Neap

VCE Specialist Mathematics Units 3&4

Suggested Solutions

2024 Trial Examination 2

Section A – Multiple-choice questions

1	A	**B**	C	D
2	A	**B**	C	D
3	A	**B**	C	D
4	**A**	B	C	D
5	A	**B**	C	D
6	**A**	B	C	D
7	A	B	C	**D**
8	A	B	**C**	D
9	A	B	C	**D**
10	A	B	C	**D**
11	A	B	C	**D**
12	A	**B**	C	D
13	**A**	B	C	D
14	A	B	**C**	D
15	A	B	C	**D**
16	**A**	B	C	D
17	A	B	**C**	D
18	A	B	C	**D**
19	**A**	B	C	D
20	A	B	**C**	D

Question 1 B

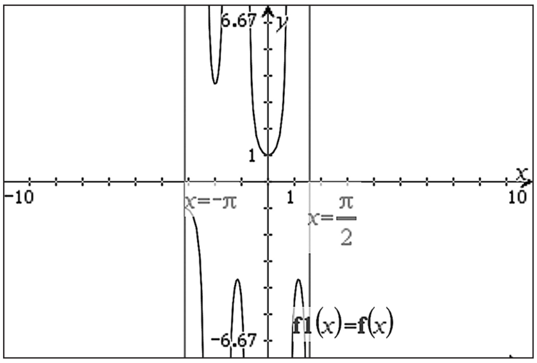
The given statement is $p \rightarrow q$, where p represents 'my plants are turning yellow' and q represents 'they are not getting enough water'.

The contrapositive of the statement is given by $\neg q \rightarrow \neg p$, where $\neg p$ represents 'my plants are not turning yellow' and $\neg q$ represents 'they are getting enough water'

Therefore, the contrapositive statement is 'If they are getting enough water, then my plants are not turning yellow.'

Question 2 B

Using a CAS calculator to sketch the graph of *f* gives:



The asymptote at $x = \frac{\pi}{2}$ is excluded since the interval is $\left(-\pi, \frac{\pi}{2}\right)$. Thus, reading from the graph, there are four vertical asymptotes in the given interval.

Question 3 B

Using a CAS calculator gives:

$$f(x) := \sin^{-1}(\ln(x)) \quad \text{Done}$$

$$\text{solve}\left(\frac{d^2}{dx^2}(f(x)) < 0, x\right) \quad e^{-1} < x < e^{\frac{\sqrt{5}}{2} - \frac{1}{2}}$$

$$\therefore \frac{1}{e} < x < e^{\frac{\sqrt{5}-1}{2}}$$

Question 4 A

Using a CAS calculator gives:

$$p(z) := 2 \cdot z^3 + b \cdot z^2 + c \cdot z + 100 \quad \text{Done}$$

$$\text{solve}\left(\begin{cases} p(2+4 \cdot i) = 0 \\ p(2-4 \cdot i) = 0 \end{cases}, b, c\right) \quad b = -3. \text{ and } c = 20.$$

$$\begin{aligned} \therefore b + c &= -3 + 20 \\ &= 17 \end{aligned}$$

Question 5 B

The subset is the perpendicular bisector of points (–3, –1) and (3, 3).

Thus, the line will pass through $\left(\frac{-3+3}{2}, \frac{-1+3}{2}\right) = (0, 1)$.

$$m = -\frac{1}{\frac{-1-3}{-3-3}}$$
$$= -\frac{3}{2}$$

Finding the equation of the path gives:

$$y - 1 = -\frac{3}{2}(x - 0)$$
$$y = -\frac{3}{2}x + 1$$

Through a process of elimination, it can be found that only (–2, 4) satisfies this equation.

Question 6 A

Since $\angle COA = 90°$, OC can be obtained by rotating OA anticlockwise at an angle of 90°.

Since $OC = 2OA$, point C can be obtained by rotating point A anticlockwise at an angle of 90° and dilating by a factor of 2.

Therefore, point C can be represented as $2iz$.

As $OABC$ is a rectangle, it is also a parallelogram where OB is its diagonal.

Point B is the sum of points A and C.

Therefore, point B is represented by the complex number $z + 2iz$.

