Review of VCAA 2010 Specialist Maths Exam 2 – Section 1

Question 1

An ellipse has a horizontal semi-axis length of 1 and a vertical semi-axis length of 2. The centre of the ellipse is at the point with coordinates (3, –5).

A possible equation for the ellipse is

A. $(x+3)^2 + 4(y-5)^2 = 4$

B. $(x-3)^2 + 4(y+5)^2 = 4$

C. $4(x+3)^2 + (y-5)^2 = 4$

D. $4(x-3)^2 + (y+5)^2 = 4$

E. $4(x-3)^2 + (y+5)^2 = 1$

Possible solution

Ellipse centre $(3,-5)$ $\quad r_x = a = 1 \quad r_y = b = 2$

$$\frac{(x-3)^2}{1^2} + \frac{(y+5)^2}{2^2} = 1$$

$$4(x-3)^2 + (y+5)^2 = 4$$

So answer D

Question	% A	% B	% C	% D	% E	% No Answer	Comments
1	1	8	2	83	5	0	

Question 2

Each of the following equations represents a hyperbola.

Which hyperbola **does not** have perpendicular asymptotes?

A. $(x-1)^2 - (y+2)^2 = 1$

B. $x^2 - 2x - y^2 + 4y = 4$

C. $(x-1)^2 - (y+2)^2 = 9$

D. $(y-1)^2 - (x+2)^2 = 1$

E. $2x^2 - 4x - y^2 - 4y = 4$

Possible solution

Gradient of asymptotes, $m = \pm\frac{b}{a}$

For perpendicular asymptotes, $m_1 m_2 = -1$

$$\frac{b}{a} \times -\frac{b}{a} = -1$$

$$\frac{b^2}{a^2} = 1$$

$\Rightarrow a = b$ as $a, b > 0$

Not the case in last option

So answer E

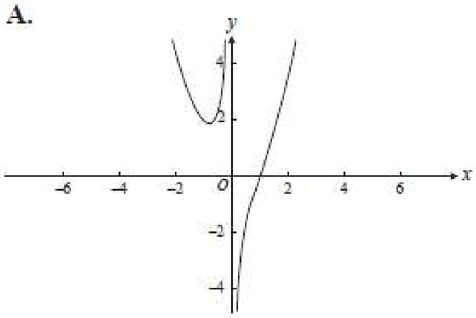
2	4	13	11	11	60	1	

Question 3

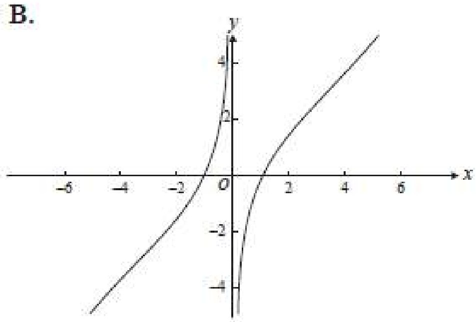
Let $f(x)=\dfrac{x^k+a}{x}$, where k and a are real constants.

If k is an odd integer which is greater than 1 and $a<0$, a possible graph of f could be

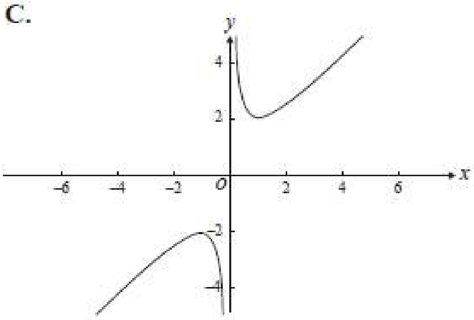
A.



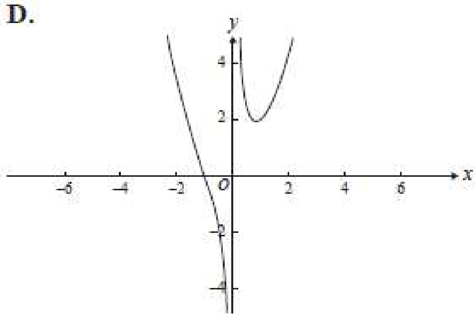
B.



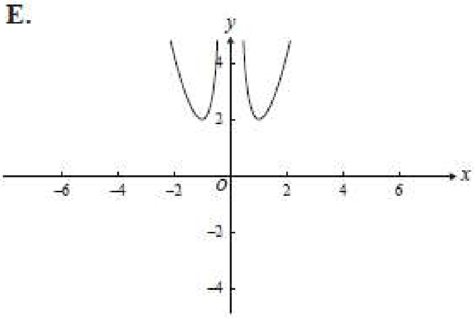
C.



