Review of VCAA 2010 Specialist Maths Exam 1

Question 1

Consider $f(z) = z^3 + 9z^2 + 28z + 20,\ z \in C$.

Given that $f(-1) = 0$, factorise $f(z)$ over C.

Possible solution

$\Rightarrow (z-(-1)) = (z+1)$ is a factor.

Coefficients are real, so remaining factors are either real or a complex conjugate pair.

$$\begin{array}{r|l} & z^2 + 8z + 20 \\ z+1 & z^3 + 9z^2 + 28z + 20 \\ & -(z^3 + z^2) \\ & \quad 8z^2 + 28z + 20 \\ & \quad -(8z^2 + 8z) \\ & \qquad 20z + 20 \\ & \qquad -(20z + 20) \end{array}$$

$f(z) = (z+1)(z^2 + 8z + 20)$ ...[M1]

$= (z+1)(z^2 + 8z + 16 - 16 + 20)$
$= (z+1)((z+4)^2 + 4)$ ...[M1]

$= (z+1)(z+4+2i)(z+4-2i)$ ...[A1]

Question 1

Marks	0	1	2	3	Average
%	8	16	16	59	2.3

$(z + 1)(z + 4 + 2i)(z + 4 - 2i)$

This question was quite well done. Errors included assuming both $(z + 1)$ and $(z - 1)$ were factors (a kind of spurious notion of a complex conjugate) and a few students misinterpreted the factor theorem and made $(z - 1)$ the factor. There were many slips in completing the square or using the quadratic formula. The factors $(z + 1)(z + 4 + 4i)(z + 4 - 4i)$ were very common. A number of students found the three roots of $f(z) = 0$ but did not mention factors. Some students omitted brackets, for example $(z + 1)(z^2 + 8z + 16) - 16 + 20 = (z + 1)(z + 4)^2 + 4 = (z + 1)(z + 4 + 2i)(z + 4 - 2i)$, and could not obtain full marks. Some students were unable to factorise $z^2 + 8z + 20$ and seemed not to realise that it was necessary to complete the square or use the quadratic formula.

Question 2

A body of mass 2 kg is initially at rest and is acted on by a resultant force of $v - 4$ newtons where v is the velocity in m/s. The body moves in a straight line as a result of the force.

a. Show that the acceleration of the body is given by $\frac{dv}{dt} = \frac{v-4}{2}$.

Possible solution

$F = ma$

$v - 4 = 2a$ ...[M1]

$a = \frac{dv}{dt} = \frac{v-4}{2}$ as required

Question 2a.

Marks	0	1	Average
%	8	92	0.9

Students had to show the given result $\frac{dv}{dt} = \frac{v-4}{2}$.

This question was very well done, with most students using $F = ma$ to get the required result.

b. Solve the differential equation in **part a.** to find v as a function of t.

Possible solution

$$\frac{dt}{dv} = \frac{2}{v-4} \qquad v \neq 4$$

$$t = \int \frac{2}{v-4}\,dv \qquad \ldots[\text{M1}]$$

$$= 2\log_e|v-4| + c$$

$$\left.\begin{aligned} &t=0, v=0, a=-4<0 \\ &\Rightarrow a<0 \text{ for } t\geq 0 \\ &\Rightarrow v<0 \text{ for } t\geq 0 \\ &v-4<0 \\ &|v-4| = 4-v \\ &t = 2\log_e(4-v)+c \\ &(0,0) \Rightarrow t = 2\log_e(4)+c \\ &c = -2\log_e(4) \\ &t = 2\log_e(4-v) - 2\log_e(4) \end{aligned}\right\}\ldots[\text{M1}]$$

$$t = 2\log_e\left(\frac{4-v}{4}\right) \qquad \ldots[\text{M1}] \text{ n.b. don't square - extraneous solutions}$$

$$\frac{4-v}{4} = e^{\frac{t}{2}}$$

$$v = 4-4e^{\frac{t}{2}} \qquad t\geq 0 \qquad \ldots[\text{A1}]$$

Question 2b.

Marks	0	1	2	3	4	Average
%	9	9	21	38	23	2.6

$v = 4 - 4e^{t/2}$

Most students were able to progress partway into this question, but many were unable to choose the solution consistent with the initial condition. Many students correctly found that $|v-4| = 4e^{t/2}$, but mistakenly assumed that this meant that $v-4 = 4e^{t/2}$ when the correct equation was $4-v = 4e^{t/2}$ to satisfy $t = 0$, $v = 0$. This approach led to the most common answer, which was $v = 4+4e^{t/2}$. It is highly advisable that students check that their final answer satisfies the given initial conditions. A significant number of students integrated to get $\log_e(v-4)$ rather than the correct $\log_e(4-v)$ or $\log_e|v-4|$. In these cases, there were many 'solutions' involving the 'constant' $\log_e(-4)$ with several of these leading to the correct answer when the negatives 'cancelled'. A few students forgot to include a constant of integration. Some correctly found the integral but left their answer with t as the subject. Some used $A = e^{-ct}$ in their working and then found that $A = -4$, which is not consistent, while some others were unable to simplify $e^{2\log_e 4}$.