Question 7 D

$$\int_0^1 \sqrt{\left(\frac{d}{dt}e^{2t}\right)^2 + \left(\frac{d}{dt}te^t\right)^2}\,dt = \int_0^1 \sqrt{\left(2e^{2t}\right)^2 + \left(e^t + te^t\right)^2}\,dt$$
$$= \int_0^1 \sqrt{4e^{4t} + \left(e^t + te^t\right)^2}\,dt$$

Question 8 C

$u = x$ $\qquad du = dx$

$dv = (x+1)^9\,dx$ $\qquad v = \frac{(x+1)^{10}}{10}$

$$\int u\,dv = uv - \int v\,du$$

$$\int x(x+1)^9\,dx = \frac{x(x+1)^{10}}{10} - \int \frac{(x+1)^{10}}{10}\,dx$$
$$= \frac{1}{10}x(x+1)^{10} - \int \frac{1}{10}(x+1)^{10}\,dx$$

Question 9 D

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \times \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

$$= (y_1z_2 - y_2z_1)\underset{\sim}{i} - (x_1z_2 - x_2z_1)\underset{\sim}{j} + (x_1y_2 - x_2y_1)\underset{\sim}{k}$$

The missing part of the algorithm is $-(x_1z_2 - x_2z_1) = x_2z_1 - x_1z_2$.

Question 10 D

Using a CAS calculator gives:

$$\text{euler}\left(3\cdot x^2+x,x,y,\{1,1.3\},0,0.1\right) \quad \begin{bmatrix} 1. & 1.1 & 1.2 & 1.3 \\ 0. & 0.4 & 0.873 & 1.425 \end{bmatrix}$$

$$\begin{aligned} y_3 &= y_2 + 0.1f(x_2) \\ &= 0.873 + 0.1f(1.2) \end{aligned}$$

Question 11 D

Using a CAS calculator gives:

$$n1 := \begin{bmatrix} 3 & -2 & 1 \end{bmatrix} \quad \begin{bmatrix} 3 & -2 & 1 \end{bmatrix}$$

$$n2 := \begin{bmatrix} -2 & 4 & 3 \end{bmatrix} \quad \begin{bmatrix} -2 & 4 & 3 \end{bmatrix}$$

$$\cos^{-1}\left(\frac{|\text{dotP}(n1,n2)|}{\text{norm}(n1)\cdot\text{norm}(n2)}\right) \quad 56.9124$$

$\therefore \angle \approx 57°$

Question 12 B

Using a CAS calculator gives:

$$v(x) := \frac{2}{4+x^2} \quad \textit{Done}$$

$$v(x)\cdot\frac{d}{dx}(v(x)) \quad \frac{-8\cdot x}{(x^2+4)^3}$$

$$\therefore \text{acceleration} = \frac{-8x}{(x^2+4)^3}$$

Question 13 A

Using a CAS calculator gives:

$v0:=0$	0
$v1:=v0+2\cdot 5$	10
$\text{solve}(0.5\cdot 2\cdot 5^2+v1\cdot t+0.5\cdot 1\cdot t^2=60,t)$	$t=-23.0384$ or $t=3.0384$
$time:=5+3.0384$	8.0384

$\therefore\ t \approx 8$ s

Question 14 C

Let x be the distance of the bottom of the ladder from the wall.

Let y be the distance of the top of the ladder from the floor.

$$x^2+y^2=100$$

$$y=\sqrt{100-x^2},\ y\geq 0$$

$$\frac{dy}{dx}=\frac{-x}{\sqrt{100-x^2}}$$

$$\frac{dy}{dt}=\frac{dy}{dx}\bigg|_{x=6}\times\frac{dx}{dt}$$

$$=\frac{-6}{8}\times 1$$

$$=-0.75$$

Question 15 D

D is correct. Using a CAS calculator gives:

$a:=[1\ \ -2\ \ 2]$	$[1\ \ -2\ \ 2]$
$b:=[-2\ \ -4\ \ 4]$	$[-2\ \ -4\ \ 4]$
$c:=[4\ \ 1\ \ -1]$	$[4\ \ 1\ \ -1]$

$\text{solve}(a=k\cdot b,k)$	false
$\text{dotP}(a,b)$	14
$\text{norm}(a)=\text{norm}(b)$	false
$\text{solve}(a=m\cdot b+n\cdot c,m,n)$	$m=\frac{9}{14}$ and $n=\frac{4}{7}$

$\underset{\sim}{a}=m\underset{\sim}{b}+n\underset{\sim}{c}$ for $m=\frac{9}{14}$ and $n=\frac{4}{7}$

A is incorrect. There is no $k\in R$ such that $\underset{\sim}{a}=k\underset{\sim}{b}$.

