\frac{-4}{x+5} + \frac{2x+3}{x^2+4}\right) dx$	A1
$= \int \left( \frac{-4}{x+5} + \frac{2x}{x^2+4} + \frac{3}{x^2+4} \right) dx$	
$= -4\log_{e} x+5  + \log_{e}(x^{2}+4) + \frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right) + c$	A1
Question 10 (3 marks)	

Assuming 
$$\sin(x) + \cos(x) < 1$$
 for some  $x \in \left[0, \frac{\pi}{2}\right]$ :  

$$(\sin(x) + \cos(x))^2 < 1^2$$

$$sin^{2}(x) + 2sin(x)cos(x) + cos^{2}(x) < 1$$
  
 $2sin(x)cos(x) + 1 < 1$   
 $2sin(x)cos(x) < 0$  M1

This is false, since  $sin(x) \ge 0$  and  $cos(x) \ge 0$  for  $x \in \left[0, \frac{\pi}{2}\right]$ . M1 Therefore, by contradiction, proof is complete.