Multiple-choice questions

1 $\frac{4}{3} - 1 + \frac{3}{4} - \ldots$ is equal to:

A $\frac{13}{12}$

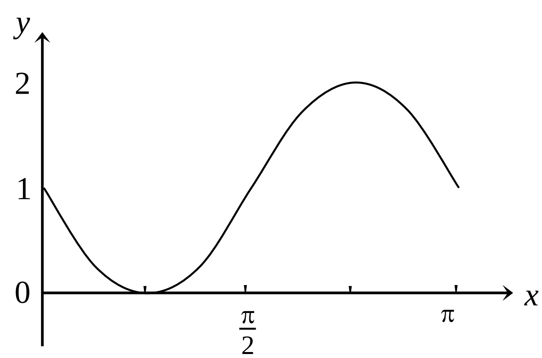
B $\frac{25}{8}$

C $\frac{16}{3}$

D $\frac{16}{21}$

E 1

2 A possible equation for the graph shown is:



A $y = 1 - \sin x$

B $y = 1 - \cos x$

C $y = 1 - \sin 2x$

D $y = 1 - \cos 2x$

E $y = 1 + \sin x$

3 The exact value of $\sin\left(\frac{103\pi}{3}\right)$ is:

A 1

B $\frac{1}{2}$

C $-\frac{1}{2}$

D $\frac{\sqrt{3}}{2}$

E $-\frac{\sqrt{3}}{2}$

4 The sum of the first 50 multiples of 7 is:

A 350

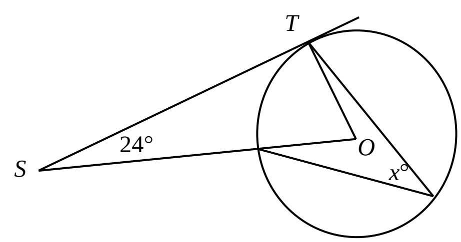
B 8925

C 9350

D 11 025

E 7.88×10^{15}

5 In the diagram, where *O* is the centre of the circle and *ST* is a tangent to the circle at *T*, *x* equals:



A 24

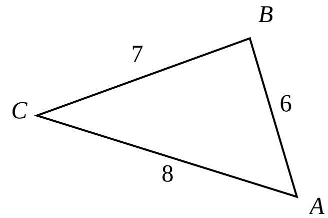
B 36

C 33

D 66

E 78

6 If triangle *ABC* has side lengths as shown, then the magnitude of angle *A* (in degrees) is closest to:



A 32

B 76

C 47

D 58

E 35

7 The coordinates of the *x*-axis intercepts of the graph of the ellipse with the equation $\frac{x^2}{9} + \frac{y^2}{25} = 1$ are:

A (–9, 0) and (9, 0)

B (–5, 0) and (5, 0)

C (0, –3) and (0, 3)

D (–3, 0) and (3, 0)

E (3, 0) and (5, 0)

8 The solutions of $\cos 3x = \frac{1}{2}$ for $x \in [0, 2\pi]$ are:

A $\frac{\pi}{3}, \frac{5\pi}{3}$ only

B $\frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}, \frac{17\pi}{3}$ only

C $\frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}$ only

D $\frac{\pi}{9}, \frac{7\pi}{9}, \frac{13\pi}{9}$ only

E $\frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9}$ only

9 The equations for the asymptotes of the hyperbola with equation

$\frac{(y+1)^2}{25} - \frac{(x-2)^2}{16} = 1$ are:

A $y = \frac{5}{4}x + \frac{8}{3}$ and $y = \frac{-3}{4}x + \frac{2}{3}$

B $y = \frac{4}{5}x + \frac{10}{5}$ and $y = \frac{-4}{5}x + \frac{7}{5}$

C $y = \frac{5}{4}x + \frac{10}{3}$ and $y = \frac{-5}{4}x + \frac{2}{3}$

D $y = \frac{4}{5}x + \frac{10}{5}$ and $y = \frac{-4}{5}x + \frac{7}{3}$

E $y = \frac{5}{4}x - \frac{7}{2}$ and $y = \frac{-5}{4}x + \frac{3}{2}$

10 The point P $(2 \sin t, 1 - 3 \cos t)$ always lies on the curve with equation:

A $\frac{x^2}{2} + \frac{(1-y)^2}{3} = 1$

B $\frac{x^2}{4} + \frac{(y-1)^2}{9} = 1$

C $\frac{x^2}{2} - \frac{(1-y)^2}{3} = 1$

D $\frac{x^2}{4} - \frac{(1-y)^2}{9} = 1$

E $\frac{x^2}{4} + \frac{y^2}{9} = 1$

11 Let $\sec x = 3$, $\frac{3\pi}{2} < x \leq 2\pi$. The exact value of $\cot x$ is:

A $-2\sqrt{2}$

B $2\sqrt{2}$

C $\frac{-1}{\sqrt{10}}$

D $\frac{\sqrt{2}}{4}$

E $\frac{-\sqrt{2}}{4}$

12 The equations of the asymptotes of $y = 2\tan^{-1}x + \pi$ are:

A $y = -\pi$ and $y = 3\pi$

B $y = 0$ and $y = 2\pi$

C $y = \frac{\pi}{2}$ and $y = \frac{3\pi}{2}$

D $y = -2 + \pi$ and $y = 2 + \pi$

E $y = -\pi + 2$ and $y = 3\pi + 2$

13 Find the rule for the image of the graph of $y = \sin^{-1} x$ after the following sequence of transformations is applied: a dilation of factor 2 from the y-axis, a reflection in the x-axis and a translation of 1 unit in the positive direction of the x-axis.

A $y = -2\sin^{-1}(x) + 1$

B $y = -\sin^{-1}\left(\frac{x-1}{2}\right)$

C $y = \frac{-1}{2}\sin^{-1}(x) + 1$

D $y = -\sin^{-1}\left(\frac{x}{2} + 1\right)$

E $y = 2\sin^{-1}(x) + \frac{1}{2}$

14 $\frac{1}{2}\sin^{-1} x + \frac{1}{2}\cos^{-1} x = a$ where a is a constant. The value of a is:

A 0

B 1

C π

D $\dfrac{\pi}{2}$

E $\dfrac{\pi}{4}$

15 The family of equations of the vertical asymptotes of the function with rule

$f(\theta) = \dfrac{1}{1+\cos\theta}$ is:

A $\theta = \dfrac{3\pi}{2} k$ where $k \in Z\backslash\{0\}$

B $\theta = \dfrac{\pi}{2}(3 - 2k)$ where $k \in Z$

C $\theta = \dfrac{\pi}{2}(3 + 2k)$ where $k \in Z\backslash\{0\}$

D $\theta = \dfrac{\pi}{2}(2 + 4k)$ where $k \in Z$

E $\theta = \dfrac{\pi}{2}(3 + 4k)$ where $k \in Z$

16 The maximal domain and the range of the function f with rule $f(x) = 2 + 4\sin^{-1} x$ is:

A domain = $\left[\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right]$ and range = $[-1, 3]$

B domain = $[-1, 1]$ and range = $\left[\dfrac{-3\pi}{2}, \dfrac{3\pi}{2}\right]$

C domain = $\left[\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right]$ and range = $[2 - 2\pi, 2 + 2\pi]$

D domain = $[-1, 1]$ and range = $[2 - 2\pi, 2 + 2\pi]$

E domain = R and range = $[-2, 6]$

17 The gradient of the tangent to the ellipse with equation $\frac{x^2}{9} + \frac{y^2}{4} = 1$ at the point $\left(1, \frac{4\sqrt{2}}{3}\right)$ is:

A $\frac{-\sqrt{2}}{6}$

B $\frac{1}{3\sqrt{2}}$

C $\frac{-2}{3}$

D $\frac{4}{9}$

E $\frac{-4}{9}$

18 Using an appropriate substitution, $\int_0^1 x\sqrt{2x+1}\, dx$ is equal to:

A $\frac{1}{4}\int_1^3 (u-1)\sqrt{u}\, du$

B $\int_1^3 (u-1)\sqrt{u}\, du$

C $\frac{1}{4}\int_0^1 (u-1)\sqrt{u}\, du$

D $\int_0^1 (u-1)\sqrt{u}\, du$

E $\frac{1}{2}\int_1^3 (u-1)\sqrt{u}\, du$

19 An antiderivative of $\frac{9}{x^2 - 9x}$ is:

A $\log_e|x^2 - 9x|$

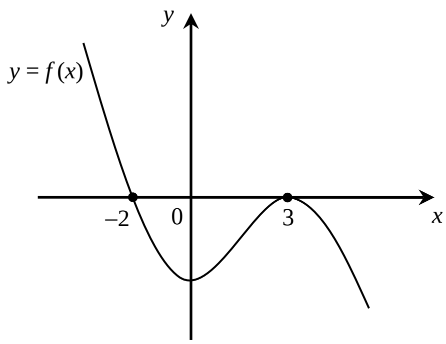
B $(2x - 9)\log_e|x^2 - 9x|$

C $\frac{-9}{x} - \log_e|x|$

D $\log_e|x| - \log_e\left|\frac{9}{x}\right|$

E $\log_e \frac{|x-9|}{|x|}$

20 The graph of $y = f(x)$ is shown below.



If $F(x)$ is an antiderivative of $f(x)$, the stationary points of the graph of $y = F(x)$ are:

A a local minimum at $x = 0$, a local maximum at $x = 3$

B stationary points of inflection at $x = 0$ and $x = 3$, a local maximum at $x = -2$

C a stationary point of inflection at $x = 3$, a local maximum at $x = -2$

D a stationary point of inflection at $x = 0$, a local maximum at $x = -2$

E a stationary point of inflection at $x = 3$, a local minimum at $x = -2$

21 If $\frac{dy}{dx} = 4 + y^2$ and $y = 0$ when $x = 0$, then y is equal to:

A $\frac{1}{3}x^2 + 4x$

B $\frac{1}{2}\tan(2x)$

C $2\tan\left(\frac{1}{2}x\right)$

D $2\tan x$

E $2\tan(2x)$

22 A particle moves in a straight line so that its position x cm from a fixed point O at time t seconds ($t \geq 0$) is given by $x = t^3 - 9t^2 + 24t - 1$. The position of the particle (in cm) the second time it is instantaneously at rest is:

A 4

B 2

C 10

D 14

E 15

23 A body is projected up from the ground with a velocity of 30 m/s. Its acceleration due to gravity is -10 m/s^2. The body's velocity is equal to zero:

A after 0 seconds

B after 1 second

C after 2 seconds

D after 3 seconds

E never

24 A car accelerating uniformly from rest reaches a speed of 50 km/h in 5 seconds. In that time the car will have travelled:

A $\frac{625}{18}$ metres

B 125 metres

C $\frac{625}{9}$ metres

D 1. 25 kilometres

E 34.72 kilometres

25 A particle is moving along Ox so that, at time t, $x = 5 \sin (2t)$. The acceleration of the particle when $t = \frac{\pi}{4}$ is:

