SPECIALIST MATHEMATICS

Units 3 & 4 – Written examination 2

2024 Trial Examination

SOLUTIONS

SECTION A

Question 1
D

Explanation:

"For $x \in N$, $t$here exists a prime number of the form: $x^2 + 3x + 2$" is false, since:
$x^2 + 3x + 2 = (x + 1)(x + 2)$, so $x^2 + 3x + 2$ has factors $1, x + 1, x + 2, (x + 1)(x + 2)$ therefore cannot be prime.

Question 2
A

Explanation:

$E \Rightarrow C$
n is a natural number $\Rightarrow n^2 + n$ is an even number.

$n \in N$

If n *is even*
$n^2 + n = (2m)^2 + 2m, m \in N$
$(2m)^2 + 2m = 2(2m^2 + m)$, which is even

If $n\ is\ odd$
$n^2 + n = (2m + 1)^2 + 2m + 1, m \in N$
$(2m + 1)^2 + 2m + 1 = 2(2m^2 + 3m + 1)$, which is also even

Question 3
B

Explanation:
$$f(x) = \frac{2x}{\sqrt{x^2 - 36}} + 2$$

As $x \to \pm\infty$, $f(x) \to \frac{2x}{x} + 2 = 4$

So, the equation of the horizontal asymptote is $y = 4$

Question 4
D

Explanation:
The table of values is:

a	b
2	7
3	20
4	44
5	81

Question 5
C

Explanation:

Consider the graphs of the functions:
Check the graph of $y = \sec 3\theta$ and $y = \operatorname{cosec} 2\theta$ on your CAS.
There are 4 intersection points every 2π

Question 6
B

Explanation:
$y = cos^{-1}((1-2x) - sin^{-1}(\frac{1}{2x})$

For $y = cos^{-1}((1-2x)$ to be defined, $-1 \leq 1-2x \leq 1$
$-2 \leq -2x \leq 0$
$-1 \leq -x \leq 0$
$0 \leq x \leq 1$

For $sin^{-1}(\frac{1}{2x})$ to be defined, $-1 \leq \frac{1}{2x} \leq 1$
$-2 \leq \frac{1}{x} \leq 2$
$x \leq -\frac{1}{2} \cup x \geq \frac{1}{2}$

$\{0 \leq x \leq 1\} \cap \left\{x \leq -\frac{1}{2} \cup x \geq \frac{1}{2}\right\} = \left\{\frac{1}{2} \leq x \leq 1\right\}$
$x \in [\frac{1}{2}, 1]$

Question 7
B

$\cot a = b$
$\tan a = \frac{1}{b}$
$\sin a = \frac{1}{\sqrt{b^2+1}}$
$\cos a = \frac{b}{\sqrt{b^2+1}}$
$\sin 2a = 2 \sin a \cos a$
$= 2 \times \frac{1}{\sqrt{b^2+1}} \times \frac{b}{\sqrt{b^2+1}}$
$= \frac{2b}{1+b^2}$

Question 8
B

Explanation:

$\overline{z_1} = 2cis\left(\frac{\pi}{6}\right)$

$\sqrt{z_2} = \sqrt{4}cis\left(\frac{\pi}{6}\right) = 2cis\left(\frac{\pi}{6}\right)$

$\overline{z_1}\sqrt{z_2} = 2cis\left(\frac{\pi}{6}\right) \times 2cis\left(\frac{\pi}{6}\right) = 4cis\left(\frac{\pi}{3}\right)$

$4cis\left(\frac{\pi}{3}\right) = 4\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