Question 3

Relative to an origin O, point A has cartesian coordinates (1, 2, 2) and point B has cartesian coordinates (–1, 3, 4).

a. Find an expression for the vector $\overrightarrow{AB}$ in the form $a\underset{\sim}{i} + b\underset{\sim}{j} + c\underset{\sim}{k}$.

Possible solution

$$\overline{OA} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \qquad \overline{OB} = \begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix}$$

$$\overline{AB} = \overline{OB} - \overline{OA} = \begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

$= -2\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}$ ...[A1]

Question 3a.

Marks	0	1	Average
%	10	90	0.9

$-2\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}$

The majority of students handled this question well. A small number found $\overrightarrow{BA}$ instead of $\overrightarrow{AB}$ and others gave the answer as (–2, 1, 2) rather than in the form requested.

b. Show that the cosine of the angle between the vectors $\overrightarrow{OA}$ and $\overrightarrow{AB}$ is $\frac{4}{9}$.

Possible solution

$$\left.\begin{aligned} \cos(\theta) &= \frac{\overline{OA} \cdot \overline{AB}}{\left|\overline{OA}\right|\left|\overline{AB}\right|} \\ &= \frac{-2+2+4}{\sqrt{9} \times \sqrt{9}} \end{aligned}\right\} \text{...[M1]}$$

$= \frac{4}{9}$ as required

Question 3b.

Marks	0	1	Average
%	28	72	0.7

Students had to show the given result $\frac{4}{9}$.

Most students were capable of using the dot product to obtain the desired result. A relatively small number seemed to be confused by the fact that the stated vectors were not 'tail to tail', not realising that vectors are free to move in space.

c. Hence find the exact area of the triangle OAB.

Possible solution

$A = \frac{1}{2}ab\sin(C)$

$$\left.\begin{aligned} &\sin^2(\theta) = 1 - \left(\frac{4}{9}\right)^2 = \pm\frac{\sqrt{65}}{9} \\ &\cos(\theta) > 0 \Rightarrow 0 < \theta < \frac{\pi}{2} \Rightarrow \sin(\theta) > 0 \\ &\sin(\theta) = \frac{\sqrt{65}}{9} \end{aligned}\right\} \ldots[\text{M1}]$$

$$\left.\begin{aligned} A &= \tfrac{1}{2}\left|\overline{OA}\right|\left|\overline{AB}\right|\sin(\theta) \\ &= \tfrac{1}{2}\sqrt{9}\sqrt{9}\frac{\sqrt{65}}{9} \end{aligned}\right\} \ldots[\text{M1}]$$

$$= \frac{\sqrt{65}}{2} \quad \ldots[\text{A1}]$$

Question 3c.

Marks	0	1	2	3	Average
%	52	10	8	30	1.2

$\frac{\sqrt{65}}{2}$

Students tended to either answer this question well or have trouble getting started. A large proportion of the cohort ignored the word 'hence' and attempted to find the area by another means, which did not attract any marks. A surprising number of students were unable to find $\sin(\theta)$ given that $\cos(\theta) = \frac{4}{9}$. Some students drew a right-angled triangle to find $\sin(\theta)$, but then found the area of this triangle. Several students correctly found that $\sin(\theta) = \frac{\sqrt{65}}{9}$, but then wrote $A = \frac{1}{2} \times 9 \times \sin\left(\frac{\sqrt{65}}{9}\right)$. There was some poor vector notation observed; for example, vectors and lengths were not carefully distinguished, leading to further confusion.