D.



E.



Possible solution

$f\left(x\right)=\dfrac{x^k+a}{x}$ $\qquad$ k is odd and $a<0$

$\Rightarrow$ vertical asymptote at $x=0$ $\qquad$ all still possible - damn!

$f\left(x\right)=x^{k-1}+\dfrac{a}{x}$ $\qquad$ $k-1\geq 0$ and even and $a<0$

$\Rightarrow$ either curved asymptote $y=x^{k-1}$ or horizontal asymptote

h.a. doesn't occur, so look for curved asymptote $y=x^{k-1}$ with even power

Eliminate B & C

$a<0$ so $f\left(x\right)$ approaches curved asymptote from:

below when $x>0$

above when $x<0$

Eliminate D & E

So answer A

3	69	9	5	15	2	0	

Question 4

The position vector of a particle at time $t \geq 0$ is given by $\underset{\sim}{r} = \sin(t)\underset{\sim}{i} + \cos(2t)\underset{\sim}{j}$.

The path of the particle has cartesian equation

A. $y = 2x^2 - 1$

B. $y = 1 - 2x^2$

C. $y = \sqrt{1 - x^2}$

D. $y = \sqrt{x^2 - 1}$

E. $y = 2x\sqrt{1 - x^2}$

Possible solution

$\underset{\sim}{r}(t) = \sin(t)\underset{\sim}{i} + \cos(2t)\underset{\sim}{j} = x\underset{\sim}{i} + y\underset{\sim}{j}$

$\cos(2t) = 1 - 2\sin^2(t)$

$y = 1 - 2x^2$

So answer B

4	4	75	11	4	7	1	

Question 5

For $-\frac{\pi}{2} < x < \frac{\pi}{2}$, the graphs of the two curves given by $y = 2\sec^2(x)$ and $y = 5|\tan(x)|$ intersect

A. only at the one point $(\arctan(2), 10)$

B. only at the two points $(\pm\arctan(2), 10)$

C. only at the one point $\left(\arctan\left(\frac{1}{2}\right), \frac{5}{2}\right)$

D. only at the two points $\left(\pm\arctan\left(\frac{1}{2}\right), \frac{5}{2}\right)$

E. at the two points $\left(\pm\arctan\left(\frac{1}{2}\right), \frac{5}{2}\right)$, as well as at the two points $(\pm\arctan(2), 10)$

Possible solution

Now $\sec^2(x) = 1 + \tan^2(x)$

$2\sec^2(x) = 5|\tan(x)|$

$2(1 + \tan^2(x)) = 5|\tan(x)|$

$2\tan^2(x) - 5|\tan(x)| + 2 = 0$

$-\frac{\pi}{2} < x < 0 \Rightarrow \tan(x) < 0 \Rightarrow |\tan(x)| = -\tan(x)$

$0 \le x < \frac{\pi}{2} \Rightarrow \tan(x) \ge 0 \Rightarrow |\tan(x)| = \tan(x)$

$2\tan^2(x) + 5\tan(x) + 2 = 0 \qquad 2\tan^2(x) - 5\tan(x) + 2 = 0$

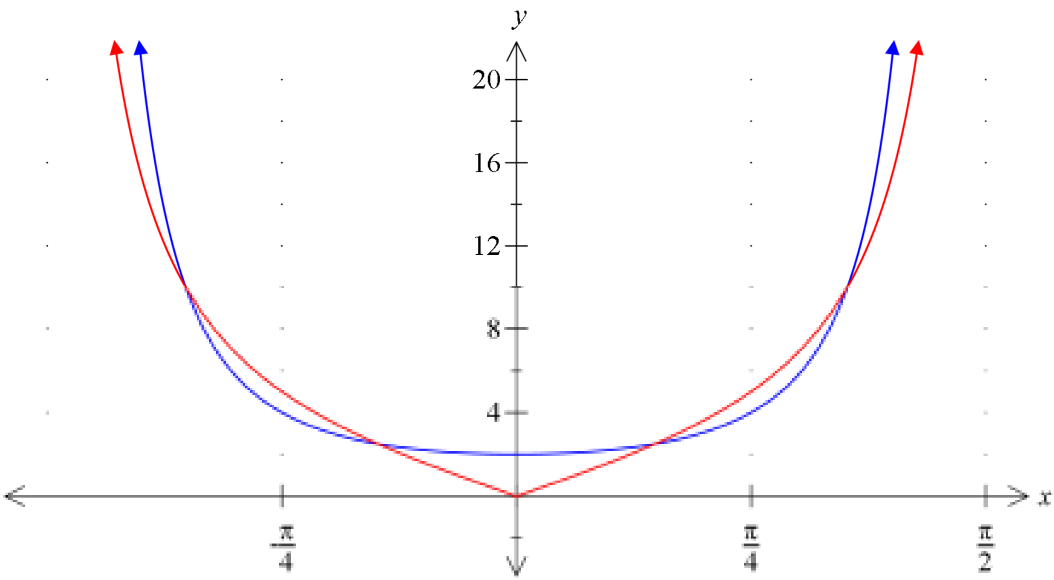
$(2\tan(x) + 1)(\tan(x) + 2) = 0 \qquad (2\tan(x) - 1)(\tan(x) - 2) = 0$