$= 2 + 2\sqrt{3}i$

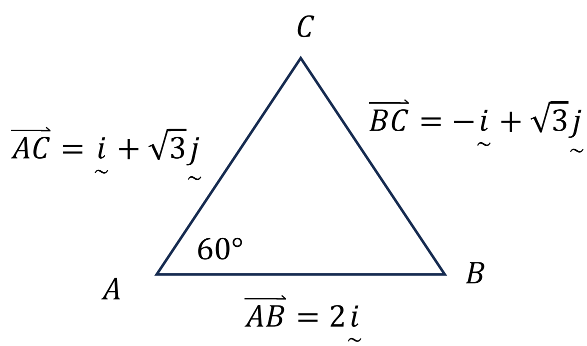
Question 9
C

Explanation:

$\overrightarrow{AB} = 2\underset{\sim}{i}$, $\overrightarrow{BC} = -\underset{\sim}{i} + m\underset{\sim}{j} = -\underset{\sim}{i} + \sqrt{3}\underset{\sim}{j}$

$m = \sqrt{3}$

$\overrightarrow{AB}.\overrightarrow{BC} = \left(2\underset{\sim}{i}\right).\left(-\underset{\sim}{i} + \sqrt{3}\underset{\sim}{j}\right) = -2$



Question 10
A

Explanation:

$\underset{\sim}{a} \parallel \underset{\sim}{b} = -\frac{1}{3}\left(\frac{1}{3}\left(-2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}\right)\right) = -\frac{1}{9}\left(-2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}\right)$

$\underset{\sim}{a} \perp \underset{\sim}{b} = \underset{\sim}{a} - \left(\underset{\sim}{a} \parallel \underset{\sim}{b}\right)$

$= \left(\underset{\sim}{i} + \underset{\sim}{k}\right) + \frac{1}{9}\left(-2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}\right)$

$= \frac{1}{9}\left(7\underset{\sim}{i} - 2\underset{\sim}{j} + 10\underset{\sim}{k}\right) = \frac{\underset{\sim}{c}}{9}$

$\underset{\sim}{c} = 7\underset{\sim}{i} - 2\underset{\sim}{j} + 10\underset{\sim}{k}$

Question 11
D

Explanation:

$\Pi_1 = \underset{\sim}{r}\,.\left(-\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}\right) = 2$ and $\Pi_2 = \underset{\sim}{r}\,.\left(2\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}\right) = 4$

Cartesian equation of Π_1: $-x + y - 2z = 2$

Cartesian equation of Π_2: $2x - y + 2z = 4$

Add the equations:

$x = 6$

So, $-6 + y - 2z = 2$

$y - 2z = 8$

Let $y = \lambda, \lambda \in R$

$\lambda - 2z = 8$

$z = \dfrac{\lambda - 8}{2}$

$x = 6, y = \lambda, z = \dfrac{\lambda - 8}{2}, \lambda \in R$

$\underset{\sim}{r} = 6\underset{\sim}{i} - 4\underset{\sim}{k} + \lambda\left(\underset{\sim}{j} + \frac{1}{2}\underset{\sim}{k}\right), \lambda \in R$

Question 12
A

Explanation:

$\int(\tan^{-1} x)dx$ where $u = \tan^{-1} x$ and $\frac{dv}{dx} = 1$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$u = \tan^{-1} x,\ \ \dfrac{dv}{dx} = 1$

$$\int(\tan^{-1} x)dx = x\tan^{-1} x - \int x\left(\frac{1}{1 + x^2}\right)dx$$

$$= x\tan^{-1} x - \frac{1}{2}log_e(1 + x^2) + c$$

Question 13
B

Explanation:

$a = \frac{dv}{dt} = (0.5v - 1)ms^{-2}.$
$t = 2\int \frac{0.5}{0.5v - 1} dv$

$t = 2log_e(0.5v - 1) + c$

$0 = 2log_e(0.5 \times 6 - 1) + c$

$c = -2log_e(2)$
$t = 2log_e(0.5v - 1) - 2log_e(2)$

$t = 2log_e\left(\frac{0.5v - 1}{2}\right)$

$0.5v - 1 = 2e^{0.5t}$
$v = 4e^{0.5t} + 2$
$x = 8e^{0.5t} + 2t - 8$
$x(2) = (8e - 4)m \approx 17.75m$