B is incorrect. $\underset{\sim}{a}\cdot\underset{\sim}{b}=14\neq 0$.

C is incorrect. $|\underset{\sim}{a}|\neq|\underset{\sim}{b}|$.

Question 16 A

Using a CAS calculator gives:

$$y(t):=h-\frac{1}{2}\cdot g\cdot t^2 \qquad \textit{Done}$$

$$\text{solve}\left(y(t)=0,t\right)$$

$$t=-\sqrt{\frac{2\cdot h}{g}} \text{ and } \frac{h}{g}\geq 0 \text{ or } t=\sqrt{\frac{2\cdot h}{g}} \text{ and } \frac{h}{g}\geq 0$$

$$\therefore T=\sqrt{\frac{2h}{g}}$$

Question 17 C

$$\underset{\sim}{\text{n}}=3\underset{\sim}{\text{i}}-7\underset{\sim}{\text{j}}+11\underset{\sim}{\text{k}}$$

$$\begin{aligned}\underset{\sim}{\text{r}}(t)&=\underset{\sim}{\text{n}}t+2\underset{\sim}{\text{i}}-5\underset{\sim}{\text{j}}+0\underset{\sim}{\text{k}}\\&=3t\underset{\sim}{\text{i}}-7t\underset{\sim}{\text{j}}+11t\underset{\sim}{\text{k}}+2\underset{\sim}{\text{i}}-5\underset{\sim}{\text{j}}+0\underset{\sim}{\text{k}}\\&=(3t+2)\underset{\sim}{\text{i}}+(-7t-5)\underset{\sim}{\text{j}}+11t\underset{\sim}{\text{k}}\end{aligned}$$

Question 18 D

$$\begin{aligned}\mu&=4\times 180+3\times 200\\&=1320\text{ g}\end{aligned}$$

$$\begin{aligned}\sigma&=\sqrt{15+15+15+15+10+10+10}\\&=\sqrt{90}\\&=3\sqrt{10}\end{aligned}$$

Question 19 A

$$\begin{aligned}\mu &= 0.4E(w)+0.2\\ &= 0.4\times 5+0.2\\ &= 2.2\end{aligned}$$

$$\begin{aligned}\text{Var}(T) &= 0.4^2\times \text{Var}(w)\\ &= 0.16\times 0.05^2\end{aligned}$$

$$\begin{aligned}\sigma &= \sqrt{0.16}\times\sqrt{0.05^2}\\ &= 0.4\times 0.05\\ &= 0.02\end{aligned}$$

Using a CAS calculator gives:

zInterval 0.02,2.2,50,0.95: *stat.results*

"Title"	"z Interval"
"CLower"	2.19446
"CUpper"	2.20554
"$\bar{x}$"	2.2
"ME"	0.005544
"n"	50.
"σ"	0.02

$\therefore$ 95% confidence interval = (2.19, 2.21)

Question 20 C

$T_1 \sim N\left(20,\ 2^2\right)$

$T_2 \sim N\left(20,\ 2^2\right)$

$T_1 - T_2 \sim N\left(0,\ \left(\sqrt{8}\right)^2\right)$

Using a CAS calculator to find $\Pr\left(\left|T_1 - T_2\right| < 1\right)$ gives:

$1-\text{normCdf}\left(-1,1,0,\sqrt{8}\right)$	0.723674

Section B

Question 1 (12 marks)

a. $\overrightarrow{AB} = -\overrightarrow{OA} + \overrightarrow{OB}$

$= -2\underset{\sim}{i} + \underset{\sim}{j} - 5\underset{\sim}{k} + 0\underset{\sim}{i} + 3\underset{\sim}{j} + 2\underset{\sim}{k}$