$\tan(x) = -\frac{1}{2}, -2, \frac{1}{2}, 2$

4 solutions, so last option, no real need to check coordinates

So answer E

OR draw the graph – 4 solutions are apparent with a suitable window.



5	2	7	9	22	60	1	

Question 6

Let $z = \text{cis}\left(\frac{5\pi}{6}\right)$.

The imaginary part of $z - i$ is

A. $-\frac{i}{2}$

B. $-\frac{1}{2}$

C. $-\frac{\sqrt{3}}{2}$

D. $-\frac{3}{2}$

E. $-\frac{3i}{2}$

Possible solution

$$z = \text{cis}\left(\frac{5\pi}{6}\right) = \cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)$$

$$= -\frac{\sqrt{3}}{2} + i\left(\frac{1}{2}\right)$$

$$z - i = -\frac{\sqrt{3}}{2} + i\left(\frac{1}{2} - 1\right) - \frac{\sqrt{3}}{2} - \frac{1}{2}i$$

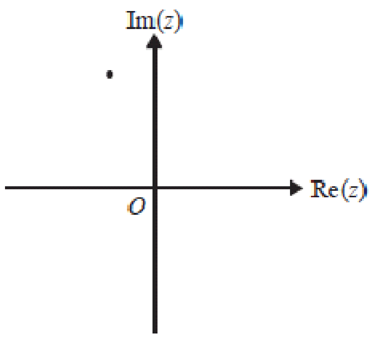
$$\text{Im}(z - i) = -\frac{1}{2}$$

So answer B

6	15	70	6	5	3	0	$\text{cis}\left(\frac{5\pi}{6}\right) - i = -\frac{\sqrt{3}}{2} - i\frac{1}{2}$. The imaginary part is $-\frac{1}{2}$.

Question 7

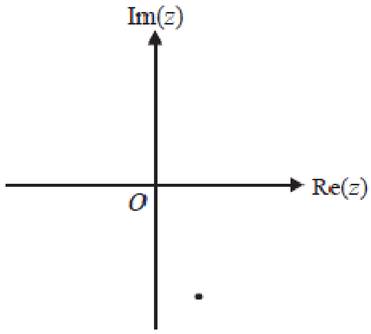
A particular complex number z is represented by the point on the following argand diagram



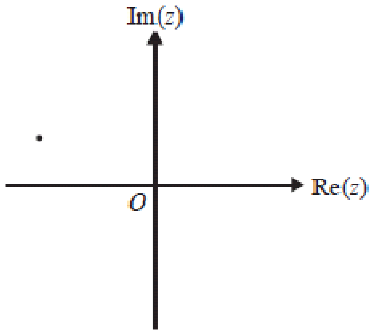
All axes below have the same scale as those in the diagram above.

The complex number $i\bar{z}$ is best represented by

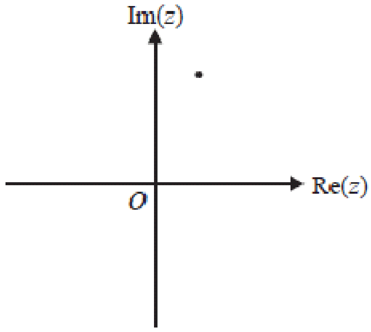
A.



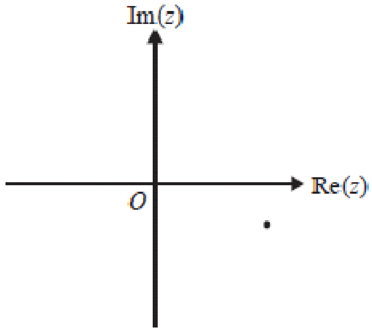
B.



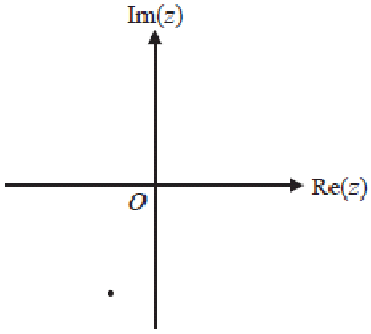
C.