A –20

B –10

C 0

D 10

E 20

26 A particle moves in a straight line. At time t, $t \geq 0$, its displacement x to the right of a fixed point O on the line is given by $x = 9t^2 - t^3$. The interval of time for which the particle is moving to the right is:

A $(0, 6)$

B $(6, \infty)$

C $(-\infty, 0)$

D $(-\infty, 6)$

E $(0, 9)$

27 A particle moves along a straight line such that at time t seconds its position in metres relative to a fixed point O on the line is given by $x(t) = 5t^2 - 4$. The velocity (in m/s) when $t = 2$ is:

A 8

B 10

C 20

D 6

E –10

28 The displacement x from the origin of a particle travelling in a straight line is given by $x = 2t^3 - 10t^2 - 44t + 112$. The average speed (in m/s) of the particle during the first 4 seconds is:

A –76

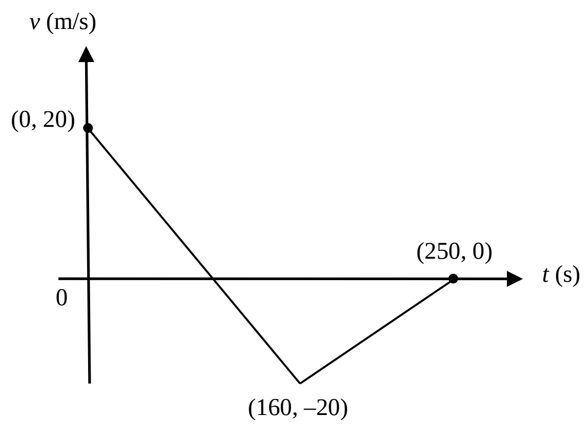
B –24

C 4

D 52

E 76

29 The velocity–time graph shown describes the motion of a particle.



The acceleration of the particle (in m/s^2) during the first 160 seconds is:

A –40

B –0.25

C 0

D 0.25

E 40

30 The displacement x metres from the origin of a particle travelling in a straight line is given by $x = 2 - 2\cos\left(\frac{3\pi}{4}t - \frac{\pi}{2}\right)$. The maximum displacement of the particle (in m) is:

A –4
B –2
C 0
D 2
E 4

31 A particle moves in a straight line so that its position x cm from a fixed point O at time t seconds ($t \geq 0$) is given by $x = t^3 - 9t^2 + 24t - 1$. The particle's initial position (in cm) is:

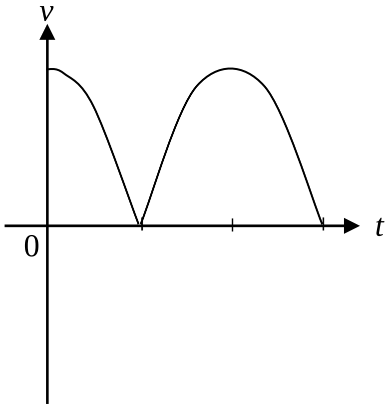
A 0
B 5
C 1
D –1
E 2

32 A particle moves in a straight line so that its position x cm from a fixed point O at time t seconds ($t \geq 0$) is given by $x = t^3 - 9t^2 + 24t - 1$. The particle's initial velocity (in cm/s) is:

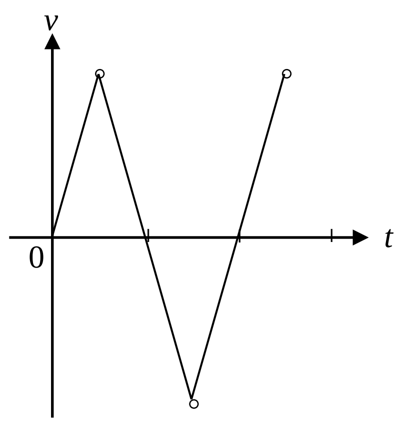
A 0
B 24
C 1
D –1
E 9

33 A ball is dropped vertically, hits the ground and bounces vertically upwards to its original height. It continues bouncing, returning to its original height after each bounce. With positive velocity indicating downwards motion, the velocity–time graph that best represents the ball's motion from when it is dropped until it hits the ground for the second time is:

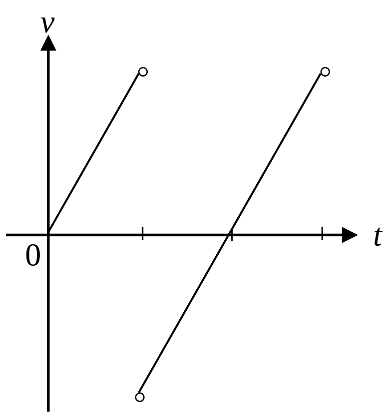
A



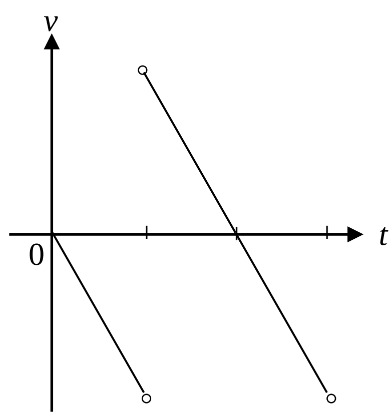
B



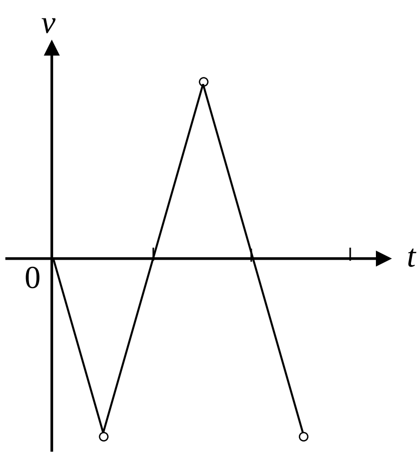
C



D



E



34 A body is projected up from the ground with a velocity of 30 m/s. Its acceleration due to gravity is -10 m/s^2. The maximum height (in m) reached by the body is:

A 90

B 30

C 6

D 45

E 3

35 A particle moves in a straight line. At time t seconds its displacement from a fixed origin is x metres and its velocity is v m/s. Given that $v = \sqrt{16x - 2x^2}$, the acceleration of the particle in m/s^2 when $x = 2$ is:

A 0

B 2

C 4

D 6

E 8

36 The displacement x from the origin of a particle travelling in a straight line is given by $x = 2t^3 - 10t^2 - 44t + 112$. The acceleration (in m/s^2) at time $t = 3$ seconds is:

A -24

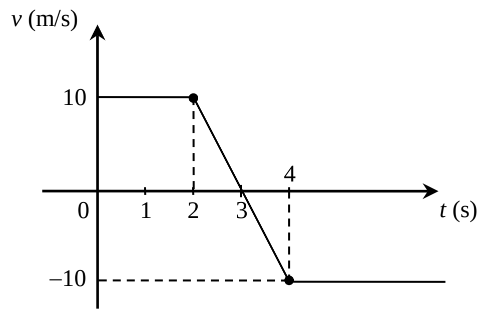
B -16

C 0

D 16

E 24

37 A particle moves with velocity v m/s as indicated in the velocity–time graph.



The distance, in metres, travelled by the particle in the first 4 seconds is:

A 10

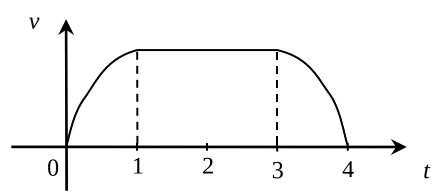
B 15

C 20

D 25

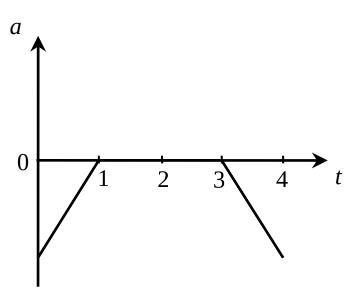
E 30

38 The following is the velocity–time graph of a racing car over a short course.

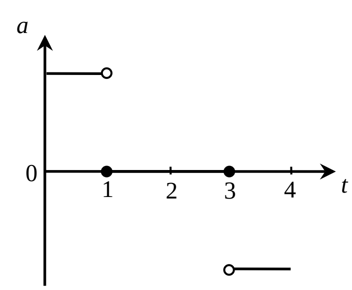


Which one of the following could be the acceleration–time graph of the car's motion?

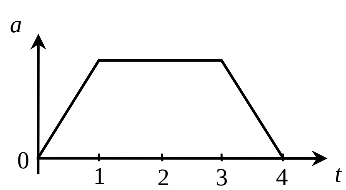
A



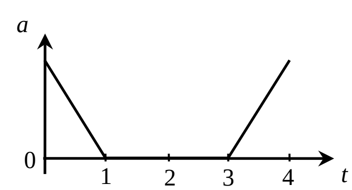
B

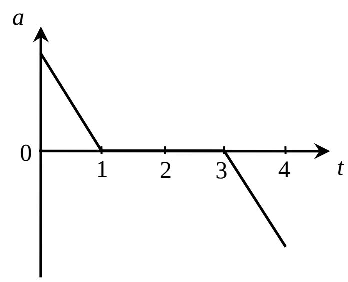


C

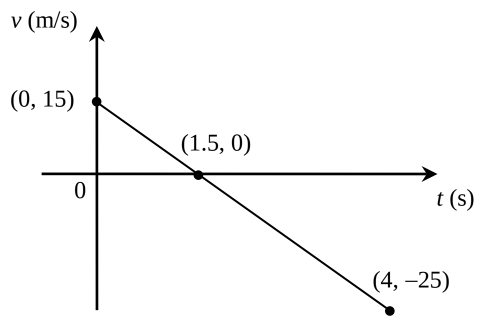


D



E 

39 This velocity–time graph represents the motion of a ball that is thrown vertically upwards from a high balcony and then falls to the ground below, landing at $t = 4$. The air resistance is negligible and acceleration due to gravity is 10 m/s^2.



The height in metres of the balcony above the ground is:

A 11.25

B 15

C 20

D 25

E 31.25

40 A horse is hauling a heavy sled of mass 400 kg across a uniformly rough horizontal surface. The horse exerts a horizontal force of 650 newtons on the sled while the resistance to the sled's motion is 500 newtons. If the sled is initially at rest, then the velocity of the sled after 3 seconds is:

A 1 m/s

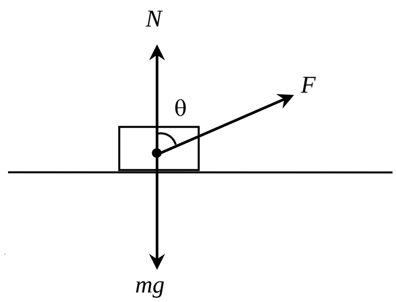
B 5 m/s

C $\frac{9}{8}$ m/s

D $\frac{5}{4}$ m/s

E 2 m/s

41 A body of mass m kg is being pulled along a smooth horizontal table by means of a string inclined at angle θ to the vertical. The diagram below indicates the forces acting on the body.