$= -2\underset{\sim}{i} + 4\underset{\sim}{j} - 3\underset{\sim}{k}$ A1

$\overrightarrow{AC} = -\overrightarrow{OA} + \overrightarrow{OC}$

$= -2\underset{\sim}{i} + \underset{\sim}{j} - 5\underset{\sim}{k} - 2\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$

$= -4\underset{\sim}{i} + 2\underset{\sim}{j} - 4\underset{\sim}{k}$ A1

b. There is no $\lambda \in R$ such that $\overrightarrow{AB} = \lambda\overrightarrow{AC}$. M1

Therefore, A, B and C are not collinear.

c. $\text{area} = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$

$= \sqrt{65}$ A1

$ab:=[-2\ \ 4\ \ -3]$	$[-2\ \ 4\ \ -3]$
$ac:=[-4\ \ 2\ \ -4]$	$[-4\ \ 2\ \ -4]$
$\frac{1}{2}\cdot \text{norm}(\text{crossP}(ab,ac))$	$\sqrt{65}$

*Note: Consequential on answer to **Question 1a.***

d. $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ -2 & 4 & -3 \\ -4 & 2 & -4 \end{vmatrix}$

$= -10\underset{\sim}{i} + 4\underset{\sim}{j} + 12\underset{\sim}{k}$ A1

$\Pi_1 : -10x + 4y + 12z = -10 \times 0 + 4 \times 3 + 12 \times 2 = 36$

$\Pi_1 : -5x + 2y + 6z = 18$ M1

$ab:=[-2\ \ 4\ \ -3]$	$[-2\ \ 4\ \ -3]$
$ac:=[-4\ \ 2\ \ -4]$	$[-4\ \ 2\ \ -4]$
$\text{crossP}(ab,ac)$	$[-10\ \ 4\ \ 12]$

*Note: Consequential on answer to **Question 1a.***

e. Letting $z = 0$ gives: M1

$$\begin{cases} -5x + 2y = 18 \\ x - 3y = -1 \end{cases} \Rightarrow x = -4,\ y = -1$$

Hence, point $P(-4, -1, 0)$ lies on line L. M1

$$\underset{\sim}{v} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ -5 & 2 & 6 \\ 1 & -3 & 1 \end{vmatrix}$$

$= 20\underset{\sim}{i} + 11\underset{\sim}{j} + 13\underset{\sim}{k}$ A1

$L : \underset{\sim}{r}(t) = (-4 + 20t)\underset{\sim}{i} + (-1 + 11t)\underset{\sim}{j} + 13t\underset{\sim}{k}$ A1

solve$\left(\begin{cases} -5\cdot x+2\cdot y=18 \\ x-3\cdot y=-1 \end{cases}, x,y\right)$	$x=-4$ and $y=-1$
crossP([-5 2 6],[1 -3 1])	[20 11 13]

Note: Accept responses that use alternate z-coordinates to find point P.

f. $\overrightarrow{AP} = -6\underset{\sim}{i} - 5\underset{\sim}{k}$ M1

$\underset{\sim}{n} = \underset{\sim}{i} - 3\underset{\sim}{j} + \underset{\sim}{k}$

$$d = \frac{\left|\overrightarrow{AP} \cdot \underset{\sim}{n}\right|}{\left|\underset{\sim}{n}\right|}$$

$= \sqrt{11}$ A1

Note: Consequential on answer to ***Question 1a.*** *The value of $\overrightarrow{AP}$ is consequential on point P.*

Question 2 (10 marks)

a. $u = 2\sqrt{2}\,\text{cis}\left(\frac{\pi}{3}\right)$ A1

$v = 2\sqrt{2}\,\text{cis}\left(-\frac{\pi}{3}\right)$ A1

$u:=\sqrt{2}+\sqrt{6}\cdot i$	$\sqrt{2}+\sqrt{6}\cdot i$
$v:=\text{conj}(u)$	$\sqrt{2}-\sqrt{6}\cdot i$
$\|u\|$	$2\cdot\sqrt{2}$
angle(u)	$\frac{\pi}{3}$
angle(v)	$\frac{-\pi}{3}$