Question 14
D

Explanation:

$a = \frac{d}{dx}\left(\frac{1}{2}v^2\right) = (-36x)ms^{-2}.$
$\frac{1}{2}v^2 = \int (-36x)dx$

$v^2 = -36x^2 + c$
$(12)^2 = -36(0)^2 + c$
$c = 144$
$v^2 = 144 - 36x^2$
$v^2 = 36(4 - x^2)$
$v = \pm 6\sqrt{(4 - x^2)}$
$x = 0, v = 6$
$v = 6\sqrt{(4 - x^2)}$

$\frac{dx}{dt} = 6\sqrt{(4 - x^2)}$
$\frac{dt}{dx} = \frac{1}{6\sqrt{(4 - x^2)}}$
$t = \frac{1}{6}\sin^{-1}\left(\frac{x}{2}\right)$
$x = 2\sin 6t$
$v = 12\cos 6t$
$a = 72\sin 6t$
$a(2.5) = 72\sin(15) \approx 47cms^{-2}$

Question 15
D

Explanation:
$E(X) = 24, Var(X) = 1, E(Y) = 30, Var(Y) = 2.$
$C = 4X - 3Y$
$E(C) = 4E(X) - 3E(Y)$
$= 4 \times 24 - 3 \times 30 = 6$
$Var(C) = 16Var(X) + 9Var(Y)$
$= 16 \times 1 + 9 \times 2 = 34$
$\Pr(C < 0) = \Pr(z < \frac{0 - 6}{\sqrt{34}}) \approx 0.15$

Question 16
B

Explanation:
$$\Pr\left(\frac{-2}{\frac{5}{\sqrt{n}}} < z < \frac{2}{\frac{5}{\sqrt{n}}}\right) = 0.90$$
$$\Pr\left(z < \frac{2}{\frac{5}{\sqrt{n}}}\right) = 0.95$$

$\frac{2}{\frac{5}{\sqrt{n}}} \approx 1.645$
$\frac{5}{\sqrt{n}} \approx 1.2159$
$n \approx 16.9$
So, 17 days.

Question 17
C

Explanation:
The chance that sample statistic is **more** extreme than the one observed is known the $p - value$.

Question 18
C

Explanation:
$\underset{\sim}{a}(t) = 2\sin 2t\,\underset{\sim}{i} - 4\cos 2t\,\underset{\sim}{j}$
$\underset{\sim}{v}(t) = -\cos 2t\,\underset{\sim}{i} - 2\sin 2t\,\underset{\sim}{j} + \underset{\sim}{c}$
$\underset{\sim}{v}(0) = -\cos 0\,\underset{\sim}{i} - 2\sin 0\,\underset{\sim}{j} + \underset{\sim}{c} = \underset{\sim}{i} + 2\underset{\sim}{j}$
$-\underset{\sim}{i} + \underset{\sim}{c} = \underset{\sim}{i} + 2\underset{\sim}{j}$
$\underset{\sim}{c} = 2\underset{\sim}{i} + 2\underset{\sim}{j}$
$\underset{\sim}{v}(t) = (2 - \cos 2t)\underset{\sim}{i} + (2 - 2\sin 2t)\underset{\sim}{j}$
$\underset{\sim}{v}(\pi) = (2 - \cos 2\pi)\underset{\sim}{i} + (2 - 2\sin 2\pi)\underset{\sim}{j}$
$\underset{\sim}{v}(\pi) = \underset{\sim}{i} + 2\underset{\sim}{j}$
$\left|\underset{\sim}{v}(\pi)\right| = \sqrt{1+4} = \sqrt{5}\,ms^{-1}$

Question 19
B

Explanation:
$$L(t) = \int_0^{\frac{\pi}{6}} \sqrt{\left(\frac{8}{(\cos 2t)^2}\right)^2 + (-\sin t)^2}\,dt \approx 6.93$$