Which one of the following statements is true?

A $N - mg = 0$

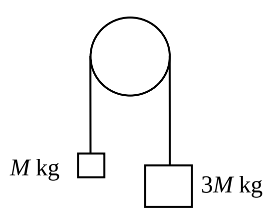
B $N + F \sin \theta - mg = 0$

C $N - F \sin \theta - mg = 0$

D $N + F \cos \theta - mg = 0$

E $N - F \cos \theta - mg = 0$

42 The diagram shows a pulley with masses of M kg and $3M$ kg attached to each end of a string.



The acceleration of the $3M$ kg mass is

A $\frac{3g}{4}$ m/s^2

B $\frac{4g}{3}$ m/s^2

C g m/s^2

D $2g$ m/s^2

E $\frac{g}{2}$ m/s^2

43 An object of mass 10 kg initially at rest is acted on by a constant force of 50 newtons.

The distance, in metres, that the object travels during the first 10 seconds of its motion is

A 25

B 50

C 100

D 250

E 2500

44 Using Euler's formula, with a step size of 0.1, for the differential equation

$\frac{dy}{dx} = \frac{1}{\log_e x}$, given that $y_0 = y(2) = 1$ then y_2 is closest to

A 1.279

B 1.144

C 1.342

D 1.183

E 1.243

45 Given the differential equation $\frac{dy}{dx} = \frac{x}{1+x^2}$, with $y_0 = y(1) = 2$ then, using Euler's formula with $h = 0.2$, the value of y_3 is closest to

A 2.1

B 2.198

C 2.293

D 2.383

E 2.468

46 If $f'(x) = \dfrac{1}{x+1}$ and $f(1) = 4$ then an approximate value of $f(1.3)$, using Euler's method with a step size of 0.1, is closest to

A 4.05

B 4.097

C 4.143

D 4.187

E 4.228

47 The solution of the differential equation $\dfrac{dy}{dx} = \dfrac{-y}{x}$ if $y = 4$ when $x = 1$, is equal to

A $y = \dfrac{4}{x^2}$

B $x^2 + y^2 = 17$

C $y^2 - x^2 = 15$

D $y = \dfrac{4}{x}$

E $y = \log_e(x+3)$

48 If $\dfrac{dy}{dx} = \dfrac{2-y}{2x}$ and at $x = 3, y = 1$ then the value of x at $y = 0$ is

A 12

B $\dfrac{3}{4}$

C $\dfrac{4}{3}$

D $\dfrac{1}{12}$

E 2

49 The differential equation $x^2\dfrac{dy}{dx} = y^3$, where $y(1) = 2$ has the solution

A $y = 2\sqrt{x}$

B $y = -2\sqrt{\dfrac{x}{8-7x}}$

C $y = 2\sqrt{\dfrac{x}{8-7x}}$

D $y = 2\sqrt{2x^2 - x}$

E $y = \sqrt{\dfrac{4x}{7x-8}}$

50 Given that a toy railway track is modelled by $\{(x, y): x = 10\sin t,\ y = 4\sin(2t),\ 0 \le t \le 2\pi\}$, where all lengths are in metres. The length of the track, in metres, is closest to

A 50

B 55

C 60

D 65

E 70

51 The length of the curve given by $y = 1 - x^2, y \ge 0$ is given by

A $\int_{-1}^{1}(1+4x^2)\,dx$

B $\int_{-1}^{1}(1+2x)\,dx$

C $\int_{0}^{1}\sqrt{1+4x^2}\,dx$

D $2\int_{0}^{1}\sqrt{1+4x^2}\,dx$

E $\int_{-1}^{1}\sqrt{1-4x^2}\,dx$

52 Which one of the following curves is the longest?

A $\{(x, y): x = \cos t, y = \sin t, 0 \le t \le \frac{\pi}{2}\}$

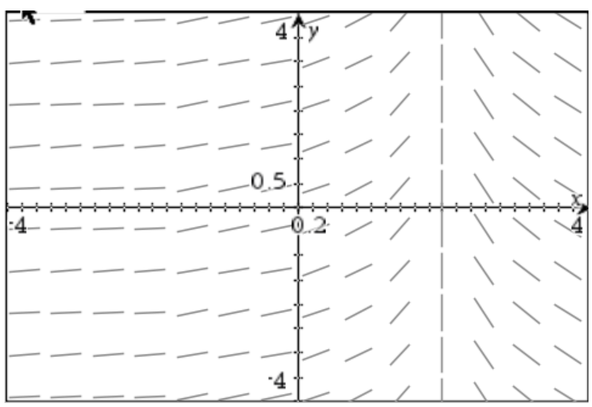
B $\{(x, y): x = 2\cos t, y = 2\sin t, 0 \le t \le \frac{\pi}{2}\}$

C $\{(x, y): x = 2\cos t, y = \sin t, 0 \le t \le \frac{\pi}{2}\}$

D $\{(x, y): x = \sin t, y = \cos t, 0 \le t \le \frac{\pi}{2}\}$

E $\{(x, y): x = \sin t, y = 2\cos t, 0 \le t \le \frac{\pi}{2}\}$

53



The differential equation represented by this slope field graph is

A $\frac{dy}{dx} = \frac{1}{x-2}$

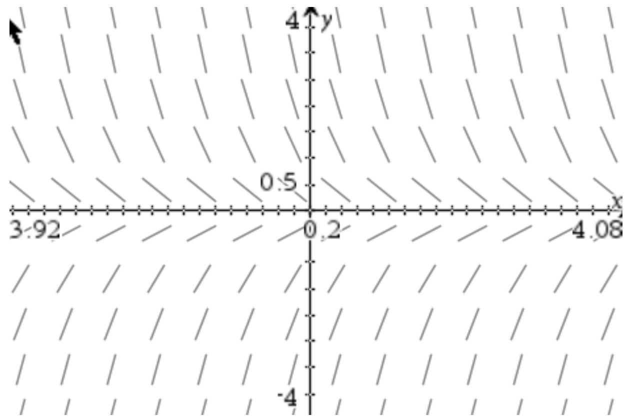
B $\frac{dy}{dx} = \frac{1}{x+2}$

C $\frac{dy}{dx} = \frac{1}{4-x}$

D $\frac{dy}{dx} = \frac{1}{2x-2}$

E $\frac{dy}{dx} = \frac{1}{2-x}$

54



The differential equation represented by this slope field graph could be

A $\dfrac{dy}{dx} = \dfrac{1}{y}$

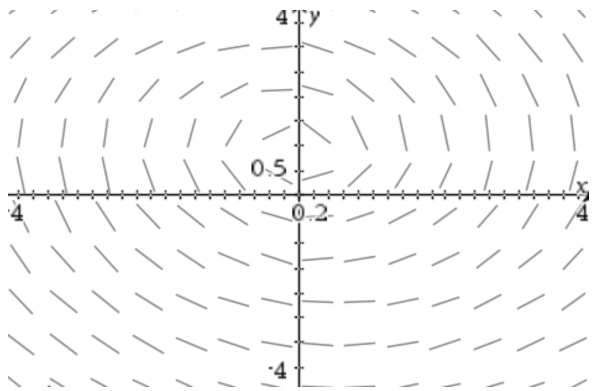
B $\dfrac{dy}{dx} = \dfrac{1}{x}$

C $\dfrac{dy}{dx} = \dfrac{1}{y^2}$

D $\dfrac{dy}{dx} = -y$

E $\dfrac{dy}{dx} = -y^2$

55



The differential equation represented by this slope field graph could be

A $\dfrac{dy}{dx} = \dfrac{x}{y-1}$

B $\dfrac{dy}{dx} = \dfrac{-4x}{y-1}$

C $\dfrac{dy}{dx} = \dfrac{y}{x}$

D $\dfrac{dy}{dx} = \dfrac{-y}{x}$

E $\dfrac{dy}{dx} = \dfrac{4x}{y+1}$

Short-answer questions (technology-free)

1 A sequence has $t_5 = 32$ and $t_{10} = 243$. Find expressions for t_n and S_n if the sequence is:

a arithmetic

b geometric.

2 Solve for x the equation $2 \sin (2x) \cos (2x) = \cos (2x)$ for $x \in [0, \pi]$.

3 **a** On the same set of axes, sketch the graphs of $f : [0, 2\pi] \to \mathbb{R}, f(x) = \sin x$ and $g: [0, 2\pi] \to \mathbb{R}, g(x) = \cos x$.

b Find the coordinates of the points of intersection of the graphs of $y = f(x)$ and $y = g(x)$.

c Hence find $\{x: \sin x < \cos x, 0 \le x \le 2\pi\}$.

4 **a** For a geometric sequence, $t_1 = \dfrac{\sqrt{32}}{3}$, $t_2 = \sqrt{8}$ and $t_3 = \sqrt{18}$, prove that

$t_6 = \dfrac{81\sqrt{32}}{32}$.

b An arithmetic sequence has $t_2 = \sqrt{8}$ and $t_3 = \sqrt{18}$. Prove that $S_{20} = \sqrt{88200}$.

5 For a triangle ABC with side lengths and angles labelled in the conventional manner,

$a = 6$, $b = 8$ and $\sin A = \dfrac{2}{3}$. Find $\sin B$.

6 If a triangle has sides of length a, b and c and $a^2 + b^2 = c^2$ prove that the triangle has a right angle.

7 Give a vector of magnitude 4 in the direction of vector $\boldsymbol{i} - 2\boldsymbol{j} + 5\boldsymbol{k}$.

8 Find the shortest distance between the point with coordinates (–2, –2, 4) and the line passing through the points with coordinates $B(2, 4 , 6)$ and $C(4, 1, 2)$.

9 Are the vectors $\boldsymbol{a} = 2\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}$, $\boldsymbol{b} = \boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{c} = -2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$ linearly independent? Prove your result.

10 OAB is an isosceles triangle with $OA = OB$. M is the midpoint of AB. Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. Use a vector proof to show that OM is perpendicular to AB.

11 Let A be the point (1, 2, 1) and let B be the point (4, 2, –1).

a Find the point on OB which is closest to A.

b What is the shortest distance between A and OB?

12 For vectors $\boldsymbol{a} = \boldsymbol{i} + 3\boldsymbol{j}$ and $\boldsymbol{b} = 2\boldsymbol{i} - \boldsymbol{j}$ describe, through a Cartesian equation, the set of points with position vector $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j}$ such that:

a $|\boldsymbol{r} - \boldsymbol{a}| = |\boldsymbol{r} - \boldsymbol{b}|$

b $\boldsymbol{r} \,.\, (\boldsymbol{r} - \boldsymbol{a}) = 0$

13 Solve the equation $\operatorname{cosec}(2x) = -\sqrt{2}$, for $x \in [0, 2\pi]$.

14 Find the exact value of:

a $\sin\left(\tan-1\left(\frac{3}{4}\right)\right)$

b $\cos\left(\tan-1\left(\frac{5}{12}\right)\right)$.