D.



E.



Possible solution

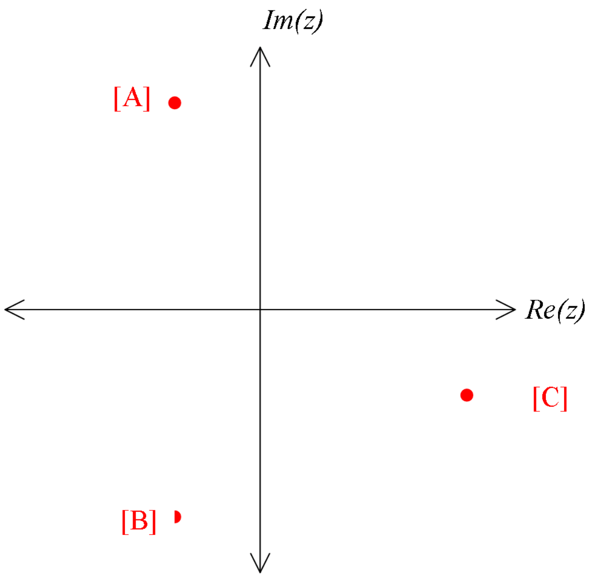
Easiest to consider this geometrically

$[\mathrm{A}]$ $\quad z = x + iy$

$[\mathrm{B}]$ $\quad \bar{z} = x - iy$ reflection in x-axis

$[\mathrm{C}]$ $\quad i\bar{z} = xi - i^2 y = y + ix$ rotation of $\dfrac{\pi}{2}$ CCW

So answer D



7	6	8	5	72	9	0	

Question 8

The polynomial equation $P(z) = 0$ has one complex coefficient. Three of the roots of this equation are $z = 3 + i$, $z = 2 - i$ and $z = 0$.

The **minimum degree** of $P(z)$ is

A. 1

B. 2

C. 3

D. 4

E. 5

Possible solution

As there are complex coefficients, there is NO requirement for roots to occur in conjugate pairs.

$\Rightarrow$ The three roots provided may be the only roots.

$\Rightarrow$ Minimum degree of $P(z)$ is 3

So answer C

8	5	8	60	14	12	0	

Question 9

Given that $z = 4\text{cis}\left(\frac{2\pi}{3}\right)$, it follows that $\text{Arg}\left(z^5\right)$ is

A. $\frac{10\pi}{3}$

B. $\frac{4\pi}{3}$

C. $\frac{7\pi}{3}$

D. $-\frac{\pi}{3}$

E. $-\frac{2\pi}{3}$

Possible solution

$z = 4\,\text{cis}\left(\frac{2\pi}{3}\right)$

$z^5 = 4^5\,\text{cis}\left(5\times\frac{2\pi}{3}\right) = 4^5\,\text{cis}\left(\frac{10\pi}{3}\right)$

$\text{Arg}\left(z^5\right) = \text{Arg}\left(\frac{10\pi}{3}\right) = -\frac{2\pi}{3}$

So answer E

9	15	8	3	5	69	0	

Question 10

On an argand diagram, a set of points which lies on a circle of radius 2 centred at the origin is

A. $\{z \in C : z\overline{z} = 2\}$

B. $\{z \in C : z^2 = 4\}$

C. $\{z \in C : \text{Re}(z^2) + \text{Im}(z^2) = 4\}$

D. $\{z \in C : (z+\overline{z})^2 - (z-\overline{z})^2 = 16\}$

E. $\{z \in C : (\text{Re}(z))^2 + (\text{Im}(z))^2 = 16\}$

N.B. This question contained a rare wording error so that 2 responses had to be accepted as correct.

Possible solution(s)

Some intuition is needed to avoid pure elimination approach.

Require $x^2 + y^2 = 2^2$

$x^2 + y^2 = 4$

$z\overline{z} = x^2 + y^2 \neq 2$ reject A

$z^2 = 4 \Rightarrow z = \pm 2$ which are both on $x^2 + y^2 = 4$ and so satisfy the request for "... **a** set of points ..." accept B

$z^2 = (x^2 - y^2) + 2xyi$

$\text{Re}(z^2) + \text{Im}(z^2) = x^2 - y^2 + 2xy = 4$ which is a hyperbola, reject C