Question 20
D

Explanation:

$\underset{\sim}{x}(t) = 20t\underset{\sim}{i} + (-4.9t^2 + 20t + 2)\underset{\sim}{j}$
$\underset{\sim}{v}(t) = 20\underset{\sim}{i} + (-9.8t + 20)\underset{\sim}{j}$
Projectile lands when $-4.9t^2 + 20t + 2 = 0$
$t \approx 4.179s$
$\underset{\sim}{v}(4.179) \approx 20\underset{\sim}{i} + (-20.957)\underset{\sim}{j}$
$\tan\theta \approx \frac{20.957}{20}$
$\theta \approx 46°20'$

SECTION B

Question 1 (12 marks)

a. (1 mark)

Answer:
$V = 400 - 5t$

1A

b. (1 mark)

Answer:
$S = 400 \times 4 = 1600g$

1A

c. (2 marks)

Answer:
$dS/_{dt} = Salt\ in - Salt\ out$
$dS/_{dt} = 0 - \frac{S}{400-5t} \times 10$

1W

$dS/_{dt} = \frac{-10S}{400-5t} = \frac{2S}{t-80}$

1A

d. (3 marks)

Answer:

$$^{dS}/_{dt} = \frac{2S}{t-80}$$
$$\int \frac{1}{S} dS = \int \frac{2}{t-80} dt$$
$$log_e S = 2log_e(|t-80|) + c$$

1W

$$t = 0, S = 1600$$
$$log_e 1600 = 2log_e(|0-80|) + c$$
$$c = log_e 1600 - 2log_e 80$$
$$c = log_e\left(\frac{1600}{6400}\right) = log_e\left(\frac{1}{4}\right)$$

1W

$$log_e S = 2log_e(|t-80|) + log_e\left(\frac{1}{4}\right)$$
$$log_e S = log_e\left(\frac{(t-80)^2}{4}\right)$$
$$S = \frac{(t-80)^2}{4} = \frac{1}{4}(t-80)^2$$

1A

e. (3 marks)

Answer:

$$S = \frac{1}{4}(t-80)^2$$
$$\frac{dS}{dt} = \frac{1}{2}(t-80) = -30$$
$$t = 20$$

1W

$$S(20) = \frac{1}{4}(20-80)^2 = 900g$$
$$V(20) = 400 - 100 = 300L$$

1W

$$Concentration\ of\ salt = \frac{900}{300} = 3gL^{-1}$$

1A

f. (2 marks)

Answer:

$\frac{dS}{dt} = -30, \qquad \frac{dV}{dt} = -5$

1W

$\frac{dV}{dS} = \frac{dt}{dS} \times \frac{dV}{dt} = -\frac{1}{30} \times -5 = \frac{1}{6}\ Lg^{-1}$

1A

Question 2 (10 marks)

$\overrightarrow{OA} = \underset{\sim}{a} = -\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}\ ,\ \overrightarrow{OB} = \underset{\sim}{b} = -2\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}$

a. (1 mark)

Answer:

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = \left(\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}\right) + \left(-2\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}\right) = -\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k}$

1A

b. (2 marks)

Answer:

$\underset{\sim}{r}(t) = \underset{\sim}{a} + t\left(\underset{\sim}{b} - \underset{\sim}{a}\right), t \in R$

$\underset{\sim}{r}(t) = \left(-\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}\right) + t\left(-\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k}\right)$

1W

When $t = 2$

$\underset{\sim}{r}(2) = \left(-\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}\right) + 2\left(-\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k}\right) = -3\underset{\sim}{i} + 0\underset{\sim}{j} + 5\underset{\sim}{k}$

So, $C = (-3,0,5)$ is on the line containing the points A and B.