15 Sketch the graph of $y = \sec\left(2x - \frac{\pi}{3}\right)$, for $x \in [-\pi, \pi]$.

16 Sketch the graph of $y = \cos^{-1}(x + 4)$.

17 **a** Given that $\sin(\theta + \alpha) = \lambda \sin(\theta - \alpha)$ show that $\tan\theta = \frac{(\lambda + 1)\tan\alpha}{\lambda - 1}$.

b If $\lambda = 2$ and $\alpha = \frac{\pi}{3}$, solve the equation $\sin(\theta + \alpha) = \lambda \sin(\theta - \alpha)$ for θ where $-2\pi \le \theta \le 2\pi$.

18 If $\sin A = \frac{12}{13}$, $\frac{\pi}{2} < A < \pi$, and $\cos B = \frac{-4}{5}$, $\pi < B < \frac{3\pi}{2}$, find the exact value of $\cos(A - B)$.

19 For each of the following, find $\text{Arg}(z_1 z_2)$ and $\text{Arg}(z_1) + \text{Arg}(z_2)$.

a $z_1 = \text{cis}\left(\frac{\pi}{4}\right)$ and $z_2 = \text{cis}\left(\frac{\pi}{3}\right)$

b $z_1 = \text{cis}\left(\frac{-2\pi}{3}\right)$ and $z_2 = \text{cis}\left(\frac{-3\pi}{4}\right)$

c $z_1 = \text{cis}\left(\frac{2\pi}{3}\right)$ and $z_2 = \text{cis}\left(\frac{\pi}{2}\right)$

20 For the transformation $z \to z + 2$, sketch the image of each of the following sets of points on an Argand diagram.

a $|z| = 3$

b $\text{Arg}(z) = \frac{\pi}{3}$

c $\frac{-\pi}{3} \le \text{Arg}(z) \le \frac{\pi}{3}$

d $|z - (1 + i)| = |z - 2|$

21 **a** If $0 < \text{Arg}(z) < \dfrac{\pi}{2}$, show that $\text{Arg}(1 - z) = -\pi + \text{Arg}(z - 1)$.

b If $\dfrac{-\pi}{2} < \text{Arg}(z) < 0$, show that $\text{Arg}(1 - z) = \text{Arg}(z - 1) + \pi$.

22 Find the locus defined by $\arg(z + i) - \arg(z + 1) = \dfrac{\pi}{2}$.

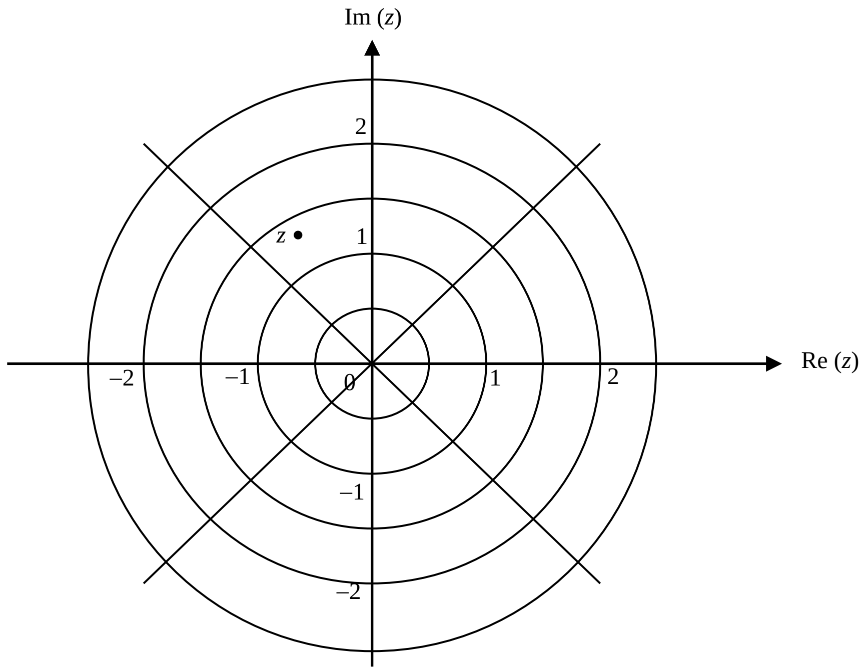
23 Shade the region of the complex plane defined by $\{z : |z - 1 + i| \geq 4\}$.

24 For the equation $P(z) = z^3 + (3 - 2i)z^2 + z + 3 - 2i$:

a show that $-3 + 2i$ is a solution of the equation $P(z) = 0$

b find all the solutions of the equation $P(z) = 0$.

25 The complex number $z = \sqrt{2} \text{ cis } \theta$ is shown on the Argand diagram below. Plot and label the complex numbers u, v and w on the same diagram, where $u = z^2$, $v = \dfrac{1}{z}$ and $w = z^2 + \dfrac{1}{z}$.



26 Shade the region of the complex plane defined by $\{z: iz - i\bar{z} < 3\}$.

27 Find the equation of the tangent to the ellipse with equation $\frac{x^2}{4} + y^2 = 1$ at the point(s) at which:

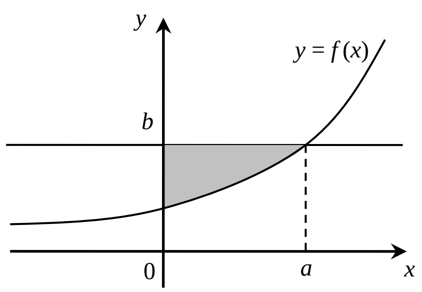
a $x = 2$

b $x = 0$

c $x = 1$.

28 If $f(x) = \log_e(\sin x)$, find $f''(x)$.

29 The shaded region is rotated around the x-axis to form a solid of revolution. Find an expression for the volume of the resultant solid.



30 **a** Show that $\frac{d}{dx}(\sin^{-1}(\sqrt{2x})) = \frac{\sqrt{2}}{2\sqrt{x-2x^2}}$.

b Hence find the exact value of $\int_{0.25}^{0.5} \frac{1}{\sqrt{x-2x^2}}\,dx$.

31 Find the area of the region bounded by the two curves $y = 4x^2 + 2x$ and $y = -2x^2 + x + 1$.

32 Verify that $y = ae^{kx^2}$ is a solution to the differential equation $x\frac{d^2y}{dx^2} - (2kx^2 + 1)\frac{dy}{dx} = 0$.

33 Solve the differential equation $(x+1)^2 \dfrac{dy}{dx} = 1$ where $y = 2$ when $x = 0$ and $y = 1$ when $x = -2$.

34 Find the values of a and b if $y = a\cos(2x) + b\sin(2x)$ satisfies the differential equation $\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} + 2y = \cos(2x) + \sin(2x)$.

35 Solve the differential equation $f'(x) = \dfrac{3x}{\sqrt{x^2+1}}$, given that $f(0) = 2$.

36 A tank contains 50 litres of a salt solution which contains 40 grams of dissolved salt. Pure water runs into the tank at the rate of 1.5 litres/minute and the mixture is kept uniform by stirring. The mixture then runs out at the same rate as the water runs in. If m grams of salt remain after t minutes, express:

a $\dfrac{dm}{dt}$ in terms of m

b m in terms of t.

37 Solve the differential equation $\dfrac{dy}{dx} = \dfrac{2}{1-x^2}$ given that $y = 0$ when $x = 0$. Specify the domain for which this information implies a unique solution.

38 Sand is poured into a conical heap so that the radius length r cm is always $\dfrac{3}{4}$ of the height h cm. The volume of sand in the heap is V cm^3 at time t minutes.

a Express V in terms of h.

b If the height is increasing at the rate of 2 cm/min, express $\dfrac{dV}{dt}$ in terms of h.

39 The velocity, v m/s, of a particle moving in a line is given by $v = e^{\frac{-x}{2}} + 4, t \geq 0$, where x metres is the position of the particle at time t seconds.

a Find the acceleration of the particle in terms of x.

b Find, correct to one decimal place, the time it takes for the particle to travel 20 metres.

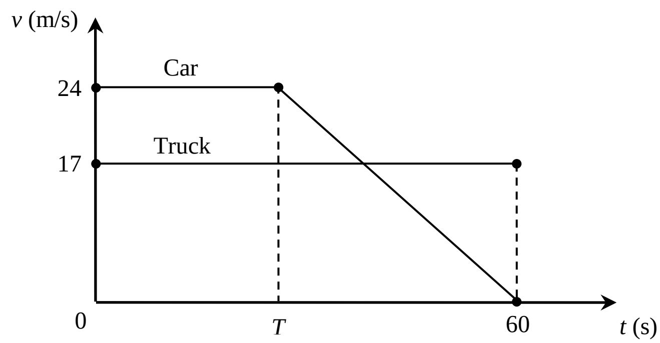
40 An object is thrown vertically upwards from the top of a building, 50 metres above ground level. The object takes 10 seconds to reach the ground. Use $g = 9.8$ m/s^2.

a Find the initial speed.

b Find the maximum height reached, correct to one decimal place.

41 A particle moves in a line. At time t seconds, $t \geq 0$, its displacement from a fixed origin O is x metres and its acceleration, a m/s^2, is given by $a = 2t - \cos t$. If the particle starts at the point where $x = 3$, with a velocity of 2 m/s towards O, express x in terms of t.

42 A car travelling at 24 m/s overtakes a truck travelling at a constant speed of 17 m/s along a straight road. T seconds later, the car decelerates uniformly to rest. The truck continues at constant speed and it passes the car at the instant the car comes to a stop. This is exactly 60 seconds after the car had passed the truck.



The velocity–time graph representing this situation is shown above. Find T.

43 A particle is moving in a line so that its displacement, x m, from a fixed origin O, at time, t seconds, is given by $x = \cos(2t) + 4\cos t$, $0 \le t \le 2\pi$. If v m/s is the velocity and a m/s^2 is the acceleration at time t, find at what time(s) the particle:

a is at rest

b has zero acceleration.

44 Find the Cartesian equation for the graph represented by the vector equation

$\boldsymbol{r}(t) = \sec(t)\,\boldsymbol{i} + (1 + \tan(t))\boldsymbol{j},\ t \in \left[0, \frac{\pi}{2}\right)$.

