FURTHER MATHEMATICS (Unit 3)

APPLICATION TASK (SAC) 2020

CORE: Data Analysis (B)

Exam Results

Start date: _____   **End date:** _____

Time: 5 periods

Student Name: _____

Student Number: _____

AN INVESTIGATION OF STUDENT RESULTS

INTRODUCTION:

Year 10 teachers have been looking at the association between students' test results and their exam results. Two year 10 classes had their average test results and their end of year exam results recorded.

1. Is there a connection between class and exam result?
2. Is there a connection between classes' test results and exam results?
3. Is it possible to predict a person's exam result from their average test result?

These questions and others are the focus of the task.

Using the data sheet given to you, answer the following questions.

Class	Average Test Result (%)	Exam Result (%)
10Y	63	67
10Y	59	57
10Y	52	53
10Y	73	63
10Y	57	59
10Y	64	33
10Y	65	74
10Y	58	42
10Y	51	65
10Y	63	43
10Y	58	17
10Y	31	12
10Y	56	72
10Y	41	17
10Y	49	38
10Y	67	62
10Y	68	57
10Y	66	62
10Y	67	59
10Y	57	40
10Z	79	52
10Z	72	79
10Z	72	62
10Z	56	39
10Z	62	68
10Z	64	58
10Z	63	56
10Z	55	49
10Z	66	63
10Z	71	66
10Z	64	57
10Z	78	69
10Z	85	68
10Z	77	62
10Z	68	59
10Z	71	66
10Z	73	76
10Z	51	69
10Z	67	70
10Z	68	61

Part One:

You have been provided with a table containing the results for 40 year 10 students.

Question 1 (4 marks)

a. Complete the two – way frequency table on the left and the associated two – way percentage frequency table on the right for students' average test results. (All percentages must be given correct to the nearest whole number)

	Class		
Result (%)	10Y	10Z	Total
0 – 69			
70+			
Total			

	Class %	
Result (%)	10Y	10Z
0 – 69		
70+		
Total		

b. Comment on what each of these tables tells you about the association between class and average test result.

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Question 2 (11 marks)

a. Construct an ordered back – to – back stem plot for the classes' average test results.

Key: 2|3 = 23%

10Y		10Z
	2	
	2*	
	3	
	3*	
	4	
	4*	
	5	
	5*	
	6	
	6*	
	7	
	7*	
	8	
	8*	

b. Calculate the mean, median, and standard deviation for the two classes' average test results. Give your standard deviation answers correct to 2 decimal places.

10Y

Mean =

Median =

Standard Deviation =

10Z

Mean =

Median =

Standard Deviation =

c.   Compare the two distributions in terms of shape and centre.

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d.   Which is a better measure of centre; the mean or the median? Justify your response.

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Part Two:

Question 1 (16 marks)

a. Calculate the five – number summary, and interquartile range for 10Y's average test results.

b. Identify if there are any outliers in 10Y's average test results, showing appropriate calculations to support your answer.

c. Calculate the five – number summary, and interquartile range for 10Y's exam results.

d. Identify if there are any outliers in 10Y's exam results, showing appropriate calculations to support your answer.

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e. Construct parallel boxplots for 10Y's average test results and exam results.

f. Explain why parallel boxplots are an appropriate way to graphically display the data.

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g. Compare the two distributions in terms of shape, centre, and spread, quoting appropriate values to support your response.

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Question 2 (4 marks)

The parallel boxplots below show 10Z's average test results and exam results.

10Z Results
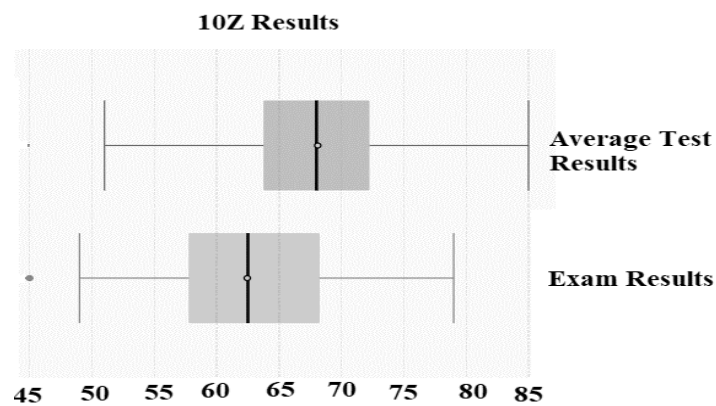
a. Approximately, what percentage of 10Z's average test results were below 73%?