1A

c. (2 marks)

Answer:

$D = (1, -2, 1)$

$\underset{\sim}{r}(t) = \left(\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}\right) + t\left(-3\underset{\sim}{i} + 0\underset{\sim}{j} + 5\underset{\sim}{k}\right), t \in R$

1W, 1A

d. (2 marks)

Answer:

$mx + y + z = n,\ \ m, n\ \in R$

$A = (-1, 2, -1)$

$-m + 2 - 1 = n \ldots (1)$

$m + n = 1 \ldots (1)$

$B = (-2, 1, 2)$

$-2m + 1 + 2 = n \ldots (2)$

$2m + n = 3 \ldots (2)$

1W

$(1) - (2)$

$-m = -2$

$m = 2$

$n = -1$

1A

e. (3 marks)

Answer:

$E = (-1, 0, 1)$

Use the cross product to find the normal to the plane containing points A and B.

$\overrightarrow{OA} = \underset{\sim}{a} = -\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}\ ,\ \overrightarrow{OB} = \underset{\sim}{b} = -2\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}$

$\overrightarrow{OA} \times \overrightarrow{OB} = (4 + 1)\underset{\sim}{i} - (-2 - 2)\underset{\sim}{j} + (-1 + 4)\underset{\sim}{k}$

$\underset{\sim}{n} = 5\underset{\sim}{i} + 4\underset{\sim}{j} + 3\underset{\sim}{k}$

1W

$\underset{\sim}{\hat{n}} = \frac{1}{\sqrt{50}}\left(5\underset{\sim}{i} + 4\underset{\sim}{j} + 3\underset{\sim}{k}\right) = \frac{\sqrt{2}}{10}\left(5\underset{\sim}{i} + 4\underset{\sim}{j} + 3\underset{\sim}{k}\right)$

$\overrightarrow{EA} = \overrightarrow{EO} + \overrightarrow{OA} = \left(\underset{\sim}{i} + 0\underset{\sim}{j} - \underset{\sim}{k}\right) + \left(-\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}\right) = 0\underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k}$

1W

$\overrightarrow{EA} \parallel \underset{\sim}{n} = \frac{\sqrt{2}}{5}\left(\underset{\sim}{\hat{n}}\right)$

$\left|\overrightarrow{EA} \parallel \underset{\sim}{n}\right| = \frac{\sqrt{2}}{5}$

1A

Question 3 (11 marks)

a. (2 marks)

Answer:

$\bar{x} = 61, \frac{\sigma}{\sqrt{n}} = \frac{6}{\sqrt{32}}$

1W

$61 \pm 1.96 \times \frac{6}{\sqrt{32}}$

58.9 to 63.1 months

1A

b. (2 marks)

Answer:

$\Pr(\mu > 63) = \Pr\left(z > \frac{63 - 61}{\frac{6}{\sqrt{32}}}\right) = \Pr(z > 1.886) \approx 0.03$

1W, 1A

c. (1 mark)

Answer:

$H_0\ \mu = 60$
$H_1\ \mu > 60$

1A

d. (2 marks)

$$z = \frac{61-60}{\frac{6}{\sqrt{32}}} \approx 0.9428$$

1W

$\Pr(z > 0.9428) \approx 0.173$
$\text{p} - \text{value} \approx 0.173$

1A

e. (1 mark)

Answer:

Since $0.173 > 0.05$ Accept H_0

1A

f. (3 marks)

Answer:

$$\Pr(\mu > 62) = \Pr\left(z > \frac{62-60}{\frac{6}{\sqrt{n}}}\right) = 0.01$$

1W

$$\frac{2}{\frac{6}{\sqrt{n}}} = 2.326$$

$$\frac{2}{\frac{6}{\sqrt{n}}} = 2.326$$

1W

$\sqrt{n} = 6.98$
$n = 49$ batteries.