45 The following vector equations each represent the position of a particle at time t, $t \ge 0$.

For each equation:

i find the corresponding Cartesian equation stating domain and range

ii sketch the path of the particle indicating the initial position and the initial direction of motion.

a $\boldsymbol{r}(t) = \cos\left(t + \frac{\pi}{4}\right)\boldsymbol{i} + \sin\left(t + \frac{\pi}{4}\right)\boldsymbol{j}$

b $\boldsymbol{r}(t) = (3 - t)\boldsymbol{i} + (t^2 + 2t)\boldsymbol{j}$

c $\boldsymbol{r}(t) = \tan(t)\,\boldsymbol{i} + \sec(t)\,\boldsymbol{j},\ t \in \left[0, \frac{\pi}{2}\right)$

46 For each of the following vector equations:

i find the corresponding Cartesian equation stating domain and range

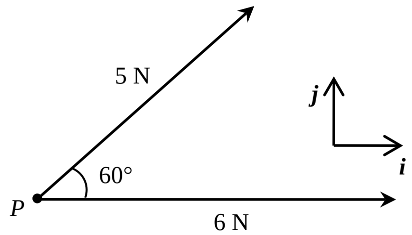
ii sketch the relation.

a $\boldsymbol{r}(t) = (3 - t)\boldsymbol{i} + 4(t + 1)\boldsymbol{j},\ t \in \mathbb{R}$

b $\boldsymbol{r}(t) = \cos(t)\,\boldsymbol{i} + (1 - \sin(t))\,\boldsymbol{j},\ t \in \mathbb{R}$

c $\boldsymbol{r}(t) = \sin^2\left(\frac{\pi t}{2}\right)\boldsymbol{i} + 2\cos^2\left(\frac{\pi t}{2}\right)\boldsymbol{j},\ t \in [0, \infty)$

47 Forces of 5 N and 6 N act at a point P as shown in the diagram below. The 6 N force acts in the direction of the unit vector $\boldsymbol{i}$.



a If the resultant force $\boldsymbol{F} = a\boldsymbol{i} + b\boldsymbol{j}$, state the values of a and b.

b Find $|\boldsymbol{F}|$, correct to two decimal places.

48 An empty lift of mass 8 tonnes, suspended by a cable, is descending.

a Draw a diagram showing the forces on the lift.

b If the lift is accelerating at 0.8 m/s^2 find the magnitude of the force, in newtons, exerted by the cable on the lift (using g = 9.8 m/s^2).

49 Forces $\boldsymbol{F}_1$, $\boldsymbol{F}_2$ and $\boldsymbol{F}_3$ act on a body of mass 4 kg. If the body is in equilibrium, and $\boldsymbol{F}_1 = 2\boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{F}_2 = 5\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k}$, find $\boldsymbol{F}_3$.

50 A block of mass 50 kg rests on a rough horizontal surface. The coefficient of friction between the surface and the block is 0.1. A horizontal force of F newtons is applied to the block. (Assume the acceleration due to gravity has magnitude 9.8 m/s^2.)

a Show that, if F = 49, the block will not move.

b Find the acceleration of the block if F = 100.

51 For the differential equation $\dfrac{dy}{dx} = \dfrac{8x^3}{3y^2}$, with $y(0) = -1$ find the value of y when $x = 1$

52 For the differential equation $y' = \dfrac{2x-4}{e^y}$, given that $y(-1) = 0$ solve, expressing y in terms of x, and sketch the graph of y against x.

53 For the differential equation $\frac{dr}{d\theta} = \frac{r^2}{\theta}$, given that $r(1) = 2$, find the value of r at $\theta = e^{\frac{-1}{2}}$.

54 Using Euler's method, find the approximate y coordinate of the point where $x = 1$ on the curve which is the solution of the differential equation $\frac{dy}{dx} = 2x - 1$ given that the point (0, 2) is on the solution curve. Use a step size of 0.2.

55 For the differential equation $\frac{dy}{dx} = -3x^2$, with $y_0 = 2$ when $x_0 = -1$ find y_3 using Euler's formula with a step size of 0.1.
Solve the differential equation to find the value of y when $x = -0.6$.

56 For the differential equation $\frac{dy}{dx} = \frac{2}{x^2}$, with $y_0 = y(1) = \frac{1}{2}$,
Use Euler's method to find y_2, in fraction form, using $h = \frac{1}{5}$.
Solve the differential equation, hence find the value of y approximated by y_2, giving your answer in fraction form.

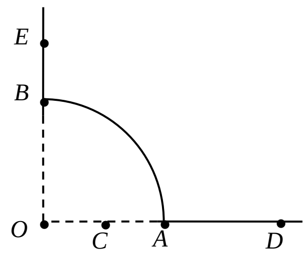
57 For the curve given by $\{(x, y) : x = 2\sin(2t),\ y = -2\cos(2t),\ \frac{\pi}{6} \le t \le \frac{\pi}{2}\}$, find the length of this curve, using integration.

58 **a** Given that $f(x) = \log_e\left(\frac{\sin x}{1 + \cos x}\right)$, find *f'(x)*.

b Hence find the length of the curve given by $y = \log_e(\sin x), \frac{\pi}{3} \le x \le \frac{\pi}{2}$.

Extended-response questions

1



The diagram shows three sections of a road. One section from A through D runs east–west so that D is east of A. Another section from B through E runs north–south so that E is north of B. The third section connecting A and B is a quarter of a circle, centred at O, with O located 4 km due south of B and 4 km due west of A.

The region is flat and covered in scrub. A hiker starts at C, which is halfway between O and A. The hiker can hike along the road at 8 km/h and off-road at 2 km/h.

a Find the time taken by the hiker, to the nearest minute, in each of the following cases. He walks:

i directly from C to B

ii due east from C to A, then along the road to B

iii due north from C to the road, to a point Q, then along the road to B

iv north-east from C to the road, to a point Y, then along the road to B

v from C to F on the road, where F is north-east of O, then along the road to B.

b **i** If the hiker walks from C to X on the road, where X is on a bearing of $\theta°$ from O, then along the road to B, find, in terms of θ, an expression for the total time, t hours, taken by the hiker.

ii Hence find θ, correct to one decimal place, if the hiker wishes to complete the walk in the shortest possible time, and determine the shortest possible time to the nearest minute.

2 ΔABC has $A = (2, 1)$, $B = (-1, 2)$ and $C = (0, 5)$. Find:

a the lengths of the sides of the triangle

b the equation of the line l_1, through B and perpendicular to AC

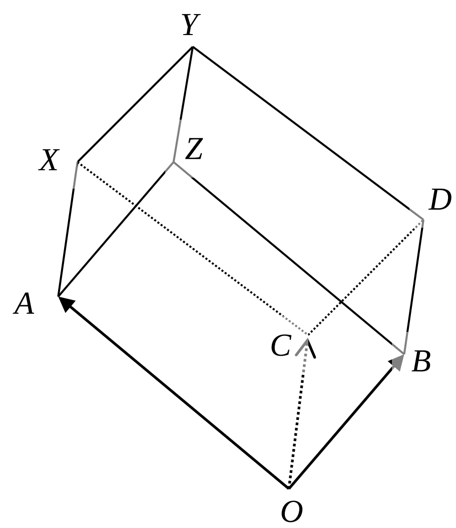
c the equation of the line l_2, the perpendicular bisector of AB

d the coordinates of P, the point of intersection of l_1 and l_2

e the equation of the circumcircle of ΔABC.

3 A cuboid is positioned on level ground so that it rests on one of its vertices, O.

$\overrightarrow{OA} = 3\boldsymbol{i} - 4\boldsymbol{j} - \boldsymbol{k}$, $\overrightarrow{OB} = \boldsymbol{i} + 2w\boldsymbol{j} - 5\boldsymbol{k}$, $\overrightarrow{OC} = x\boldsymbol{i} + y\boldsymbol{j} + 5\boldsymbol{k}$.



a **i** Find $\overrightarrow{OA} \bullet \overrightarrow{OB}$ in terms of w.

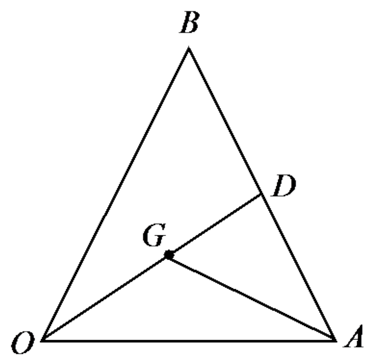
ii Hence find the value of w.

b **i** Use the fact that OA is perpendicular to OC to write an equation relating x and y.

ii Find another equation relating x and y and hence find the values of x and y.

c Hence find the exact volume of this cuboid.

4 In the figure OAB is a triangle with D the midpoint of AB. Let $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$. G is a point on OD such that $\overrightarrow{OG} = \frac{2}{3}\overrightarrow{OD}$.



a Find $\overrightarrow{OG}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

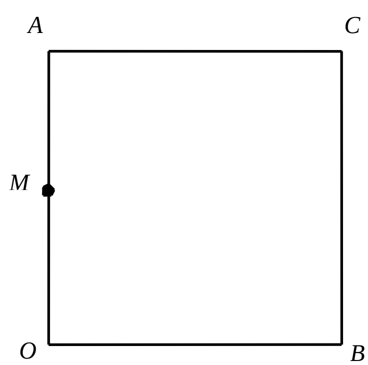
b Find $\overrightarrow{GA}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

c Find $\overrightarrow{GA} \bullet \overrightarrow{OG}$.

d **i** If GA is perpendicular to OG show that angle BOA has magnitude θ where $\cos\theta = \dfrac{|\boldsymbol{b}|^2 - 2|\boldsymbol{a}|^2}{|\boldsymbol{a}|\,|\boldsymbol{b}|}$

ii If $|\boldsymbol{b}| = \sqrt{3}\,|\boldsymbol{a}|$, give the magnitude of angle BOA correct to two decimal places.

5 $OACB$ is a square with $\overrightarrow{OA} = a\boldsymbol{j}$ and $\overrightarrow{OB} = a\boldsymbol{i}$. M is the midpoint of OA.



a Find, in terms of a:

i $\overrightarrow{OM}$

ii $\overrightarrow{MC}$.

b P is a point on MC such that $\overrightarrow{MP} = \lambda\overrightarrow{MC}$. Find $\overrightarrow{MP}$, $\overrightarrow{BP}$ and $\overrightarrow{OP}$ in terms of λ and a.

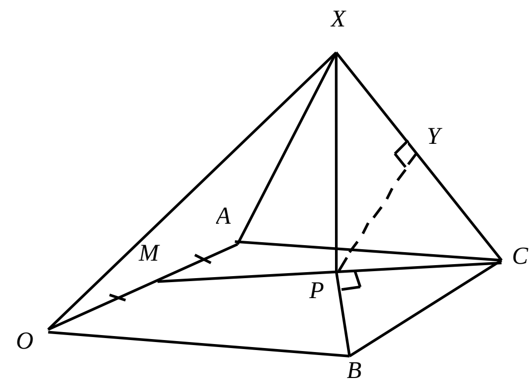
c If BP is perpendicular to MC:

i find the value of λ and also find $|\overrightarrow{BP}|$, $|\overrightarrow{OP}|$ and $|\overrightarrow{OB}|$. Comment.

ii if $\theta = \angle PBO$, evaluate $\cos\theta$.

d If $|\overrightarrow{OP}| = |\overrightarrow{OB}|$ find the possible values of λ and illustrate these two cases carefully.

In the diagram, $\overrightarrow{OA} = a\boldsymbol{j}$, $\overrightarrow{OB} = a\boldsymbol{i}$ and BP is perpendicular to MC where M is the midpoint of OA. $\overrightarrow{PX} = a\boldsymbol{k}$. Y is a point on XC such that PY is perpendicular to XC.



e Find $\overrightarrow{OY}$.