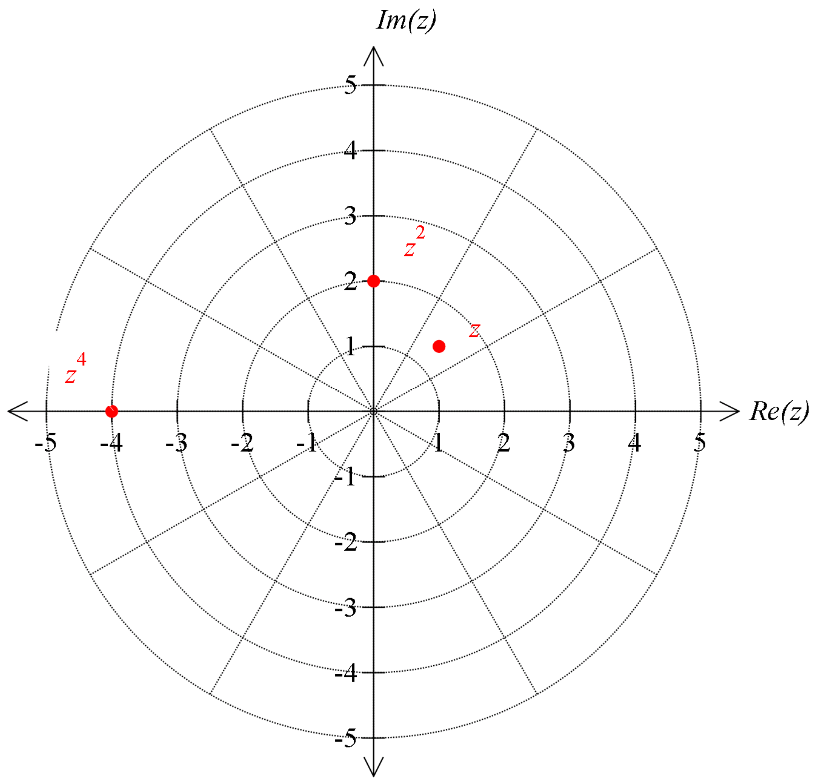
Question 4

Given that $z = 1 + i$, plot and label points for each of the following on the argand diagram below.

i. z

ii. z^2

iii. z^4



Possible solution

$z = 1 + i = \sqrt{2}\,\text{cis}\left(\frac{\pi}{4}\right)$ …[A1] plotted

$z^2 = \left(\sqrt{2}\right)^2 \text{cis}\left(2 \times \frac{\pi}{4}\right) = 2\,\text{cis}\left(\frac{\pi}{2}\right) = 0 + 2i$ …[A1] plotted

$z^4 = \left(\sqrt{2}\right)^4 \text{cis}\left(4 \times \frac{\pi}{4}\right) = 4\,\text{cis}(\pi) = -4 + 0i$ …[A1] plotted

Question 4

Marks	**0**	**1**	**2**	**3**	**Average**
%	6	9	15	70	**2.5**

Points at (1, 1), (0, 2), (–4, 0)

This question was very well done by most students. The most common error was to place z on the unit circle. Other errors were z^4 being placed at (4, 0) or at (0, –4), z^2 being placed in two locations and z^4 being placed in four locations. Quite a few students drew a line segment from the origin to the point, instead of drawing a dot at the point or a cross at the required location.

Question 5

Given that $f(x) = \arctan(2x)$, find $f''\left(\frac{\pi}{2}\right)$.

Possible solution

$f(x) = \arctan(2x)$

$f'(x) = \dfrac{2}{1+(2x)^2}$

$= \dfrac{2}{1+4x^2}$ ...[A1]

$= 2\left(1+4x^2\right)^{-1}$

$f''(x) = 2 \times -\left(1+4x^2\right)^{-2} \times 8x$

$= -\dfrac{16x}{\left(1+4x^2\right)^2}$ ...[M1]

$f''\left(\dfrac{\pi}{2}\right) = -\dfrac{16\left(\frac{\pi}{2}\right)}{\left(1+4\left(\frac{\pi}{2}\right)^2\right)^2} = -\dfrac{8\pi}{\left(1+\pi^2\right)^2}$ ...[A1]

Question 5

Marks	0	1	2	3	Average
%	25	33	7	35	**1.5**

$f''\left(\dfrac{\pi}{2}\right) = -\dfrac{8\pi}{(1+\pi^2)^2}$

A large proportion of students were unable to find the first derivative correctly, despite the assistance of the formula sheet. The chain rule was often not used, with the derivative of $\arctan(2x)$ commonly being given as $\dfrac{1}{1+4x^2}$. A number of students did not recognise $\arctan(2x)$ as being an inverse circular function but interpreted it as being either a reciprocal tan function, a tan function or tried to use an invented trigonometric identity. It was relatively common to see $\log_e\left(1+4x^2\right)$ as the second derivative. The issue with the use of the chain rule was also seen in attempts to find the second derivative. Some students used the quotient rule in obtaining the second derivative, with varying success; however, this was an unnecessary complication. Several students found substituting $\dfrac{\pi}{2}$ to be quite challenging.

Question 6

Evaluate $\displaystyle\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \cos^2(2x)\sin(2x)\,dx$.