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b. Approximate the interquartile range for 10Z's exam results.

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c. Explain why 10Z's results are associated with the type of assessment. Quote appropriate statistics in your response.

Part Three:

Question 1 (11 marks)

a. Construct a histogram of the average test results, for both classes, on the same set of axes, with a step width of 5%.

b. Using the histogram, complete the grouped frequency table below.

Interval	Frequency	Interval	Frequency
20 - < 25		55 - < 60	
25 - < 30		60 - < 65	
30 - < 35		65 - < 70	
35 - < 40		70 - < 75	
40 - < 45		75 - < 80	
45 - < 50		80 - < 85	
50 - < 55		85 - < 90	

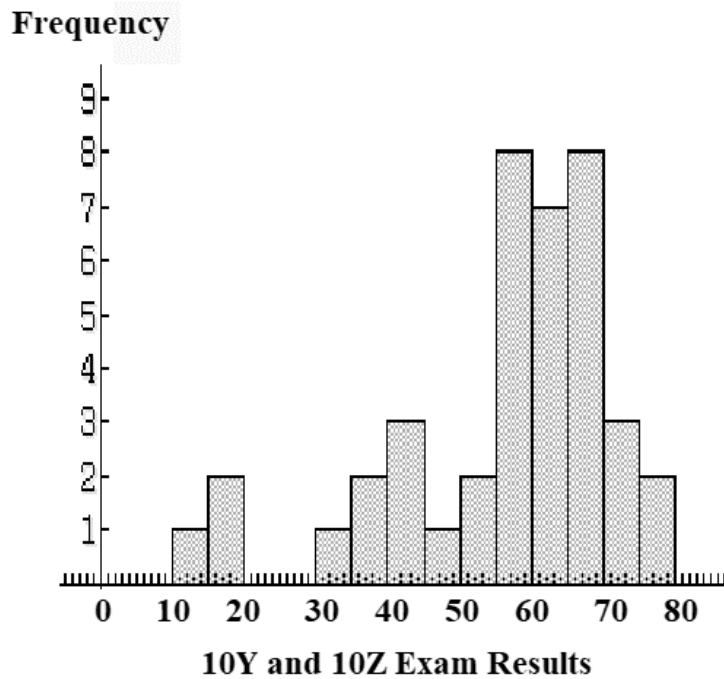c. Describe the shape of the histogram.

d. What percentage of average test results are above 65%?

e. Calculate the mean and standard deviation for average test results. Give your answers rounded to 3 significant figures.

f. Is the mean an appropriate measure of centre for average test results? Explain your answer.

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Question 2 (5 marks)

A histogram showing two of the twelve year 10 classes exam results is shown below:


a. Describe the histogram above in terms of shape, centre, and spread. Quote appropriate statistics in your response.

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b. Is it appropriate to apply the 68 – 95 – 99.7% rule to the year 10 exam results shown in the histogram above? Give a reason to justify your response.

Part Four:

The year 10 mathematics teachers are investigating whether students' exam results are related to their average test results.

While investigating this, they created a scatterplot, from the raw data you were given, showing the relationship between average test results and exam results, which can be seen below:
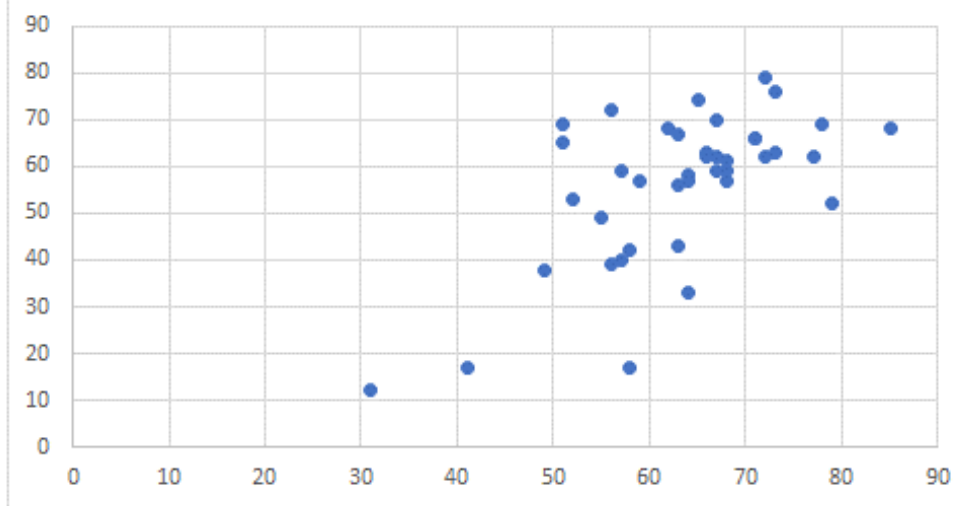
Question 1 (11 marks)