1A

Question 4 (12 marks)

a. (3 marks)

Answer:
$z^4 = -64 = 64cis(\pi)$

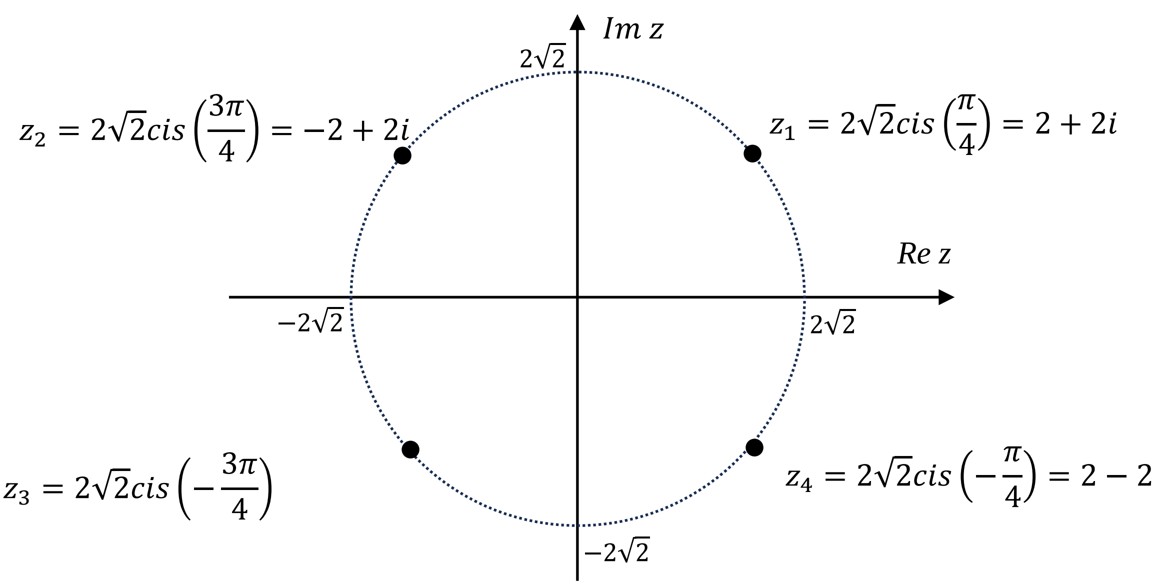
1W

$z_1 = 64^{\frac{1}{4}}cis\left(\frac{\pi}{4}\right) = 2\sqrt{2}cis\left(\frac{\pi}{4}\right) = 2 + 2i$
$z_2 = 2\sqrt{2}c\backslash is\left(\frac{\pi}{4} + \frac{\pi}{2}\right) = 2\sqrt{2}cis\left(\frac{3\pi}{4}\right) = -2 + 2i$
$z_3 = 2\sqrt{2}cis\left(\frac{3\pi}{4} + \frac{\pi}{2}\right) = 2\sqrt{2}cis\left(\frac{5\pi}{4}\right) = 2\sqrt{2}cis\left(-\frac{3\pi}{4}\right) = -2 - 2i$
$z_4 = 2\sqrt{2}cis\left(\frac{5\pi}{4} + \frac{\pi}{2}\right) = 2\sqrt{2}cis\left(\frac{7\pi}{4}\right) = 2\sqrt{2}cis\left(-\frac{\pi}{4}\right) = 2 - 2i$

1W, 1A

b. (3 marks)

Answer:



2W, 1A

c. (2 marks)

Answer:
Equation 1:
$z_1 = 2 + 2i, \qquad z_4 = 2 - 2i$
$(z - 2 - 2i)(z - 2 + 2i) = 0$
$z^2 - 4z + 8 = 0$

1A

Equation 2:
$z_2 = -2 + 2i, \qquad z_3 = -2 - 2i$
$(z + 2 - 2i)(z + 2 + 2i) = 0$
$z^2 + 4z + 8 = 0$

1A

d. (2 marks)

Answer:
$x^2 + y^2 = 8$

1A

$|z| = 2\sqrt{2}$

1A

e. (2 marks)