Possible solution

$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \cos^2(2x)\sin(2x)\,dx \qquad \text{let } u=\cos(2x) \quad \frac{du}{dx}=-2\sin(2x)$$

$$=-\frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \cos^2(2x)\times -2\sin(2x)\,dx \qquad \ldots[\text{M1}]$$

$$=-\frac{1}{2}\int_{x=\frac{\pi}{2}}^{\frac{3\pi}{4}} u^2\frac{du}{dx}dx \qquad x=\frac{\pi}{2},\ u=-1$$

$$=-\frac{1}{2}\int_{u=-1}^{0} u^2\,du \qquad x=\frac{3\pi}{4}, u=0$$

$$=-\frac{1}{2}\left[\frac{1}{3}u^3\right]_{-1}^{0} \qquad \ldots[\text{M1}]$$

$$=-\frac{1}{2}\left[0+\frac{1}{3}\right]=-\frac{1}{6} \qquad \ldots[\text{A1}]$$

Question 6

Marks	0	1	2	3	Average
%	32	20	15	34	1.5

$-\frac{1}{6}$

Most students recognised that an appropriate substitution was $u=\cos(2x)$, although there were some errors with the -2 factor (for example, the integrand becoming $-2u^2$). Other students tried to use $u=\sin(2x)$ or $u=\cos^2(2x)$, or expanded using double angle formulas and then used $u=\sin(x)$ or $u=\cos(x)$. Some of these are possible but are very time-consuming, and students who ventured down these paths rarely obtained the correct answer. Too many students changed the variable correctly but left the terminals unchanged; it should be emphasised that this is not logically correct, even if changing back to the original variable later enables them to obtain a correct answer. Some changed to a new variable u including changing the terminals, integrated, and then changed back to the original variable but without changing the terminals back. A very large proportion of students gave $\frac{1}{6}$ as their final answer. There were several reasons for this: many students simply made a sign error due to the proliferation of negatives involved, and quite a few students got the correct answer but then dropped the negative sign, seemingly assuming that a definite integral had to represent an area.

Question 7

Consider the differential equation

$$\frac{d^2y}{dx^2} = \frac{4x}{(1-x^2)^2}, \; -1 < x < 1,$$

for which $\frac{dy}{dx} = 3$ when $x = 0$, and $y = 4$ when $x = 0$.

Given that $\frac{d}{dx}\left(\frac{2}{1-x^2}\right) = \frac{4x}{(1-x^2)^2}$, find the solution of this differential equation.

Possible solution

$$\frac{dy}{dx} = \int \frac{d^2y}{dx^2}dx = \int \frac{4x}{\left(1-x^2\right)^2}dx = \frac{2}{1-x^2} + c$$

$y'(0) = 3$

$3 = \frac{2}{1-0^2} + c \Rightarrow c = 1$ Assume $\frac{2}{1-x^2} \equiv \frac{A}{1+x} + \frac{B}{1-x}$

$\frac{dy}{dx} = \frac{2}{1-x^2} + 1$ $\quad 2 \equiv A(1-x) + B(1+x)$

$y = \int \frac{2}{1-x^2} + 1\,dx$ $\quad x = 1 \Rightarrow B = 1$

$= \int \frac{1}{1+x} + \frac{1}{1-x} + 1\,dx$ $\quad x = -1 \Rightarrow A = 1$ ...[M1]

$= \log_e|1+x| - \log_e|1-x| + x + d$

$= \log_e(1+x) - \log_e(1-x) + x + d$ as $-1 < x < 1$ ...[M1]

$y(0) = 4$

$4 = \log_e(1+0) - \log_e(1-0) + 0 + d$

$\Rightarrow d = 4$

$y = \log_e(1+x) - \log_e(1-x) + x + 4$ ...[A1] or equivalent

Question 7

Marks	0	1	2	3	Average
%	26	21	20	33	1.6

$y = x + \log_e\left(\frac{1+x}{1-x}\right) + 4$ (other equivalent answers were accepted)

Beginning this question was difficult for a number of students but the question was reasonably well done by students who understood how to begin. A number of students ignored the fact that the first derivative was given and wasted a great deal of time and effort attempting to independently determine the result. A number of these students obtained a different answer and then used their incorrect result to try to complete the question. A few students omitted a constant of integration at this stage. Most students then correctly used partial fractions, although there were still those who gave the integral of the rational function as $2\log_e\left(\left|1-x^2\right|\right)$. Quite a few students failed to realise that $\int \frac{1}{(1-x)}dx$ is $-\log_e\left(|1-x|\right)$ and not $+\log_e\left(|1-x|\right)$. A large number of students omitted the x term from their final answer. Some of these errors were due to the earlier omission of the constant of integration, and others were due to students simply forgetting to incorporate it back into their answer after working on the latter part of the question.

Question 8

The path of a particle is given by $\underset{\sim}{r}(t) = t\sin(t)\underset{\sim}{i} - t\cos(t)\underset{\sim}{j}$, $t \geq 0$. The particle leaves the origin at $t = 0$ and then spirals outwards.

a. Show that the **second time** the particle crosses the x-axis after leaving the origin occurs when $t = \frac{3\pi}{2}$.