b. $u^3 = 8\sqrt{8}\,\text{cis}(\pi)$

$= -16\sqrt{2}$

$v^3 = 8\sqrt{8}\,\text{cis}(-\pi)$

$= -16\sqrt{2}$

solving u^3 or v^3 M1

Therefore, $u^3 + 16\sqrt{2} = v^3 + 16\sqrt{2} = 0.$ M1

c. $z^3 + 16\sqrt{2} = (z-u)(z-v)(z-a)$, where $a \in R$ M1

$(z-u)(z-v) = z^2 - (u+v)z + uv$

$= z^2 - 2\sqrt{2}z + 8$

$\left(z^2 - 2\sqrt{2}z + 8\right)(z-a) = z^3 + 16\sqrt{2}$

$a = -2\sqrt{2}$

$z^3 + 16\sqrt{2} = \left(z^2 - 2\sqrt{2}z + 8\right)\left(z + 2\sqrt{2}\right)$ M1

Note: All working must be shown to obtain full marks.

d. Let X be a point on the positive real axis.

$\angle COX = \angle AOX - \angle AOC$

$= \frac{\pi}{3} - \frac{2\pi}{9}$

$= \frac{\pi}{9}$

$\angle COB = \angle COX + \angle BOX$

$= \frac{\pi}{9} + \frac{\pi}{3}$

$= \frac{4\pi}{9}$ M1

e. $\text{Arg}(z) = \frac{\pi}{9}$ A1

f. $A = \frac{1}{2} \times 2\sqrt{2} \times 2\sqrt{2} \times \sin\left(\frac{2\pi}{9}\right) + \frac{1}{2} \times 2\sqrt{2} \times 2\sqrt{2} \times \sin\left(\frac{4\pi}{9}\right)$ M1

$= 4\sin\left(\frac{2\pi}{9}\right) + 4 \times 2\sin\left(\frac{2\pi}{9}\right)\cos\left(\frac{2\pi}{9}\right)$

$= 4\sin\left(\frac{2\pi}{9}\right)\left(1 + 2\cos\left(\frac{2\pi}{9}\right)\right)$ A1

Question 3 (11 marks)

a. Finding the area of the inner surface gives:

$$x = \sqrt{y}$$

$$\frac{dx}{dy} = \frac{1}{2\sqrt{y}}$$

$$S_1 = \int_0^8 2\pi x\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

$$= 98.7353\ ...$$ A1

Finding the area of the outer surface gives:

$$x = \sqrt{2y}$$

$$\frac{dx}{dy} = \frac{1}{\sqrt{2y}}$$

$$S_2 = \int_0^8 2\pi x\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

$$= 144.708\ ...$$ A1

Finding the area of the top ring gives:

$$r_1 = 2\sqrt{2}$$

$$r_2 = 4$$

$$S_3 = \pi\left(r_2^2 - r_1^2\right)$$

$$= 8\pi$$ A1

$$\text{total area} = S_1 + S_2 + S_3$$

$$= 268.58\ \text{m}^2$$ A1

$$s1 := \int_0^8 \left(2\cdot\pi\cdot\sqrt{2\cdot y}\cdot\sqrt{1+\left(\frac{1}{\sqrt{2\cdot y}}\right)^2}\right)dy \qquad \frac{2\cdot\pi\cdot\left(17\cdot\sqrt{17}-1\right)}{3}$$

$$s2 := \int_0^8 \left(2\cdot\pi\cdot\sqrt{y}\cdot\sqrt{1+\left(\frac{1}{2\cdot\sqrt{y}}\right)^2}\right)dy \qquad \pi\cdot\left(\frac{11\cdot\sqrt{33}}{2}-\frac{1}{6}\right)$$

$\text{solve}\left(8=x^2,x\right)\mid x>0$	$x=2\cdot\sqrt{2}$
$\text{solve}\left(8=\frac{1}{2}\cdot x^2,x\right)\mid x>0$	$x=4$
$s1+s2+8\cdot\pi$	268.576

b. $\pi\int_0^h\left(\left(\sqrt{2y}\right)^2-\left(\sqrt{y}\right)^2\right)dy$ M1