Answer:
$(z - 2 + 2i)^4 = -64$

1A

$z_1 = 4 + 0i$
$z_2 = 0 + 0i$
$z_3 = 0 - 4i$
$z_4 = 4 - 4i$

1A

Question 5 (15 marks)

a. (2 marks)

$f: R \to R$ where $f(x) = \dfrac{x-4}{x^2+4}$
x −intercept: (4,0)
y −intercept: $(0, -1)$

2A

b. (2 marks)

$f(x) = \dfrac{x-4}{x^2+4}$
Stationary points: $f'(x) = \frac{-(x^2-8x-4)}{(x^2+4)^2} = 0$
$x^2 - 8x - 4 = 0$
$x = 4 \pm 2\sqrt{5}$
$\left(4 - 2\sqrt{5}, \dfrac{-\sqrt{5}-2}{4}\right), \left(4 + 2\sqrt{5}, \dfrac{\sqrt{5}-2}{4}\right)$

1W

Range of $f = [\frac{-\sqrt{5}-2}{4}, \frac{\sqrt{5}-2}{4}]$

1A

c. (2 marks)

$f''(x) = \dfrac{-(x^3 - 12x^2 - 12x + 16)}{(x^2+4)^3} = 0$
$x \approx 0.7735$

1W

$f'(0.7735) \approx 0.45$

1A

d. (2 marks)

$$\int\left(\frac{x-4}{x^2+4}\right)dx=\int\left(\frac{x}{x^2+4}\right)dx-\int\left(\frac{4}{x^2+4}\right)dx$$

$$=\frac{1}{2}\int\left(\frac{2x}{x^2+4}\right)dx-2\int\left(\frac{2}{x^2+4}\right)dx$$

1W

$$=\frac{1}{2}log_e(x^2+4)-2\tan^{-1}\left(\frac{x}{2}\right)+c$$

1A

e. (2 marks)

$x-5y=4$ intersects $y=f(x)$ at $(-1,-1)$ and $(4,0)$ and $(1,-\frac{3}{5})$

Area bound:

$$=\left[\left(\frac{x^2}{10}-\frac{4x}{5}\right)-\left(\frac{1}{2}log_e(x^2+4)-2\tan^{-1}\left(\frac{x}{2}\right)\right)\right]_{-1}^{1}$$
$$+\left[\left(\frac{1}{2}log_e(x^2+4)-2\tan^{-1}\left(\frac{x}{2}\right)\right)-\left(\frac{x^2}{10}-\frac{4x}{5}\right)\right]_{1}^{4}$$