Possible solution

$\underset{\sim}{r}(t) = t\sin(t)\underset{\sim}{i} - t\cos(t)\underset{\sim}{j} = x\underset{\sim}{i} - y\underset{\sim}{j}$

Particle crosses x-axis when $y = 0$

$-t\cos(t) = 0$

$t = 0 \quad \cos(t) = 0 \quad t \geq 0$

$$\left.\begin{array}{l} t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots \\ \text{Initial position at O, first crosses at } t = \frac{\pi}{2} \end{array}\right\} \ldots[\text{M1}]$$

Second occasion when $t = \frac{3\pi}{2}$ as required

Question 8a.

Marks	0	1	Average
%	45	55	0.6

Students needed to show the given result $t = \frac{3\pi}{2}$.

Students tended to either answer this question correctly or have trouble getting started. In this question, many students were unable to show that the given value was the second occurrence; just substituting the value $\frac{3\pi}{2}$ and showing that the coefficient of $\underset{\sim}{j}$ was zero was not sufficient. A number attempted to solve $\sin(t) = 0$ rather than $\cos(t) = 0$. Other students gave $t = 0$ rather than $t = \frac{\pi}{2}$ as the first occurrence. It was evident that these students had not read the question carefully enough.

b. Find the speed of the particle when $t = \frac{3\pi}{2}$.

Possible solution

$\dot{\underset{\sim}{r}}(t) = [t\cos(t) + 1\sin(t)]\underset{\sim}{i} - [-t\sin(t) + 1\cos(t)]\underset{\sim}{j}$

$= [t\cos(t) + \sin(t)]\underset{\sim}{i} + [t\sin(t) - \cos(t)]\underset{\sim}{j}$ ...[M1]

$\dot{\underset{\sim}{r}}\left(\frac{3\pi}{2}\right) = \left[\frac{3\pi}{2}\cos\left(\frac{3\pi}{2}\right) + \sin\left(\frac{3\pi}{2}\right)\right]\underset{\sim}{i} + \left[\frac{3\pi}{2}\sin\left(\frac{3\pi}{2}\right) - \cos\left(\frac{3\pi}{2}\right)\right]\underset{\sim}{j}$

$= -\underset{\sim}{i} - \frac{3\pi}{2}\underset{\sim}{j}$ ...[M1]

$\left|\dot{\underset{\sim}{r}}\left(\frac{3\pi}{2}\right)\right| = \sqrt{(-1)^2 + \left(\frac{3\pi}{2}\right)^2} = \sqrt{1 + \frac{9\pi^2}{4}} = \frac{1}{2}\sqrt{4 + 9\pi^2}$ ...[A1]

Question 8b.

Marks	0	1	2	3	Average
%	39	12	12	37	1.5

$$\sqrt{1 + \left(\frac{3\pi}{2}\right)^2} = \sqrt{1 + \frac{9\pi^2}{4}} = \frac{1}{2}\sqrt{4 + 9\pi^2}$$

Attempts to solve this question were quite varied, ranging from excellent to very disappointing. There were some poor attempts at finding the velocity vector. Many students did not use the product rule, while others gave incorrect derivatives, usually involving incorrect signs. Even when the correct velocity vector was found, substitution of $t = \frac{3\pi}{2}$ proved to be difficult for many students, with incorrect signs again being common. Several students who were correct to this stage made some surprising errors involving incorrect 'simplification' of the square root answer. These included $\sqrt{1 + \left(\frac{3\pi}{2}\right)^2} = 1 + \frac{3\pi}{2}$ and $\sqrt{1 + \left(\frac{3\pi}{2}\right)^2} = \sqrt{\frac{13\pi^2}{4}}$. There was again an issue with students not reading the question carefully enough, with a number of students correctly finding a vector expression for the velocity but not going on to find the speed. Notation was often poor, with several students omitting $\underset{\sim}{i}$ and $\underset{\sim}{j}$, and many not using brackets properly.

A few very successful students found a general expression for the speed, $|V(t)| = \sqrt{1 + t^2}$, and then substituted in $t = \frac{3\pi}{2}$.

Let θ be the acute angle at which the path of the particle crosses the x-axis.

c. Find $\tan(\theta)$ when $t = \frac{3\pi}{2}$.

Possible solution

$\dot{\underset{\sim}{r}}\left(\frac{3\pi}{2}\right) = -\underset{\sim}{i} - \frac{3\pi}{2}\underset{\sim}{j} = \dot{x}\underset{\sim}{i} + \dot{y}\underset{\sim}{j}$ from part b.