$=\dfrac{\pi h^2}{2}$ A1

c. **i.** $\dfrac{dV}{dt}=\dfrac{dV}{dh}\times\dfrac{dh}{dt}$

$\sqrt{h}=\pi h\times\dfrac{dh}{dt}$ M1

$dt=\pi\sqrt{h}dh$

$\int_0^t dt=\pi\int_0^8\sqrt{h}dh$

$t=47.4$ minutes A1

$$\text{solve}\left(\int_0^t 1\,dt=\pi\cdot\int_0^8\sqrt{h}\,dh,t\right) \quad t=47.3908$$

ii. $\int_0^t dt=\pi\int_0^h\sqrt{h}dh$ M1

$h=\dfrac{1}{2}\sqrt[3]{\dfrac{18t^2}{\pi^2}}$ A1

$V=\dfrac{\pi h^2}{2}$

$=\dfrac{3\pi}{8}\sqrt[3]{\dfrac{12t^4}{\pi^4}}$

$=\sqrt[3]{\dfrac{81t^4}{128\pi}}$ A1

$$\text{solve}\left(\int_0^t 1\,dt=\pi\cdot\int_0^h\sqrt{h}\,dh,h\right)$$
$$h=\frac{2^{\frac{1}{3}}\cdot(3\cdot t)^{\frac{2}{3}}}{2\cdot\pi^{\frac{2}{3}}} \text{ and } t\geq 0$$

$$v(h):=\frac{1}{2}\cdot\pi\cdot h^2 \quad \textit{Done}$$
$$v\left(\frac{3^{\frac{2}{3}}\cdot 2^{\frac{1}{3}}\cdot t^{\frac{2}{3}}}{2\cdot\pi^{\frac{2}{3}}}\right) \quad \frac{3\cdot 3^{\frac{1}{3}}\cdot 2^{\frac{2}{3}}\cdot t^{\frac{4}{3}}}{8\cdot\pi^{\frac{1}{3}}}$$

Note: Consequential on answers to ***Questions 3b.*** *and* ***3c.i.***

Question 4 (9 marks)

a.

$i(n):=\displaystyle\int_0^1 \left(1-x^2\right)^{\frac{n}{2}}\,dx$	*Done*
$i(1)$	$\frac{\pi}{4}$

$I_1 = \frac{\pi}{4}$ A1

b. $I_n = \int_0^1 (1-x^2)^{\frac{n}{2}} dx$

$u = (1-x^2)^{\frac{n}{2}} \quad du = -nx(1-x^2)^{\frac{n}{2}-1} dx$

$dv = dx \quad v = x$

M1

$I_n = \underbrace{\left[x(1-x^2)^{\frac{n}{2}}\right]_0^1}_{0} + n\int_0^1 x^2 (1-x^2)^{\frac{n}{2}-1} dx$ A1

$I_n = n\int_0^1 (x^2-1+1)(1-x^2)^{\frac{n-2}{2}} dx$

$= n\int_0^1 -(1-x^2-1)(1-x^2)^{\frac{n-2}{2}} dx$ M1

$= -n\int_0^1 \left[(1-x^2)(1-x^2)^{\frac{n-2}{2}} - (1-x^2)^{\frac{n-2}{2}}\right] dx$

$= -n\int_0^1 (1-x^2)^{\frac{n}{2}} dx + n\int_0^1 (1-x^2)^{\frac{n-2}{2}} dx$

$= -nI_n + nI_{n-2}$

$= \frac{n}{n+1} I_{n-2}$ M1

c. $I_5 = \frac{5}{6} I_3$

$= \frac{5}{6} \times \frac{3}{4} I_1$ M1

$= \frac{5}{6} \times \frac{3}{4} \times \frac{\pi}{4}$

$= \frac{5\pi}{32}$ A1

d. $\pi \int_0^1 y^2 dx = \pi \int_0^1 \left(1 - x^2\right)^n dx$ A1

$= \pi I_{2n}$

$\therefore\ p = \pi,\ q = 2$ A1

Question 5 (8 marks)

a. $x = \left(\frac{1}{\cos t} + a\cos t\right)\cos t$

$= 1 + a\cos^2 t$

$y = \left(\frac{1}{\cos t} + a\cos t\right)\sin t$

$= \frac{1 + a\cos^2 t}{\cos t} \times \sin t$

$= \left(1 + a\cos^2 t\right)\tan t$ M1

$y = x\tan t$

$x^2 + y^2 = x^2 + x^2\tan^2 t$

$= x^2\left(1 + \tan^2 t\right)$

$= x^2\sec^2 t$ M1

$x - 1 = a\cos^2 t$

$\therefore (x-1)\left(x^2 + y^2\right) = \left(a\cos^2 t\right)\left(x^2\sec^2 t\right)$