1W

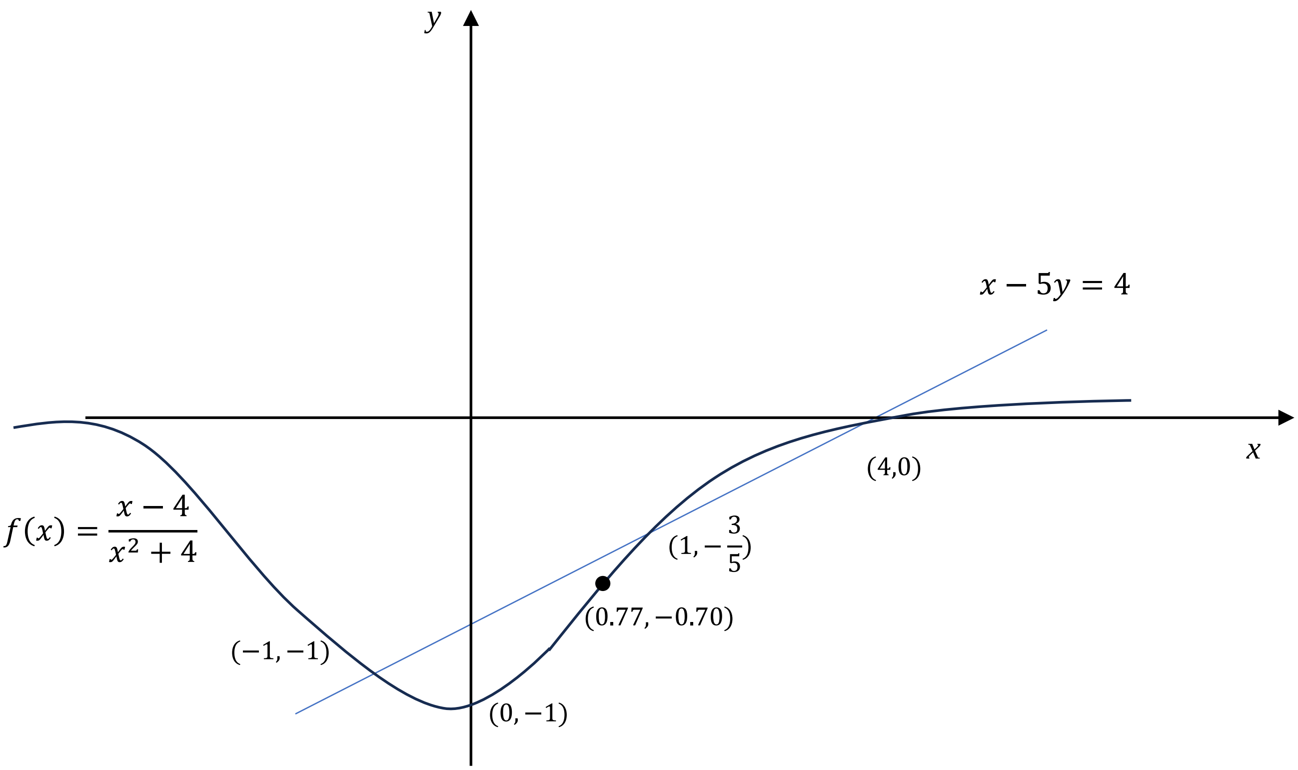
$$=\left(\frac{1}{10}-\frac{4}{5}\right)-\left(\frac{1}{2}log_e(5)-2\tan^{-1}\left(\frac{1}{2}\right)\right)-\left(\left(\frac{1}{10}+\frac{4}{5}\right)-\left(\frac{1}{2}log_e(5)-2\tan^{-1}\left(-\frac{1}{2}\right)\right)\right)$$
$$+\left(\frac{1}{2}log_e(20)-2\tan^{-1}(2)\right)-\left(\frac{8}{5}-\frac{16}{5}\right)-\left(\left(\frac{1}{2}log_e(5)-2\tan^{-1}\left(\frac{1}{2}\right)\right)-\left(\frac{1}{10}-\frac{4}{5}\right)\right)$$

$$=(-\frac{16}{10}+4\tan^{-1}\left(\frac{1}{2}\right))+\left(\frac{1}{2}log_e(4)+4\tan^{-1}\left(\frac{1}{2}\right)-\pi+\frac{9}{10}\right)$$

$$=log_e(2)+8\tan^{-1}\left(\frac{1}{2}\right)-\pi-\frac{7}{10}$$

1A

f. (3 marks)



1 shape, 1 POI, 1 straight line with points of intersection

3 marks

g. (2 marks)

$$V_x = \pi \int_0^a \left(\frac{x-4}{x^2+4}\right)^2 dx = \frac{5\pi^2}{16} - \frac{\pi}{8}$$

$$V_x = \pi \int_0^a \left(\frac{x-4}{x^2+4}\right)^2 dx = \frac{\left(5\tan^{-1}\left(\frac{a}{2}\right)\right)\pi}{4} + \frac{(3a+8)\pi}{2(a^2+4)} - \pi$$

1W

$$\frac{\left(5\tan^{-1}\left(\frac{a}{2}\right)\right)\pi}{4} + \frac{(3a+8)\pi}{2(a^2+4)} - \pi = \frac{5\pi^2}{16} - \frac{\pi}{8}$$

$a = 2$

1A