$\tan(\theta) = \frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = \frac{\dot{y}}{\dot{x}}$

$= \frac{-\frac{3\pi}{2}}{-1} = \frac{3\pi}{2}$ …[A1]

Question 8c.

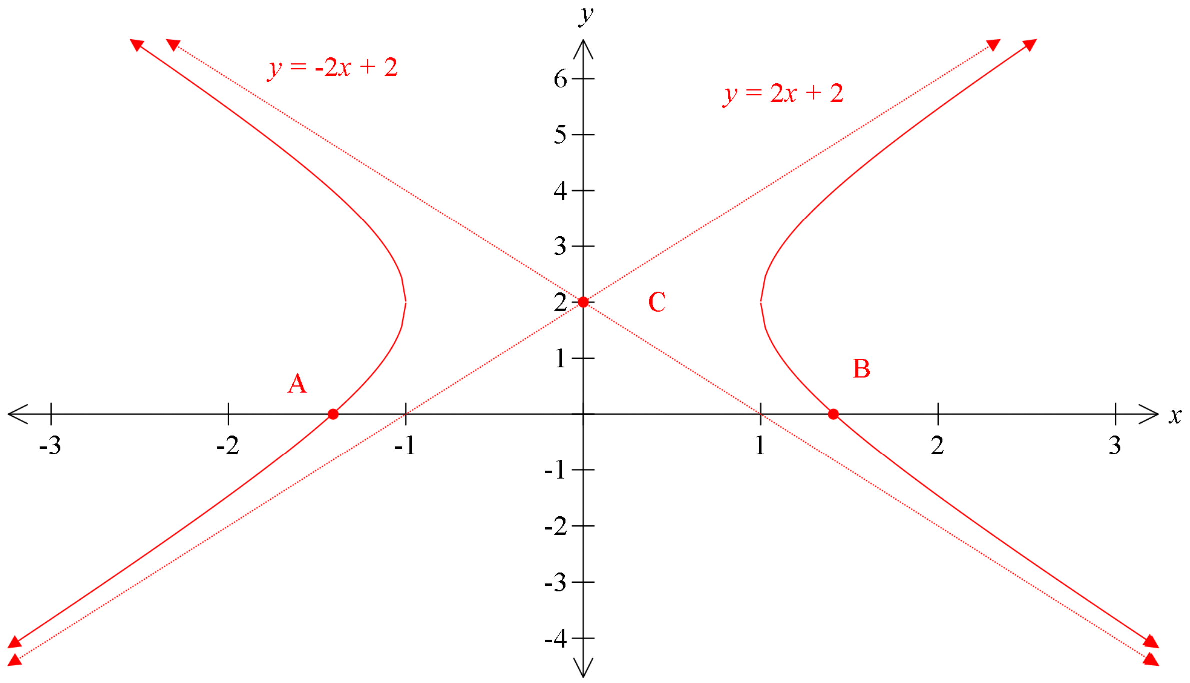
Marks	0	1	Average
%	80	20	0.2

Students needed to find $\tan(\theta)$ when $t = \frac{3\pi}{2}$.

Very few students were able to complete this question. Few realised that this was an application of the chain rule and could be solved using $\tan(\theta) = \frac{dy}{dx} = \frac{\dot{y}(t)}{\dot{x}(t)}$. Of those who successfully obtained the correct answer of $\frac{3\pi}{2}$, a number of students then discarded their answer on the basis that 'since θ is acute, $\tan(\theta) = \frac{\pi}{2}$' or 'so $\tan(\theta)$ is undefined'. A number of students attempted to find θ rather than $\tan(\theta)$. A large proportion of students attempted to answer this question using the position vector instead of the velocity vector, while some others tried to find $\tan\left(\frac{3\pi}{2}\right)$.

Question 9

a. On the axes below sketch the graph with equation $x^2 - \frac{(y-2)^2}{4} = 1$. State all intercepts with the coordinate axes and give the equations of any asymptotes.



Possible solution

$\frac{(x-0)^2}{1^2} - \frac{(y-2)^2}{2^2} = 1$ horizontally orientated hyperbola ...[A1] correct shape