$= ax^2$ M1

b. Since $a < -1$ and $x = 1 + a\cos t < 0$:

$(x-1)\left(x^2 + y^2\right) = ax^2$

$x - 1 = \frac{ax^2}{x^2 + y^2}$

$\neq 0$

Therefore, $x = 1$ is an asymptote. A1

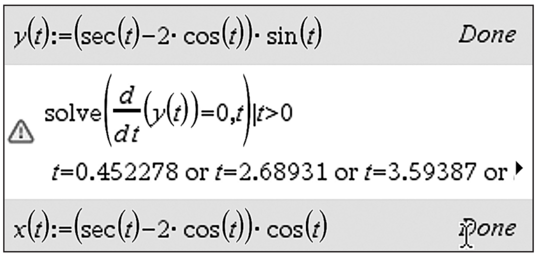
c. $\frac{d}{dt}y(t) = 0$ M1

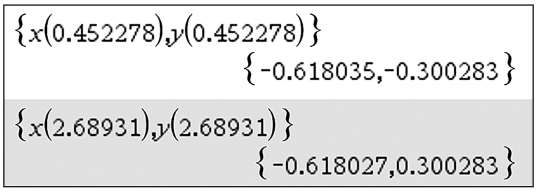
Since the graph indicates that there are two stationary points, the first two *t*-values should be used.

$t = 0.452278\ \ldots$, $t = 2.68931\ \ldots$

Substituting these values into the parametric equations gives:

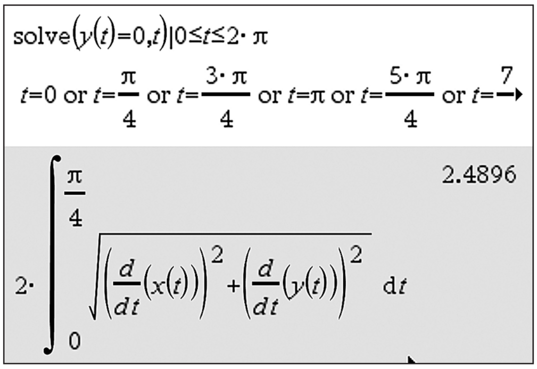
maximum = (–0.62, 0.30), minimum = (–0.62, –0.30) A1





d. $2\int_0^{\frac{\pi}{4}} \sqrt{\left(\frac{d}{dt}x(t)\right)^2 + \left(\frac{d}{dt}y(t)\right)^2}\, dt$ A1

$= 2.49$ A1



Question 6 (10 marks)

a. 0.2685 A1

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normCdf(3000,∞,42·70,15·√42)
                                0.268547
```

b. 39 A1

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f(n):=normCdf(3000,∞,n·70,15·√n)    Done
f(40)                               0.017507
f(39)                               0.001974
```

c. X – travel time (excluding waiting times)

$E(X) = 55 - 6 \times 2$

$= 43$ A1

$SD(X) = \sqrt{3 + 6 \times \frac{24}{60}}$

$= \sqrt{\frac{27}{5}}$ **OR** $\frac{3\sqrt{15}}{5}$ A1

d. 0.0271 A1

$$\text{normCdf}\left(-\infty, 41, 43, \frac{\sqrt{\frac{27}{5}}}{\sqrt{5}}\right) \quad 0.027146$$

Note: Consequential on answer to ***Question 6c.***

e. $H_0 : \mu = 55$

$H_1 : \mu > 55$ A1

f. p-value $= 0.0570$ A1

As p-value > 0.05, accept H_0. A1

$$\text{normCdf}\left(57, \infty, 55, \frac{4}{\sqrt{10}}\right) \quad 0.056923$$

g. 57.081 minutes A2

$$\text{invNorm}\left(0.95, 55, \frac{4}{\sqrt{10}}\right) \quad 57.0806$$