$z + \overline{z} = 2x \qquad z - \overline{z} = 2y$

$(2x)^2 + (2y)^2 = 16 \Leftrightarrow x^2 + y^2 = 4$ accept D

$\text{Re}(z) = x \qquad \text{Im}(z) = y$

$x^2 + y^2 = 16 \neq 4$ reject E

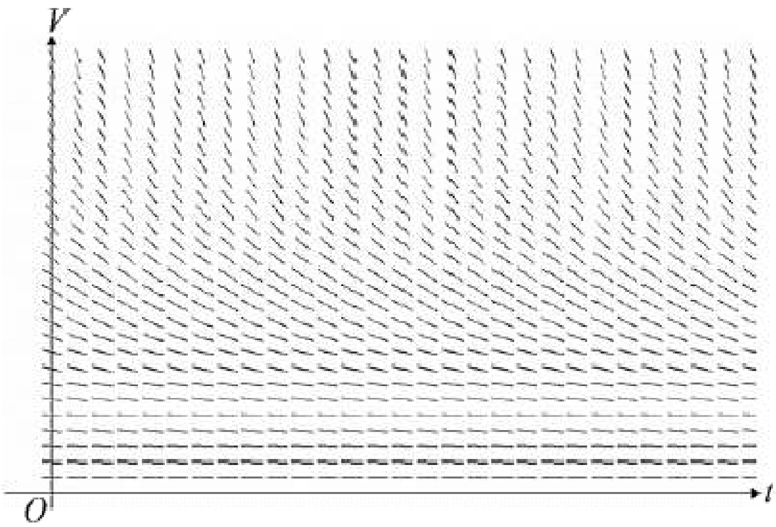
So answer B or D

Don't count on this happening again this year!!!

10	12	12	42	28	6	0	Option D gave $(2x)^2 - (i2y)^2 = 16$, which is $x^2 + y^2 = 4$ – the equation to **all** points on the given circle. As the question nominated a set of points rather than **the** set of points, the two points $z = \pm 2$ specified by option B were also accepted.

Question 11

A direction field for the volume of water, V megalitres, in a reservoir t years after 2010 is shown below.



According to this model, for $k > 0$, $\frac{dV}{dt}$ is equal to

A. $-kt^2$

B. $\frac{k}{V}$

C. $-kV^2$

D. kV^2

E. $-\frac{k}{V}$

Possible solution

Horizontal isoclines $\Rightarrow \frac{dV}{dt} \neq f(t)$ reject A

$\frac{dV}{dt}$ is defined for $V = 0$ reject B & E

if $V > 0, \frac{dV}{dt} < 0$ reject D

$\therefore$ accept C

So answer C

11	9	19	40	10	22	1	Considering $t = 0$ and checking gradients of line segments eliminates option A. Considering $V = 0$ eliminates options B and E. Option D provided gradients of the incorrect sign. Hence option C was correct.

Question 12

Let $\frac{dy}{dx} = \frac{x+2}{x^2+2x+1}$ and $(x_0, y_0) = (0, 2)$.

Using Euler's method, with a step size of 0.1, the value of y_1 correct to two decimal places is

A. 0.17

B. 0.20

C. 1.70

D. 2.17

E. 2.20

Possible solution

Do NOT use an Euler's program for something this easy!

$x_0 = 0 \quad y_0 = 2 \quad h = 0.1 \quad \frac{dy}{dx} = f'(x) = \frac{x+2}{x^2+2x+1} = \frac{x+2}{(x+1)^2}$

$y_1 = y_0 + h \times f'(x_0)$

$= 2 + 0.1 \times \frac{0+2}{(0+1)^2}$

$= 2 + 0.2$

$= 2.20$

So answer E

12	2	3	6	11	77	1	

Question 13

The amount of a drug, x mg, remaining in a patient's bloodstream t hours after taking the drug is given by the differential equation

$$\frac{dx}{dt} = -0.15x.$$

The number of hours needed for the amount x to halve is

A. $2\log_e\left(\frac{20}{3}\right)$

B. $\frac{20}{3}\log_e(2)$

C. $2\log_e(15)$

D. $15\log_e\left(\frac{3}{2}\right)$

E. $\frac{3}{2}\log_e(200)$

Possible solution - by hand

$$\frac{dx}{dt} = -0.15x = -\frac{3x}{20}$$

$$\frac{dt}{dx} = -\frac{20}{3x}$$

$$t = -\frac{20}{3}\int \frac{1}{x}\,dx = -\frac{20}{3}\log_e|x| + c$$

$$t = 0, x > 0 \Rightarrow t = -\frac{20}{3}\log_e(x) + c$$

Assume when $t = 0, x = I$

$$0 = -\frac{20}{3}\log_e(I) + c$$

$$c = \frac{20}{3}\log_e(I)$$

$$t = \frac{20}{3}\log_e(I) - \frac{20}{3}\log_e(x) = \frac{20}{3}\log_e\left(\frac{I}{x}\right)$$

if $x = \frac{1}{2}I$

$$t = \frac{20}{3}\log_e\left(\frac{I}{\frac{1}{2}I}\right) = \frac{20}{3}\log_e(2)$$