6 Let $P(z) = -z^3 - z^2 + 2z - 12$, $z \in C$.

a **i** Find $P(u)$, where $u = 1 - \sqrt{3}\,i$.

ii What can be deduced about u?

b **i** Find all the roots of the equation $P(z) = 0$, expressing your answers in Cartesian form.

ii Plot the roots on an Argand diagram.

c Express u in polar form, and hence find Arg (iu).

7 Let $u = -4\sqrt{2} - 4\sqrt{2}\,i$ and $v = 2\text{ cis}\left(\frac{-\pi}{4}\right)$.

a Express u in exact polar form.

b Show that one of the cube roots of u is v.

c Find the remaining two cube roots of u in exact polar form.

d Express v in exact Cartesian form.

e Plot the three cube roots of u on an Argand diagram.

f Show that the equation $z^3 - 3\sqrt{2}\,z^2 i - 6z = -4\sqrt{2} - 6\sqrt{2}\,i$ can be expressed in the form $(z - w)^3 = -4\sqrt{2} - 4\sqrt{2}\,i$ where $w \in C$.

g Hence find one root of the equation $z^3 - 3\sqrt{2}\,z^2 i - 6z = -4\sqrt{2} - 6\sqrt{2}\,i$ in exact Cartesian form.

8 **a** Plot the complex numbers $u = 8 - 6i$ and $v = -1 - 7i$ on an Argand diagram.

b Verify that u is a member of the subset S, where

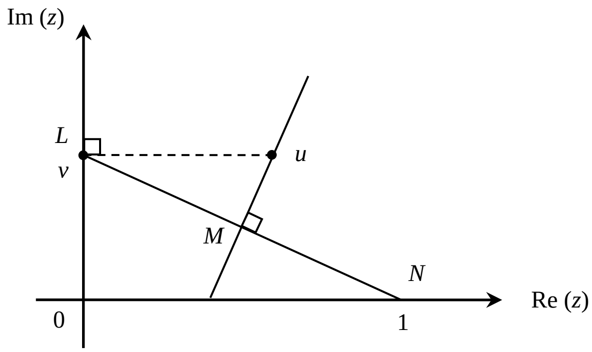
$S = \{z: |z| = 10, z \in C\}$.

c Sketch S on the Argand diagram in part **a**.

d Let w be such that $w + i\bar{v} = \bar{v}$. Find w in Cartesian form.

e Sketch $T = \{z: |z| \leq 10\} \cap \{z: |z - w| = |z - u|\}$ on the Argand diagram in part **a**.

9 In the Argand diagram shown, M is the midpoint of LN. Point L lies at vi.



a If $|z - 1| = |z - vi|$, show that the locus of z is given by the relation $2vy = 2x + v^2 - 1$ where $z = x + iy$.

b Show that $u = \dfrac{v^2 + 1}{2} + vi$.

c As v moves along the positive Im (z)-axis, u moves along a curve. Find the Cartesian equation of this curve.

10 Let $u = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$.

a Express u in polar form, where $-\pi < \text{Arg } u \le \pi$.

b Using an Argand diagram, show that $\text{Arg }(u + 1) = \frac{\pi}{8}$.

c **i** Express $u + 1$ in Cartesian form, and in polar form.

ii Hence show that the exact value of $\sin\left(\frac{\pi}{8}\right) = \frac{\sqrt{2-\sqrt{2}}}{2}$.

d Using an appropriate compound angle formula, verify your exact value for $\sin\left(\frac{\pi}{8}\right)$.

11 **a** For $g(x) = \frac{1}{f(x)}$, find the rule for $g''(x)$ in terms of $f'(x)$, $f''(x)$ and $f(x)$.

b Show that if there is a point of inflection at $(a, g(a))$ then $[f'(a)]^2 = \frac{1}{2}f''(a)f(a)$.

Consider $f(x) = x^2 - 2bx + 16$.

c Given that Δ is the discriminant of $f(x)$, find the values of b for which:

i $\Delta = 0$

ii $\Delta < 0$

iii $\Delta > 0$.

d Find the coordinates of stationary points and points of inflection for the graph of $y = \frac{1}{f(x)}$ in terms of b when the discriminant of $f(x) < 0$.

e Sketch the graph of $y = \frac{1}{f(x)}$ when the discriminant of $f(x) = 0$.

f Find an antiderivative for $\frac{1}{f(x)}$ in each of the cases discussed in part **c**.

12 **a** For $y = \tan x + \frac{1}{3}\tan^3 x$, find $\frac{dy}{dx}$.

b Solve the equation $\tan x + \frac{1}{3}\tan^3 x = 0$ for $x \in \left[0, \frac{\pi}{2}\right]$.

c Find the equation of the tangent to the curve with equation $y = \tan x + \frac{1}{3}\tan^3 x$ at the point where $x = \frac{\pi}{4}$.

d Find the area between the curve with equation $y = \tan x + \frac{1}{3}\tan^3 x$, the axes and the line $x = \frac{\pi}{4}$.

e The region contained between the axes, the line $x = \frac{\pi}{4}$ and the curve $y = \sec^2 x$ is rotated around the x-axis to form a solid of revolution. Find the volume of this solid.

13 The volume, v litres, of oil in an irregularly shaped tank, when the oil depth is h metres, is given by $v = 8000h\tan^{-1} h$.

a **i** Find the exact volume of oil in the tank, in litres, when the oil depth is 1 metre.

ii Find the oil depth, correct to the nearest centimetre, when the volume is 10 000 litres.

The tank is initially empty. Oil is then poured into the tank at a constant rate of 2000 litres per minute.

b Find, in terms of h, an expression for the rate at which the oil depth is increasing, in metres per minute, when the depth is h metres.

c **i** Write a definite integral, the value of which gives the time it takes in minutes for the oil depth in the tank to reach $\sqrt{3}$ metres.

ii Show that the exact time taken for the oil depth to reach $\sqrt{3}$ metres is $\frac{4\pi}{\sqrt{3}}$ minutes.

14 In a small town of population 1000, the rate of infection of a type of influenza is modelled by the differential equation $\frac{dN}{dt} = kN(1000 - N)$ where N is the number of people infected after t days and k is an unknown constant.

a If the rate of infection is 100 people per day when the number already infected is 500, show that the differential equation can be expressed as

$$\frac{dt}{dN} = \frac{2500}{N(1000 - N)}.$$

b Express t in terms of N, given that initially 10 people are infected.

c How many full days will it take before the number of people infected first exceeds 750?

15 A helicopter is hovering 25 m above the ground and drops a package of food to people below. The acceleration a m/s^2 of this package is given by $a = 9.8 - 0.05v^2$, where v m/s is the vertical speed at time t s. If x metres is the distance fallen at time t s, find:

a the terminal velocity of the package

b the speed of the package when it hits the ground, in m/s, correct to one decimal place

c the time it takes the package to reach the ground, in seconds, correct to one decimal place.

16 After brakes are applied in a car, under the influence of ABS (anti-lock brakes), the car comes to rest, and two different models have been conjectured. In a controlled experiment the brakes are applied when the car is moving at 25 m/s.

a In the first model, the acceleration, in m/s^2, is given by $a = \frac{-52}{(t+1)^3}, t \geq 0,$ where t seconds is the time since the brakes were applied.

i Express v in terms of t, where v m/s is the speed at time t.

ii Find the time taken for the car to come to rest using this model.

b In the second model, $a = \frac{-(900 + v^2)}{60}$.

i Express t in terms of v for this situation.

ii Find the time taken for the car to come to rest using this second model.

c What model takes longer, and by how much?

17 A dragster racing car accelerates uniformly over a straight line course and completes a 'standing' (that is, starting from rest) 400 metres in 8 seconds.

a **i** Find the acceleration, in m/s^2, of the dragster over 400 metres.

ii Show that the dragster reaches a speed of 100 m/s at the end of the 400 metre course.

At the 400 metre mark, the dragster stops accelerating. At this instant, the dragster's brakes are applied and, in addition, a small parachute opens at the rear to slow the car down. Due to these factors the deceleration of the dragster during this stage of the motion is $\dfrac{5000+0.5v^2}{400}$ m/s^2.

b **i** Show that the differential equation relating v to x, where v m/s is the velocity of the dragster x metres beyond the 400 metre mark, is

$$\frac{dv}{dx}=\frac{-(10^4+v^2)}{800v}.$$

ii Hence find, to the nearest metre, the distance the dragster takes to stop from the instant the brakes are applied.

c Use calculus to find the time, in seconds, taken to bring the dragster to rest from the 400 metre mark.

Answers to Semester 1 additional exercises

Answers to multiple-choice questions

1	D
2	C
3	D
4	B
5	C
6	D
7	D
8	E
9	E
10	B
11	E
12	B
13	B
14	E
15	D
16	D
17	A
18	A
19	E
20	C
21	E
22	E
23	D
24	A
25	A
26	A
27	C
28	D
29	B
30	E
31	D
32	B
33	C
34	D
35	C
36	D
37	E
38	E
39	C
40	C
41	D
42	E
43	D
44	A
45	C
46	C
47	D
48	B
49	C
50	B
51	D
52	B
53	E
54	D
55	B

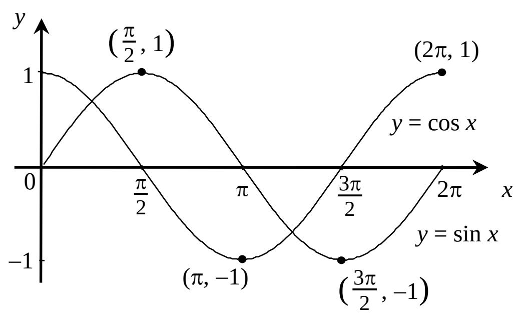
Answers to short-answer (technology-free) questions

1 **a** $\dfrac{211n}{5} - 179,\ \dfrac{211n^2}{10} - \dfrac{1579n}{10}$

b $3^{n-5} \times 2^{10-n},\ \dfrac{1024}{81}\left(\left(\dfrac{3}{2}\right)^n - 1\right)$

2 $\dfrac{\pi}{12}, \dfrac{\pi}{4}, \dfrac{5\pi}{12}, \dfrac{3\pi}{4}$

3 **a**



b $\left(\dfrac{\pi}{4}, \dfrac{\sqrt{2}}{2}\right), \left(\dfrac{5\pi}{4}, \dfrac{-\sqrt{2}}{2}\right)$

c $\left\{x : 0 \le x < \dfrac{\pi}{4}\right\} \cup \left\{x : \dfrac{5\pi}{4} < x \le 2\pi\right\}$