centre C$(0,2)$ $r_x = a = 1$ $r_y = b = 2$

vertices $(1,2)$ $(-1,2)$

gradient of asymptotes, $m = \pm\frac{r_y}{r_x} = \pm 2$

equations to asymptotes

$y - 2 = 2(x-0)$ $y - 2 = -2(x-0)$

$y = 2x + 2$ $y = -2x + 2$ ...[A1] for both

no y-intercepts

x-intercepts

$x^2 - \frac{(0-2)^2}{4} = 1$

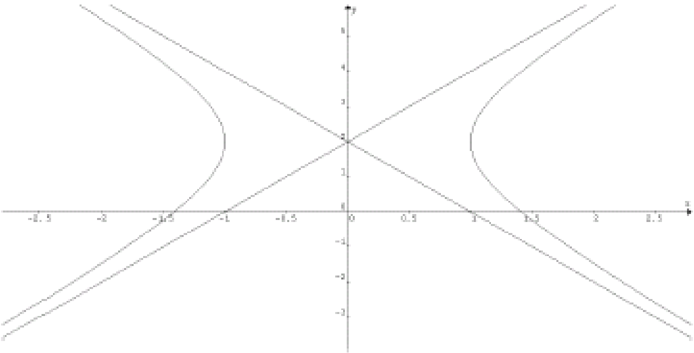
$x^2 = 2$

$x = \pm\sqrt{2}$

x-intercepts $A(-\sqrt{2},0)$ $B(\sqrt{2},0)$ ...[A1] for both

Question 9a.

Marks	0	1	2	3	Average
%	18	18	19	44	1.9



Asymptotes at $y = 2x + 2$ and $y = -2x + 2$

Intercepts at $\left(\sqrt{2}, 0\right)$ and $\left(-\sqrt{2}, 0\right)$

This question was done well by many students. Most were able to correctly identify the x-intercepts, but quite a few seemed to have little idea of the approximate size of $\sqrt{2}$, placing their intercepts a long way away from the true value. There was a large number of students who submitted graphs which were unacceptable. A number of graphs were not smooth and were roughly sketched with multiple lines. The worst of these were drawn in ink. Students should be strongly advised to use a pencil to draw graphs so that they can erase incorrect sketch attempts and present one smooth curve as their final answer. Many graphs did not show asymptotic behaviour, with some graphs looking like horizontal parabolas or semi-circles. Common errors included stating equations of the asymptotes as $y = \pm 2x$ or $y = \pm 4x + 2$. Some students seemed to believe that all hyperbolas have vertical and horizontal asymptotes. Also seen were circles, ellipses and parabolas.

b. Find the gradient of the curve with equation $x^2 - \frac{(y-2)^2}{4} = 1$ at the point where $x = 2$ and $y < 0$.

Possible solution

$x^2 - \frac{(y-2)^2}{4} = 1$

$x = 2$ and $y < 0$

$4 - \frac{(y-2)^2}{4} = 1$

$\frac{(y-2)^2}{4} = 3$

$y - 2 = \pm\sqrt{12}$

$y = 2 - \sqrt{12}$ as $y < 0$

So, $\left(2, 2-\sqrt{12}\right)$ ...[A1]

$\frac{d}{dx}\left(x^2 - \frac{(y-2)^2}{4}\right) = \frac{d}{dx}(1)$

$2x - \frac{1}{4} \times 2(y-2) \times 1 \times \frac{dy}{dx} = 0$

$2x - \frac{1}{2}(y-2)\frac{dy}{dx} = 0$

$4x - (y-2)\frac{dy}{dx} = 0$ ...[M1]

At $\left(2, 2-\sqrt{12}\right)$

$4(2) - \left(2 - \sqrt{12} - 2\right)\frac{dy}{dx} = 0$

$8 + \sqrt{12}\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{8}{\sqrt{12}} = -\frac{4}{\sqrt{3}} = -\frac{4\sqrt{3}}{3}$ ...[A1] for any of these

Question 9b.

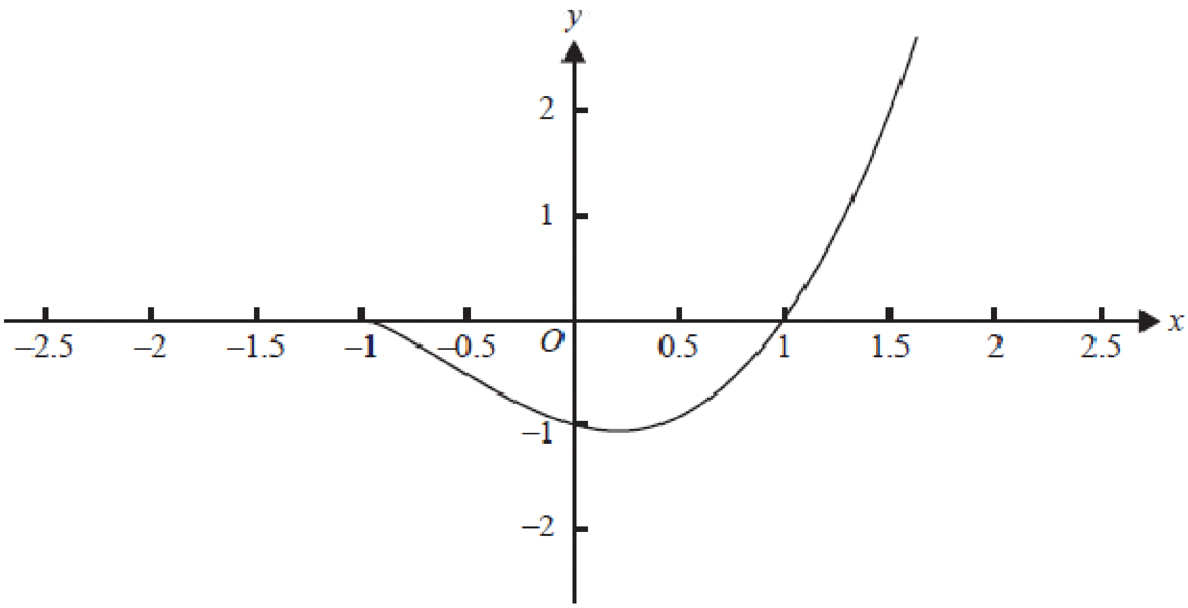
Marks	**0**	**1**	**2**	**3**	**Average**
%	29	28	11	33	**1.5**