So answer B

Possible solution - CAS

Use Exact mode

Choose $t = 0,\ x = 2$ actual x-value doesn't matter as it gets halved

Use deSolve, then solve for $x = 1$

So answer B

deSolve$(x' = -0.15 \cdot x$ and $x(0) = 2, t, x)$ $\quad x = 2 \cdot e^{\frac{-3 \cdot t}{20}}$

solve$\left(1 = 2 \cdot e^{\frac{-3 \cdot t}{20}}, t\right)$ $\quad t = \frac{20 \cdot \ln(2)}{3}$

13	9	65	11	10	3	1	

Question 14

Use a suitable substitution to show that the definite integral $\int_0^2 \frac{x}{\sqrt{x^2-1}}dx$ can be simplified to

A. $\frac{1}{2}\int_{-1}^{3} u^{-\frac{1}{2}}\,du$

B. $2\int_{-1}^{3} u^{-\frac{1}{2}}\,du$

C. $\frac{1}{2}\int_{0}^{2} u^{-\frac{1}{2}}\,du$

D. $2\int_{0}^{2} u^{-\frac{1}{2}}\,du$

E. $\int_{0}^{2} u^{-\frac{1}{2}}\,du$

Possible solution

$\int_0^2 \frac{x}{\sqrt{x^2-1}}dx \qquad u = x^2-1 \qquad \frac{du}{dx} = 2x$

$= \frac{1}{2}\int_0^2 \frac{1}{\sqrt{x^2-1}} \times 2x\,dx \qquad x=0, u=-1 \qquad x=2, u=3$

$= \frac{1}{2}\int_{x=0}^{2} u^{-\frac{1}{2}} \times \frac{du}{dx}dx$

$= \frac{1}{2}\int_{u=-1}^{3} u^{-\frac{1}{2}}du$

So answer A

14	80	6	9	3	1	0	Let $u = x^2-1$, $\frac{du}{dx} = 2x$, $x=0 \Rightarrow u=-1$, $x=2 \Rightarrow u=3$, hence option A was correct.

Question 15

The scalar resolute of $\underset{\sim}{a} = 3\underset{\sim}{i} - \underset{\sim}{k}$ in the direction of $\underset{\sim}{b} = 2\underset{\sim}{i} - \underset{\sim}{j} - 2\underset{\sim}{k}$ is

A. $\dfrac{8}{\sqrt{10}}$

B. $\dfrac{8}{9}\left(2\underset{\sim}{i} - \underset{\sim}{j} - 2\underset{\sim}{k}\right)$

C. 8

D. $\dfrac{4}{5}\left(3\underset{\sim}{i} - \underset{\sim}{k}\right)$

E. $\dfrac{8}{3}$

Possible solution

$\underset{\sim}{a} = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} \qquad \underset{\sim}{b} = \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix} \qquad |\underset{\sim}{b}| = \sqrt{4+1+4} = 3$

scalar resolute of $\underset{\sim}{a}$ on direction of $\underset{\sim}{b}$

$= \underset{\sim}{a} \cdot \hat{\underset{\sim}{b}} = \dfrac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{b}|}$

$= \dfrac{6+0+2}{3} = \dfrac{8}{3}$

So answer E

15	5	12	4	4	74	0	

Question 16

The square of the magnitude of the vector $\underset{\sim}{d} = 5\underset{\sim}{i} - \underset{\sim}{j} + \sqrt{10}\underset{\sim}{k}$ is

A. 6

B. 34

C. 36

D. 51.3

E. $\sqrt{34}$

Possible solution

$$|\underset{\sim}{d}|^2 = \underset{\sim}{d} \cdot \underset{\sim}{d} = \begin{bmatrix} 5 \\ -1 \\ \sqrt{10} \end{bmatrix} \cdot \begin{bmatrix} 5 \\ -1 \\ \sqrt{10} \end{bmatrix}$$

$= 25 + 1 + 10$

$= 36$

So answer C

16	6	3	88	1	1	0	

Question 17

The angle between the vectors $\underset{\sim}{a} = \underset{\sim}{i} + \underset{\sim}{k}$ and $\underset{\sim}{b} = \underset{\sim}{i} + \underset{\sim}{j}$ is exactly

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

E. π

Possible solution

$\underset{\sim}{a} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ $\quad \underset{\sim}{b} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $\quad |\underset{\sim}{a}| = \sqrt{2}$ $\quad |\underset{\sim}{b}| = \sqrt{2}$