5 $\dfrac{8}{9}$

7 $\dfrac{4}{\sqrt{30}}(\boldsymbol{i} - 2\boldsymbol{j} + 5\boldsymbol{k})$

8 $10\sqrt{\dfrac{13}{29}}$

9 Yes

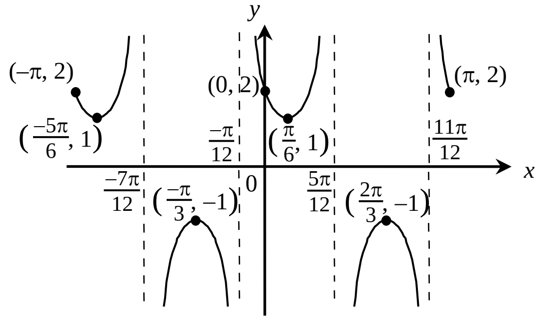
11 **a** $\left(\dfrac{4}{3}, \dfrac{2}{3}, \dfrac{-1}{3}\right)$ **b** $\dfrac{\sqrt{33}}{3}$

12 **a** $y = \dfrac{1}{4}x + \dfrac{5}{8}$ **b** $\left(x - \dfrac{1}{2}\right)^2 + \left(y - \dfrac{3}{2}\right)^2 = \dfrac{5}{2}$

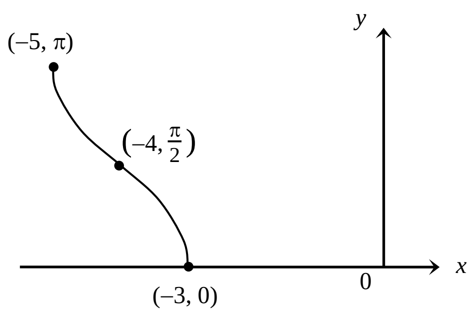
13 $x = \dfrac{5\pi}{8}, \dfrac{7\pi}{8}, \dfrac{13\pi}{8}, \dfrac{15\pi}{8}$

14 **a** $\frac{3}{5}$ **b** $\frac{12}{13}$

15



16



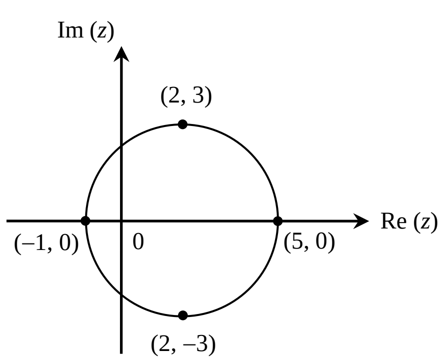
17 **b** $\theta = -2\pi + \tan^{-1}(3\sqrt{3}), -\pi + \tan^{-1}(3\sqrt{3}), \tan^{-1}(3\sqrt{3}), \pi + \tan^{-1}(3\sqrt{3})$

18 $\frac{-16}{65}$

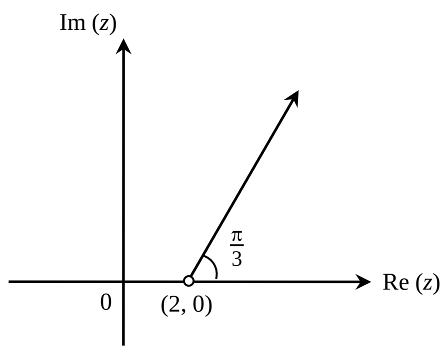
19 **a** $\frac{7\pi}{12}, \frac{7\pi}{12}$ **b** $\frac{7\pi}{12}, \frac{-17\pi}{12}$

c $\frac{-5\pi}{6}, \frac{7\pi}{6}$

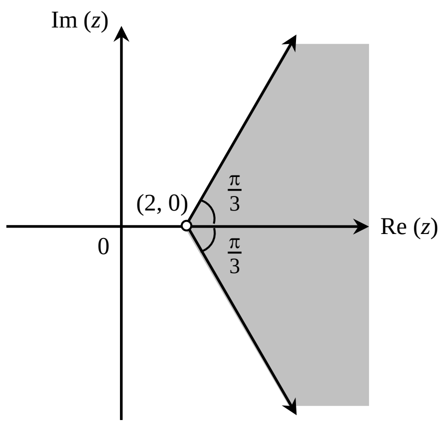
20 **a**



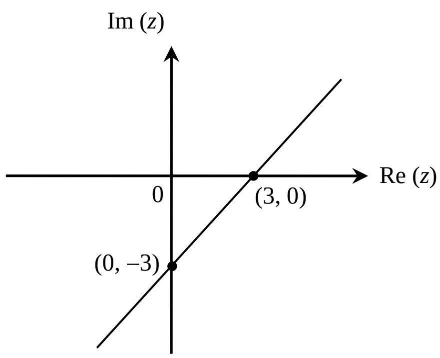
b



c



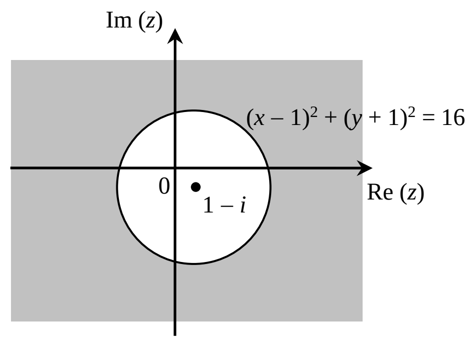
d



22 $z \in \{0\} \cup \left\{x + iy : \left(x + \frac{1}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{1}{2},\ x > 0\right\} \cup$

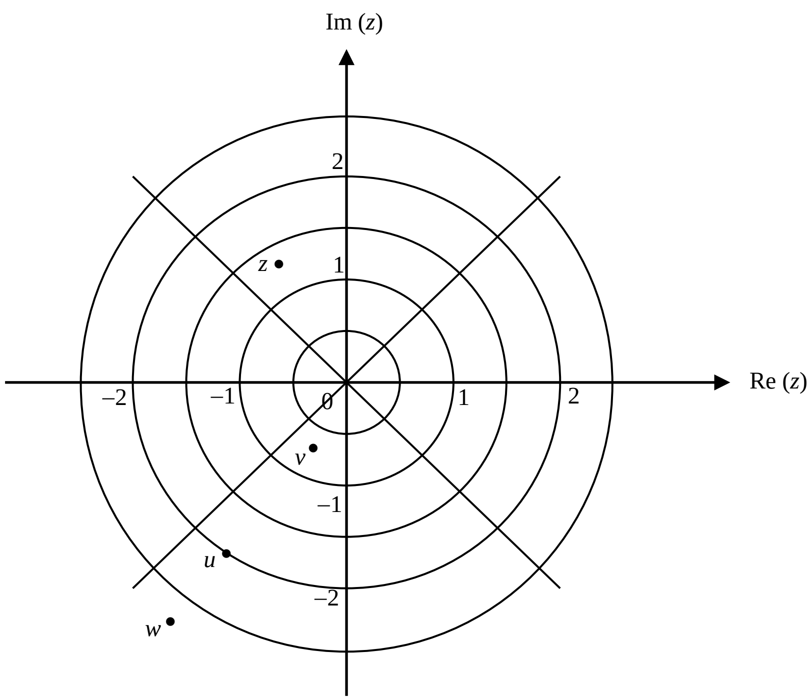
$\left\{x + iy : \left(x + \frac{1}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{1}{2},\ y > 0\right\}$

23

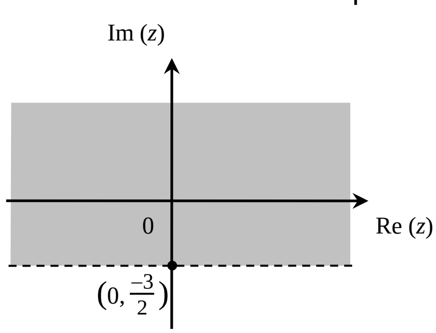


24 **b** $-3 + 2i, \pm i$

25



26



27 **a** $x = 2$ **b** $y = \pm 1$

c $y = \pm \frac{\sqrt{3}}{6}(x - 4)$

28 $-\text{cosec}^2 x$

29 $\pi b^2 a - \pi \int_0^a (f(x))^2 \, dx$

30 **b** $\frac{\pi\sqrt{2}}{4}$

31 **a** $\frac{125}{216}$ square units

33 $y = \begin{cases} \frac{3x+2}{x+1}, x > -1 \\ \frac{-(x+2)}{x+1}, x < -1 \end{cases}$

34 $a = 0$, $b = \frac{-1}{2}$

35 $f(x) = 3\sqrt{x^2+1} - 1$

36 **a** $\frac{dm}{dt} = \frac{-3m}{100}$ **b** $m = 40e^{\frac{-3t}{100}}$

37 $y = \log_e\left(\frac{1+x}{1-x}\right)$, $-1 < x < 1$.

38 **a** $V = \frac{3\pi}{16} h^3$ **b** $\frac{dV}{dt} = \frac{9\pi}{8} h^2$

39 **a** $a = \frac{-1}{2}e^{-x} - 2e^{\frac{-x}{2}}$ **b** 4.9 seconds

40 **a** 44 m/s **b** 148.8 m

41 $x = \frac{1}{3}t^3 - 2t + 2 + \cos t$

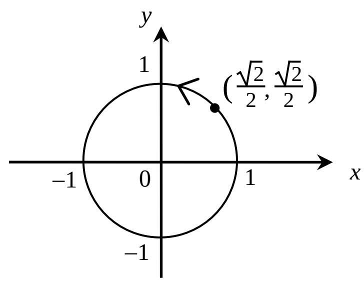
42 $T = 25$

43 **a** $t = 0, \pi, 2\pi$ **b** $t = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

44 $x^2 - (y - 1)^2 = 1$, $x, y \geq 1$

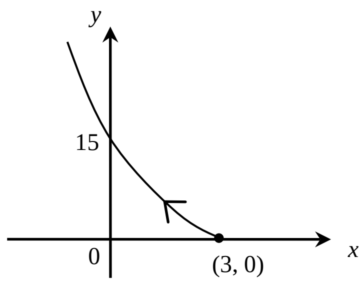
45 **a** **i** $x^2 + y^2 = 1$, dom: $[-1, 1]$, ran: $[-1, 1]$

ii



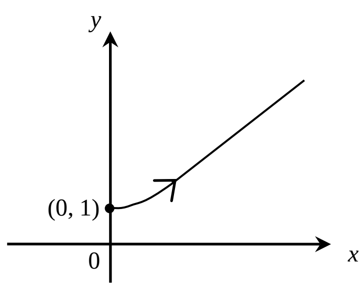
b **i** $y = x^2 - 8x + 15$, dom: $(-\infty, 3]$, ran: $[0, \infty)$

ii



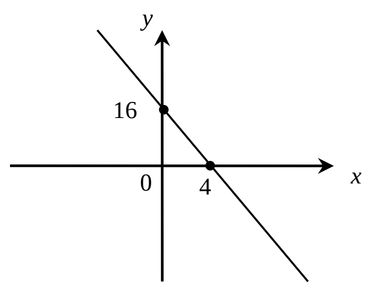
c **i** $y^2 - x^2 = 1$, dom: $[0, \infty)$, ran: $[1, \infty)$

ii



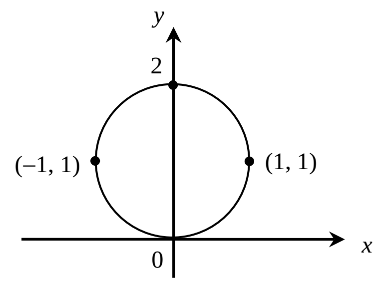
46 **a** **i** $y = 16 - 4x$, dom: $\mathbb{R}$, ran: R

ii



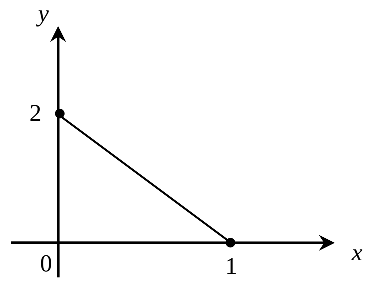
b **i** $x^2 + (1 - y)^2 = 1$, dom: $[-1, 1]$, ran: $[0, 2]$

ii



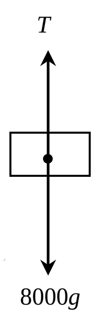
c **i** $y = -2x + 2$, dom: [0, 1], ran: [0, 2]

ii



47 **a** $a = \frac{17}{2}$, $b = \frac{5\sqrt{3}}{2}$ **b** 9.54 N

48 **a**



b 72 000 N

49 $-7\boldsymbol{i} + 4\boldsymbol{j} - 5\boldsymbol{k}$

50 **b** 1.02 m/s^2

51 1

52 $y = \log_e(x^2 - 4x - 4)$, graph is a decreasing curve, passing through (–1, 0) with asymptote at $x = 2 - 2\sqrt{2}$

