Unit 4 Further Mathematics: Matrices Revision 'Sport'

TASK ONE

Two Australian Rules football teams from the same Victorian country town have completed an 18 game season. The seasons results were

Kites: 10 wins, 1 draw, 7 losses
Goblins: 8 wins, 2 draws, 8 losses

For a win they received four (4) points, a draw two (2) points and a loss one (1) point.

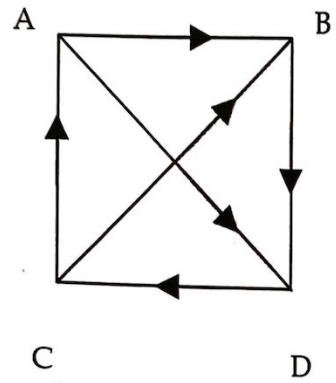
(a) Represent the season results as a 2×3 matrix

(b) Represent the points awarded as a 3×1 matrix

(c) Using matrices from **(a)** and **(b)**, calculate the total premiership points for each team.

TASK TWO

During the season, the coach of the Kites divided his team into four squads – named for the captain of each squad (Andy, Brett, Chas and Donny) – and ran a series of head-to-head competitive training sessions, where each squad competed once against each other squad in a series of skill tests. The competition results are summarised in the directed graph below.



Each arrow shows the winner of a session played in the competition. For example, the arrow from *C* to *A* shows that Chas' squad defeated Andy's squad.
In the competition, each squad was given a ranking that was determined by calculating the sum of their one-step and two-step dominances.

(a) Construct a one-step dominance matrix for this competition, with rows representing winners and columns representing losers.

(b) Construct a two-step dominance matrix for this competition, with rows representing winners and columns representing losers.

(c) Construct a matrix containing the sum of the one-step and two-step dominance matrices, and hence determine the ranking sum for each squad. Whose squad was the winner?

TASK THREE

Historical records of the first season for the Kites and Goblins in 1975 showed they played each other three times and the number of tickets sold for those matches was summarised as follows.

Match	Adults	Children	Pensioners	Total Match Takings $
Round 2	245	310	76	720.40
Round 9	120	44	0	311.00
Round 16	321	410	102	945.80

However no records of ticket prices were found.

(a) Three simultaneous equations were set up to find the ticket prices charged back in 1975. Complete the final two equations, given

x = adult ticket prices
y = children ticket prices
z = pensioner ticket prices

First Equation: $245x + 310y + 76z = \$720.40$

Second Equation:

Third Equation:

(b) Write the three simultaneous equations in matrix form.

(c) Determine the prices of the tickets in 1975 using your matrix equation from **(b)**.

TASK FOUR

The town folks were constantly changing allegiance for their two local teams when clearly one team performed far better than the other team in a season. Over the years the degree of change was determined and can be summarised as follows.

If Kites was the better performing team

- 90% of Kites' supporters remained Kites' supporters the following season.
- 40% of Goblins' supporters became Kites' supporters the following year.

If Goblins was the better performing team

- 95% of Goblins' supporters remained Goblins' supporters the following season.
- 30% of Kites' supporters became Goblins' supporters the following year.

In the town of 20 000 supporters, assume the supporters initially are equally divided between the two teams.

(a) Write down the initial state matrix concerning supporter numbers for each team.

Now consider that Kites were to perform better than Goblins for a three year period.

(b) Write the transition matrix for when Kites are the better side.

(c) Set up an appropriate matrix equation and find the number of supporters for the two teams after three (3) years of the Kites performing better than the Goblins.

Now consider that Goblins were to perform better than Kites for a three year period.

(d) Write the transition matrix for when Goblins are the better side.

(e) Set up an appropriate matrix equation and find the number of supporters for the two teams after three (3) years of the Goblins performing better than the Kites.

(f) Which team would in the long term have the largest supporter base if they continued to perform better?
Justify your answer with calculations.

TASK FIVE

The gym at the Goblins Football and Netball Club has a number of fitness activities.

The football players are advised to vary their training each week to maximise their fitness outcomes, and can choose between the treadmills (T), the weight circuit (W) or football skills training (F).

During the first week of the season, 100 players from various levels attended training. All of the players started on the treadmill. The state matrix for week 1, is:

$$W_1 = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} T \\ W \\ F \end{matrix}$$

The transition matrix showing the movement between training regimes each week is:

$$\begin{matrix} & \textit{From} \\ & \begin{matrix} T & W & F \end{matrix} \\ T = & \begin{bmatrix} \boxed{0} & 0.2 & 0.4 \\ 0.5 & \boxed{0} & 0.6 \\ 0.5 & 0.8 & \boxed{0} \end{bmatrix} \begin{matrix} T \\ W \\ F \end{matrix} \quad \textit{To} \end{matrix}$$

(a) Three elements in the transition matrix have been highlighted. Explain the meaning of these three elements.

(b) Determine the number of players who will do football skills training in the second week of training.

(c) The pattern continues. Show that in the long run, 41 players will do football skills training each week if values are rounded to the nearest whole number.

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The Goblin Football and Netball Club has football teams playing at each of three levels; firsts, seconds and thirds.

Some players are promoted or dropped a level during the season and others are unavailable due to injury or return from injury.

The number of players, P_n, at each level for the nth week of one particular season is modeled by the equation :

$$P_{n+1} = \begin{bmatrix} 0.8 & 0.2 & 0 \\ 0.2 & 0.6 & 0.4 \\ 0 & 0.2 & 0.6 \end{bmatrix} \times P_n + \begin{bmatrix} -2 \\ 5 \\ -3 \end{bmatrix}, \quad P_1 = \begin{bmatrix} 40 \\ 40 \\ 40 \end{bmatrix} \begin{matrix} \textit{Firsts} \\ \textit{Seconds} \\ \textit{Thirds} \end{matrix}$$

(d) How many players would be available at each level during the third week?

(e) The club would like to retain the same number of players that they had at each level during week 2.

$\begin{bmatrix} -2 \\ 5 \\ -3 \end{bmatrix}$ represents the number of players lost or returned due to injury at that level each week.

Using the matrix $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ instead of $\begin{bmatrix} -2 \\ 5 \\ -3 \end{bmatrix}$ will ensure that the numbers do not change after week 2.

What are the values of x, y and z?

TASK SIX

The coach of the Goblins Football team loves playing around with anagrams – letter combinations that can generate a number of different words. For example, the letters *A*, *E*, *M* and *N* can form the words *AMEN*, *MANE*, *MEAN* or *NAME*.

He also knows that permutation matrices can be used to rearrange the letters in a word.

(a) If matrix $W = \begin{bmatrix} T \\ R \\ A \\ C \\ E \end{bmatrix}$ and matrix $P = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$, then what word is formed by the matrix product $P \times W$?

(b) In the matrix provided below, fill in the element values for matrix Q so that the matrix product $Q \times W$ gives the word REACT.

$$\begin{bmatrix} \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix} \times \begin{bmatrix} T \\ R \\ A \\ C \\ E \end{bmatrix} = \begin{bmatrix} R \\ E \\ A \\ C \\ T \end{bmatrix}$$

(c) Explain why the matrix product $Q^3 \times W$ gives the matrix $\begin{bmatrix} T \\ R \\ A \\ C \\ E \end{bmatrix}$.