a. Determine the equation of the least squares regression line for the scatterplot above. Write the equation in terms of the variables, and state the coefficients correct to 3 significant figures.

b. Describe the relationship between average test result and exam result in terms of strength and direction, quoting an appropriate statistic to support your answer.

c. Draw the least squares regression line on the scatterplot above.

d. Interpret the slope and intercept of the least squares regression equation.

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e. One of the 10Y students had an average test result of 63% and an exam result of 43%. Calculate the residual when the least squares regression equation is used to predict exam result from average test result. Give your answer correct to 1 decimal place.

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f. Calculate and interpret the coefficient of determination, rounded to 4 decimal places.

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Question 2 (14 marks)

The students' teachers would like to improve the linearity of the data by applying an appropriate transformation.

a. Apply each of the 3 possible transformations to the data. Complete the table below for the transformed data values. Use y to represent the exam result and x to represent the average test result. Round all values to 3 decimals places.

Transformation	Equation of least squares regression line	r - value
y^2		
$\log x$		
$\dfrac{1}{x}$		

b. State which transformed equation has improved the linearity the most, or state whether the original equation has the best linear relation. Use mathematical reasoning to support your comments.

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c. Use your chosen equation from 2b. to predict the exam result of a student who has an average test result of 65%. Round your answer to the nearest whole percentage.

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d. Use your chosen equation from 2b. to predict the average test result of a student who had an exam result of 70%. Round your answer to the nearest whole percentage.

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Question 3 (5 marks)

The year 10 students, disappointed by their exam results, undertook a 2 – week online exam skills course to improve their performance in exams.

a. Use the table below to smooth the data using a 3 – point moving mean. Round all values to 1 decimal place

Day	Time spent completing coursework (in minutes)	3 – point moving mean
Monday	42	
Tuesday	34	
Wednesday	41	
Thursday	38	
Friday	32	
Monday	35	
Tuesday	33	
Wednesday	28	
Thursday	30	
Friday	25	

b. Plot the 3 – point moving mean data onto the time series plot below.
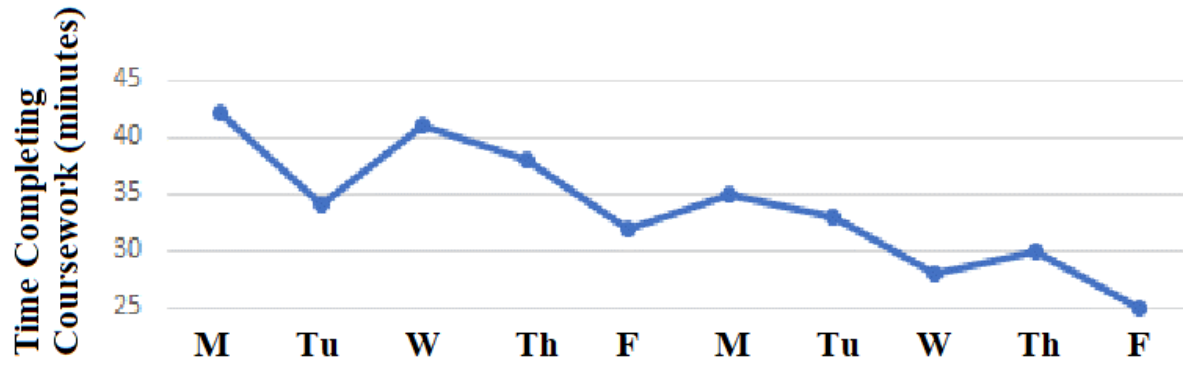
c. Comment on the effectiveness of the smoothing technique to identify trends.