53 1

54 1.80

55 **a** 1.265 **b** 1.216

56 **a** $\frac{53}{45}$ **b** $\frac{15}{14}$

57 $\frac{4\pi}{3}$

58 **a** cosec x **b** $\frac{1}{2}\log_e 3$

Answers to extended-response questions

1 **a** **i** 2 hours 14 minutes

ii 1 hour 47 minutes

iii 2 hours 0 minutes

iv 1 hour 23 minutes

v 1 hour 52 minutes

b **i** $t = \sqrt{5 - 4\sin\theta^\circ} + \frac{\pi\theta}{360}$

ii $\theta = 74.5^\circ$, shortest possible time is 1 hour 43 minutes

2 **a** $AB = \sqrt{10}$, $AC = \sqrt{20}$, $BC = \sqrt{10}$

b $y = \frac{1}{2}x + \frac{5}{2}$

c $y = 3x$

d (1, 3)

e $(x - 1)^2 + (y - 3)^2 = 5$

3 **a** **i** $8 - 8w$

ii $w = 1$

b **i** $3x - 4y - 5 = 0$

ii $x + 2y - 25 = 0$, $x = 11$, $y = 7$

c 390 units^3

4 **a** $\frac{1}{3}(\boldsymbol{a} + \boldsymbol{b})$

b $\frac{2}{3}\boldsymbol{a} - \frac{1}{3}\boldsymbol{b}$

c $\frac{1}{9}(2|\boldsymbol{a}|^2 - |\boldsymbol{b}|^2 + \boldsymbol{a}\cdot\boldsymbol{b})$

d **ii** $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx 54.74^\circ$

5 **a** **i** $\frac{a}{2}\boldsymbol{j}$

ii $a\,\boldsymbol{i} + \frac{a}{2}\boldsymbol{j}$

b $\lambda\left(a\boldsymbol{i} + \frac{a}{2}\boldsymbol{j}\right)$, $a(\lambda - 1)\,\boldsymbol{i} + \frac{a}{2}(1+\lambda)\boldsymbol{j}$, $\lambda a\,\boldsymbol{i} + \frac{a}{2}(1+\lambda)\boldsymbol{j}$

c **i** $\lambda = \frac{3}{5}$, $|\overrightarrow{BP}| = \frac{2}{\sqrt{5}}a$, $|\overrightarrow{OP}| = a$, $|\overrightarrow{OB}| = a$, isosceles triangle

ii $\frac{1}{\sqrt{5}}$

d $\lambda = \frac{3}{5}, -1$

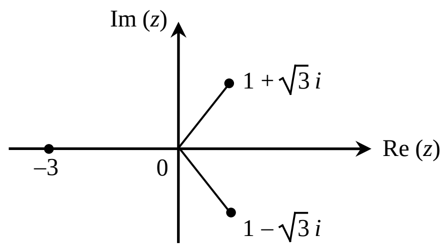
e $\frac{a}{30}(28\boldsymbol{i} + 29\boldsymbol{j} + 5\boldsymbol{k})$

6 **a** **i** 0

ii u is a root of $P(z)$

b **i** $1 \pm \sqrt{3}\,i, -3$

i



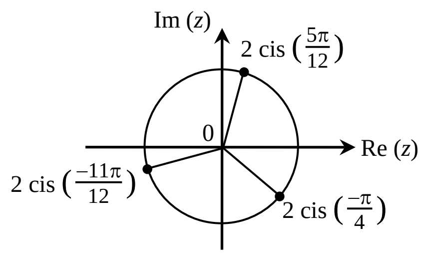
c $u = 2\text{ cis}\left(\frac{-\pi}{3}\right)$, $\text{Arg}(iu) = \frac{\pi}{6}$

7 **a** $8\text{ cis}\left(\frac{-3\pi}{4}\right)$

c $2 \text{ cis } \frac{5\pi}{12}$, $2 \text{ cis } \left(\frac{-11\pi}{12}\right)$

d $\sqrt{2} - \sqrt{2}\, i$

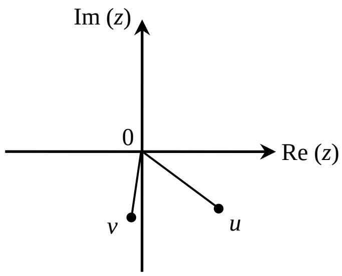
e



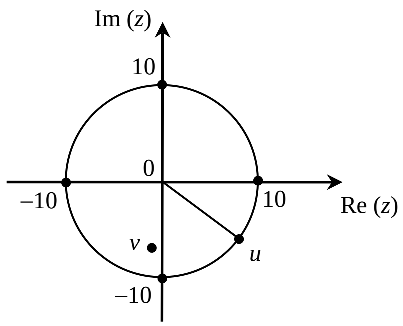
f $w = \sqrt{2}\, i$

g $\sqrt{2}$

8 **a**

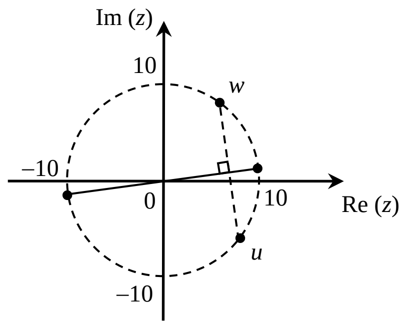


c



d $6 + 8i$

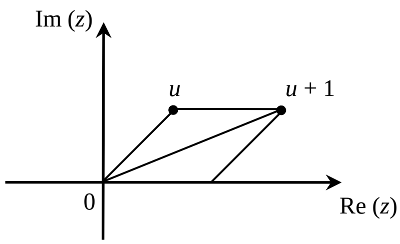
e



9 **c** $2x = y^2 + 1$

10 **a** $u = \text{cis}\ \dfrac{\pi}{4}$

b



c **i** $\left(1+\dfrac{1}{\sqrt{2}}\right) + \dfrac{1}{\sqrt{2}}\, i,\ \sqrt{2+\sqrt{2}}\ \text{cis}\ \dfrac{\pi}{8}$

11 **a** $g''(x) = \dfrac{2}{(f(x))^3}\,(f'(x))^2 - \dfrac{f''(x)}{(f(x))^2}$

c **i** $b = \pm 4$

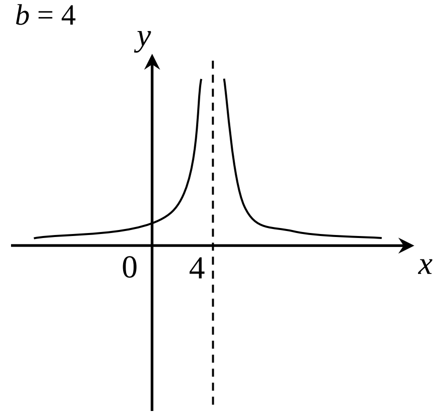
ii $-4 < b < 4$

iii $b > 4$ or $b < -4$

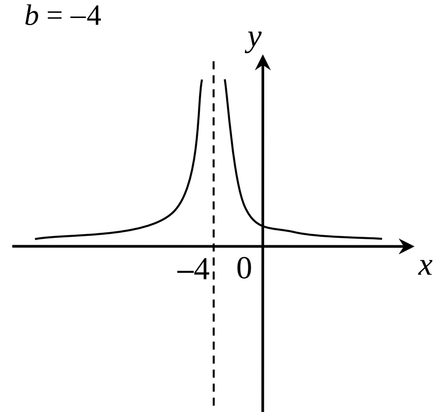
d Stationary point $\left(b, \dfrac{1}{16-b^2}\right)$; points of inflection $\left(b+\sqrt{16-b^2}, 32-2b^2\right)$ and $\left(b-\sqrt{16-b^2}, 32-2b^2\right)$

e

$b = 4$



$b = -4$



Value of *b*	**Antiderivative**
$b = 4$	$\dfrac{1}{4-x}$
$b = -4$	$\dfrac{-1}{4+x}$
$b > 4$ or $b < -4$	$\dfrac{1}{2\sqrt{b^2-16}}\log_e\left(\dfrac{x-\sqrt{b^2-16}-b}{x+\sqrt{b^2-16}-b}\right)$
$-4 < b < 4$	$\dfrac{1}{\sqrt{16-b^2}}\tan^{-1}\left(\dfrac{x-b}{\sqrt{16-b^2}}\right)$

12 **a** $\sec^4 x$

b $x = 0$

c $y = 4x - \pi + \dfrac{4}{3}$

d $\dfrac{1}{6}(1 + 2\log_e 2)$

e $\dfrac{4\pi}{3}$

13 **a** **i** 2000π litres

ii 134 cm

b $\frac{dh}{dt} = \frac{1+h^2}{4(h+(1+h^2)\tan^{-1} h)}$

c **i** $\int_0^{\sqrt{3}} \frac{4(h+(1+h^2)\tan^{-1} h)}{1+h^2}\,dh$

14 **b** $t = 2.5 \log_e\left(\frac{99N}{1000-N}\right)$

c 15 days

15 **a** 14 m/s

b 13.4 m/s

c 2.7 s

16 **a** **i** $v = \frac{26}{(t+1)^2} - 1$

ii $\sqrt{26} - 1$ seconds

b **i** $t = 2\tan^{-1}\left(\frac{5}{6}\right) - 2\tan^{-1}\left(\frac{v}{30}\right)$

ii $2\tan^{-1}\left(\frac{5}{6}\right)$ seconds

c Model 1 takes longer by 2.7 s

17 **a** **i** 12.5 m/s^2

b **ii** 277 metres

c 2π seconds