$\cos(\theta) = \frac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{a}| \cdot |\underset{\sim}{b}|}$

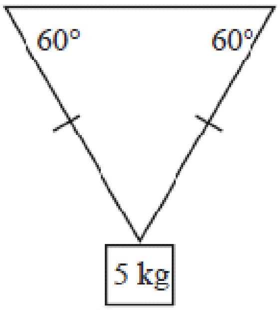
$= \frac{1+0+0}{\sqrt{2}\sqrt{2}} = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{3}$

So answer C

17	2	9	75	9	5	1	

Question 18



A 5 kg mass is suspended from a horizontal ceiling by two strings of equal length. Each string makes an angle of 60° to the ceiling, as shown in the above diagram. Correct to one decimal place, the tension in each string is

A. 24.5 newtons

B. 28.3 newtons

C. 34.6 newtons

D. 49.0 newtons

E. 84.9 newtons

Possible solution

Resolve vertically

$T\sin(60°) + T\sin(60°) - 5g = 0$

$\sqrt{3} \times T = 5g$

$T = \dfrac{5g}{\sqrt{3}} \approx 28.3$

So answer B

18	14	62	6	14	2	1	Let T be the tension in a string, $2T\sin 60^\circ = 5g$, $T = 28.3$ N, hence option B was correct.

Question 19

An object is moving in a northerly direction with a constant acceleration of 2 ms^{-2}. When the object is 100 m due north of its starting point, its velocity is 30 ms^{-1} in the northerly direction.

The exact initial velocity of the object could have been

A. $10\sqrt{5}$ ms^{-1}

B. $5\sqrt{10}$ ms^{-1}

C. $10\sqrt{7}$ ms^{-1}

D. $-10\sqrt{7}$ ms^{-1}

E. $7\sqrt{10}$ ms^{-1}

Possible solution

$u = ?,\ v = 30,\ a = 2,\ s = 100$

$v^2 = u^2 + 2as$

$30^2 = u^2 + 2 \times 2 \times 100$

$900 = u^2 + 400$

$u^2 = 500$

$u = \sqrt{500}$ as $u > 0$

$= 10\sqrt{5}$

So answer A

19	81	5	8	3	2	1	

Question 20

The acceleration, a ms^{-2}, of a particle moving in a straight line is given by $a = \dfrac{v}{\log_e(v)}$, where v is the velocity of the particle in ms^{-1} at time t s. The initial velocity of the particle was 5 ms^{-1}.

The velocity of the particle, in terms of t, is given by

A. $v = e^{2t}$

B. $v = e^{2t} + 4$

C. $v = e^{\sqrt{2t} + \log_e(5)}$

D. $v = e^{\sqrt{2t + (\log_e 5)^2}}$

E. $v = e^{-\sqrt{2t + (\log_e 5)^2}}$

Possible solution - by hand

$$a = \frac{dv}{dt} = \frac{v}{\log_e(v)} \qquad t = 0,\ v = 5$$

$$\frac{dt}{dv} = \frac{\log_e(v)}{v}$$

$$t = \int \frac{\log_e(v)}{v}\,dv \qquad u = \log_e(v) \qquad \frac{du}{dv} = \frac{1}{v}$$

$$= \int \log_e(v) \times \frac{1}{v}\,dv$$

$$= \int u \times \frac{du}{dv}\,dv = \int u\,du = \frac{1}{2}u^2 + c$$

$$t = \frac{1}{2}\left(\log_e(v)\right)^2 + c$$

$$v(0) = 5 \Rightarrow 0 = \frac{1}{2}\left(\log_e(5)\right)^2 + c$$

$$t = \frac{1}{2}\left(\log_e(v)\right)^2 - \frac{1}{2}\left(\log_e(5)\right)^2$$

$$\left(\log_e(v)\right)^2 = 2t + \left(\log_e(5)\right)^2$$

$$\log_e(v) = \sqrt{2t + \left(\log_e(5)\right)^2} \text{ given } v(0) = 5$$

$$v = e^{\sqrt{2t + \left(\log_e(5)\right)^2}}$$

So answer D

Possible solution - CAS

Use deSolve, then solve for v

So answer D

$$\text{deSolve}\left(v' = \frac{v}{\ln(v)} \text{ and } v(0) = 5, t, v\right)$$

$$\frac{(\ln(v))^2}{2} - \frac{(\ln(5))^2}{2} = t$$

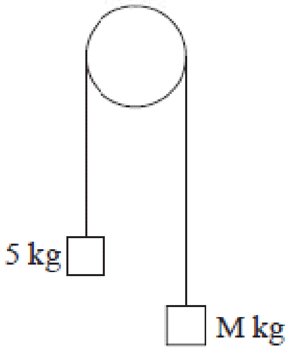
$$\text{solve}\left(\frac{(\ln(v))^2}{2} - \frac{(\ln(5))^2}{2} = t, v\right)$$

$$v = e^{\sqrt{2 \cdot t + (\ln(5))^2}} \text{ and } t \geq \frac{-(\ln(5))^2}{2} \text{ or } v = e^{\sqrt{2 \cdot ▸}}$$