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Part Five:

Question 1 (18 marks)

As part of their work to improve their exam results, the year 10 students spend time on a program that helps them develop their quick addition, subtraction, multiplication, and division skills. Below is a table that shows how many minutes the students spent on the computer using the program, on average.

	Monday	Tuesday	Wednesday	Thursday	Friday
Week 1	6.5	3.5	10	6.5	4
Week 2	5.5	2	12.5	7	3.5
Week 3	6	3	14	8.5	5.5
Week 4	7	4.5	13.5	9	6

a. Plot the data as a time series plot using an appropriate time code.

Minutes on program
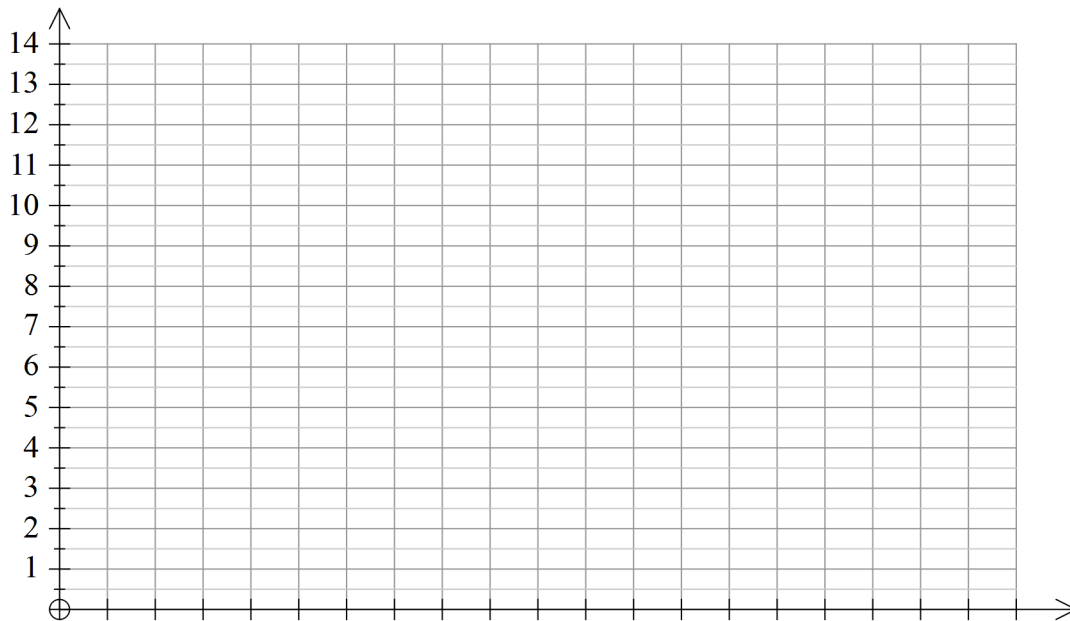
b. Describe the time series plot.

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c. The Year 10 teachers have decided to deseasonalise the data to see if there are any trends. Complete the tables below to complete the deseasonalisation.

h.   Round values to 1 decimal place.

	Monday	Tuesday	Wednesday	Thursday	Friday	Weekly Average
Week 1	6.5	3.5	10	6.5	4	6.1
Week 2	5.5	2.5	12.5	7	3.5	6.2
Week 3	6	3	14	8.5	5.5	
Week 4	7	4.5	13.5	9	6	8.0

ii.   Round values to 3 decimal places.

	Monday	Tuesday	Wednesday	Thursday	Friday
Week 1	1.066	0.574	1.639	1.066	0.656
Week 2	0.887	0.323	2.016	1.129	0.565
Week 3					
Week 4	0.875	0.563	1.688	1.125	0.750

iii.   Seasonal Indices: Round to 2 decimal places.

Monday	Tuesday	Wednesday	Thursday	Friday

iv.   Deseasonalise the original data, entering your values in the table below. Round all values to 1 decimal place.

	Monday	Tuesday	Wednesday	Thursday	Friday
Week 1					
Week 2					
Week 3					
Week 4					

d. Using a different colour, add the deseasonalised data to the time series plot in part a. Label the new graph.

e. Comment on the effectiveness of deseasonalising the data in revealing any underlying trends.

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f. Determine the equation of least square regression line for the deseasonalised data, giving all values correct to 3 significant figures.

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g. Use the equation above to predict the **actual** average time students would spend on the program in week 5 on Wednesday, giving your answer rounded to 1 decimal place.

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