$-\frac{4}{\sqrt{3}} = -\frac{4\sqrt{3}}{3} = -\frac{8}{\sqrt{12}}$

Students generally applied implicit differentiation well, although some did not get zero when differentiating the constant on the right side of the equation. Students who unnecessarily expanded the term $(y-2)^2$ before differentiating implicitly often did so incorrectly and were more likely to make algebraic and arithmetic errors. The most frequent error involved not distributing the minus sign. Many students substituted $x = 2$ into the original function to arrive at the y values of $2 \pm \sqrt{12}$. Several then failed to choose the negative root as required. Some stated the negative y value, but then substituted the positive one.

Question 10

Part of the graph with equation $y=(x^2-1)\sqrt{x+1}$ is shown below.



Find the area that is bounded by the curve and the x-axis. Give your answer in the form $\frac{a\sqrt{b}}{c}$ where a, b and c are integers.

Possible solution

$y=(x^2-1)\sqrt{x+1}$

$y=0 \Rightarrow x=\pm 1$ by inspection

$I=\int_{-1}^{1}(x^2-1)\sqrt{x+1}\,dx$ let $u=x+1$ $\quad \frac{du}{dx}=1$

$=\int_{-1}^{1}(x^2-1)\sqrt{x+1}\times 1\,dx$ $\quad x=u-1$ $\quad x^2-1=u^2-2u$

$=\int_{x=-1}^{1}(u^2-2u)\sqrt{u}\times\frac{du}{dx}dx$ $\quad x=-1, u=0$ $\quad x=1, u=2$

$=\int_{u=0}^{2}(u^2-2u)u^{\frac{1}{2}}\,du=\int_{u=0}^{2}u^{\frac{5}{2}}-2u^{\frac{3}{2}}\,du$

$=\left[\frac{2}{7}u^{\frac{7}{2}}-2\times\frac{2}{5}u^{\frac{5}{2}}\right]_0^2$

$=\frac{2}{7}\times 2^{\frac{7}{2}}-\frac{4}{5}\times 2^{\frac{5}{2}}=2^{\frac{5}{2}}\left(\frac{4}{7}-\frac{4}{5}\right)=4\sqrt{2}\left(-\frac{8}{35}\right)$

$I=-\frac{32\sqrt{2}}{35}$

$A=|I|=\frac{32\sqrt{2}}{35}$ ...[A1]

Question 10

Marks	0	1	2	3	4	Average
%	29	16	17	15	23	**1.9**

$\frac{32\sqrt{2}}{35}$

Attempts at this question were quite varied. Many students handled it very well but a large number did not recognise that area is a positive quantity, and did not place a negative sign in front of their integral, use absolute values, or reverse the terminals. As the area was below the x-axis, it was essential that one of these techniques be applied – this is assumed knowledge from Mathematical Methods CAS Units 3 and 4. Some students presented a negative answer as the area.

Some simply dropped the negative sign or wrote $A = -\frac{32\sqrt{2}}{35} = \frac{32\sqrt{2}}{35}$, which is not a reasonable statement. Most students applied the obvious substitution $u = x + 1$; however, others were attempted, generally without success. There were some algebraic errors seen, involving index laws and fractional powers, specifically $u^2 \times u^{1/2} = u^1$. The integration involving fractional powers also caused difficulties for some students. Again, there were many who did not change terminals at the substitution step. Quite a few split the area into two separate parts unnecessarily, integrating from -1 to the y-axis and adding the integral from the y-axis to 1. There were some students who integrated from -1 to 1.5 and a few integrated from -1 to infinity. Several students found the area correctly but were not able to express it in the required form, often not able to manipulate the surds or simplify their answer.