20	2	6	15	67	8	1	

Question 21

A light inextensible string passes over a smooth, light pulley. A mass of 5 kg is attached to one end of the string and a mass of M kg is attached to the other end, as shown below.



The M kg mass accelerates downwards at $\frac{7}{5}$ ms^{-2}.

The value of M is

A. $5\frac{2}{7}$

B. $6\frac{2}{5}$

C. $6\frac{2}{3}$

D. $8\frac{2}{5}$

E. $9\frac{4}{5}$

Possible solution

Resolving system in direction of motion

$$Mg - T + T - 5g = (M+5) \times \frac{7}{5}$$

$$Mg - 5g = \frac{7(M+5)}{5}$$

$$5Mg - 25g = 7M + 35$$

$$5gM - 7M = 35 + 25g$$

$$M = \frac{35 + 25g}{5g - 7} = \frac{20}{3} = 6\frac{2}{3}$$

So answer C

Possible solution - CAS

Solve for M at this stage

So answer C

$$9.8 \to g \qquad \frac{49}{5}$$

$$\text{solve}\left(m \cdot g - 5 \cdot g = \frac{7 \cdot (m+5)}{5}, m\right) \qquad m = \frac{20}{3}$$

21	11	16	63	6	3	1	

Question 22

A particle of mass m moves in a straight line under the action of a resultant force F where $F = F(x)$. Given that the velocity v is v_0 where the position x is x_0, and that v is v_1 where x is x_1, it follows that

A. $v_1 = \sqrt{\frac{2}{m}} \int_{x_0}^{x_1} \sqrt{F(x)}\, dx + v_0$

B. $v_1 = \sqrt{2} \sqrt{\int_{x_0}^{x_1} F(x)\, dx + v_0^2}$

C. $v_1 = \sqrt{2} \int_{\sqrt{x_0}}^{\sqrt{x_1}} F(x)\, dx + v_0$

D. $v_1 = \sqrt{\frac{2}{m} \int_{x_0}^{x_1} F(x)\, dx + v_0^2}$

E. $v_1 = \sqrt{\frac{2}{m} \int_{x_0}^{x_1} \left(F(x) + v_0^2\right) dx}$

Possible solution

This problem looks hideous, but careful reading should make you realise that it's just a variation on

$F(b) = F(a) + \int_a^b f(x)\,dx$ this $F(x)$ is not the force, just a generic statement!!

In this case, $a = \frac{F}{m} = \frac{F(x)}{m} = \frac{d}{dx}\left(\frac{1}{2}v^2\right) \ldots(1)$

with $v(x_0) = v_0$ and $v(x_1) = v_1$ as known points

$$(1) \Rightarrow \frac{1}{2}v^2 = \int \frac{F(x)}{m}\,dx$$

$$v^2 = \frac{2}{m}\int F(x)\,dx$$

$$\frac{2}{m}\int_{x_0}^{x_1} F(x)\,dx = v^2(x_1) - v^2(x_0)$$

$v^2(x_1) = \frac{2}{m}\int_{x_0}^{x_1} F(x)\,dx + v^2(x_0)$ here's the variation on the generic statement

but $v(x_0) = v_0$ and $v(x_1) = v_1$

$$\Rightarrow v_1^2 = \frac{2}{m}\int_{x_0}^{x_1} F(x)\,dx + v_0^2$$

$v_1 = \sqrt{\frac{2}{m}\int_{x_0}^{x_1} F(x)\,dx + v_0^2}$ for $v_1 \geq 0$ Why???

So answer D

22	23	11	14	40	10	2	Integrating $F = m \times \frac{d\left(\frac{1}{2}v^2\right)}{dx}$ where $v = v_0$ at $x = x_0$, and $v = v_1$ at $x = x_1$ gives option D.

The mean score for the multiple-choice section was 14.6 out of 22 and the standard deviation was 4.64, slightly higher than in 2009. There were three questions (Questions 10, 11 and 22) that were answered correctly by less than 50 per cent of students. Students seemed to find the 2010 multiple-choice questions accessible.