FURTHER MATHEMATICS

UNIT 3

Name_____

35 marks

ASSIGNMENT

UNIT **3** – **0** **5**

Multiple-choice questions
Each question is worth 1 mark. Circle the letter beside the correct answer.
No working needs to be shown in this section.

1 The value of V_3 in the sequence defined by the recurrence relation
$V_0 = 5$, $V_{n+1} = V_n + 6$ is:

A 5

B 11

C 17

D 23

E 29

2 The recurrence relation that generates the sequence 52, 44, 36, 28, 20, … is

A $V_0 = 52$, $V_{n+1} = 1 + 0.08V_n$

B $V_0 = 52$, $V_{n+1} = (1 + 0.08)V_n$

C $V_0 = 52$, $V_{n+1} = 8V_n$

D $V_0 = 52$, $V_{n+1} = V_n + 8$

E $V_0 = 52$, $V_{n+1} = V_n - 8$

3 A simple interest loan has an interest rate of 3.6% per annum.
If the amount borrowed is $5000, the interest charged per year is

A $36

B $50

C $72

D $180

E $1388

**4** A sum of $3500 is invested in an account that pays 2.4% per annum simple interest.
A recurrence relation that models the value of the investment after n years, V_n, is

A $V_0 = 3500, \ V_{n+1} = V_n + 84$

B $V_0 = 3500, \ V_{n+1} = V_n - 84$

C $V_0 = 3500, \ V_{n+1} = V_n + 24$

D $V_0 = 3500, \ V_{n+1} = V_n - 24$

E $V_0 = 3500, \ V_{n+1} = 2.4V_n$

5 The value of a simple interest investment after n years, V_n, is modelled by the recurrence relation $V_o = 8400, \ V_{n+1} = V_n + 285$.
After how many years will the value of the investment first exceed $10 000?

A 4

B 5

C 6

D 7

E 8

6 The graph below shows the depreciation in the value of a car over a period of 5 years. In the graph, V_n is the value of the car after n years.
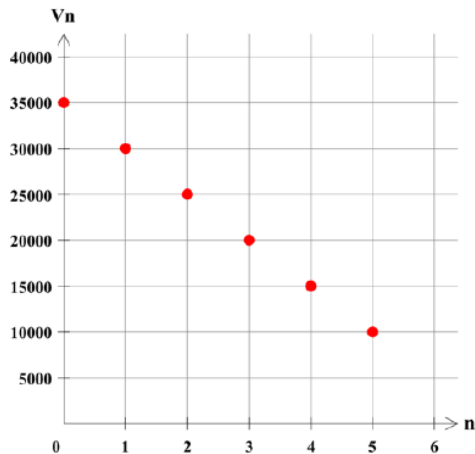
A rule for the value of the car after n years is

A $V_n = 35000 - n$

B $V_n = 35000 - 5000n$

C $V_n = 35000n - 5000$

D $V_n = 5000n - 35000$

E $V_n = 5000 - 35000n$

7 The amount of money in a bank account after n years, V_n, can be modelled by the recurrence relation $V_0 = 12000$, $V_{n+1} = 0.92V_n$. The amount of money is

A growing at the rate of 9.2% per annum

B growing at the rate of 8% per annum

C neither growing nor decaying

D decaying at the rate of 9.2% per annum

E decaying at the rate of 8% per annum

8 Alistair borrows $4500 from a bank and will pay interest at the rate of 3.6% per annum, compounding monthly. A recurrence relation that models the value of Alistair's loan after n months, V_n is

A $V_0 = 4500$, $V_{n+1} = 1.003V_n$

B $V_0 = 4500$, $V_{n+1} = 1.036V_n$

C $V_0 = 4500$, $V_{n+1} = 1.36V_n$

D $V_0 = 4500$, $V_{n+1} = 1 + 3.6V_n$

E $V_0 = 4500$, $V_{n+1} = (1 + 3.6)V_n$

9 A musician purchased a new grand piano for $17 990. After 8 years it will have an estimated value of $12 500. If the value of the piano is depreciated using a reducing-balance method, the annual rate of depreciation, is closest to

A 4.45%

B 4.66%

C 30.5%

D 43.9%

E 69%

10 An amount of money is deposited into an account with BankAus. BankAus will pay simple interest at the rate of 4.5% per annum. Which of the following investment conditions, offered by BankNZ will earn more interest in one year than the account with BankAus?

A 4.2% per annum simple interest

B 4.4% per annum, compounding quarterly

C 4.45% per annum, compounding monthly

D 4.3% per annum, compounding fortnightly

E 4.4% per annum, compounding weekly

Extended-response questions
Question 1
Andrea wants to buy a second-hand car. She starts with $100 in a money box and every month will add another $40. The amount of money in the money box each month forms the sequence 100, 140, 180, 220, . . .

Let M_n be the amount of money in the money box after n months.

a) Write down a recurrence relation that models the amount of money in the money box each month.

2 marks

b) Use the recurrence relation to determine:
   i) the amount of money in the money box after 8 months

   ii) the number of months it takes for Andrea to save $700.

1+1= 2 marks

Andrea has found a car to buy, but needs $5000 to buy it. Her parents will lend her the money, but will charge her simple interest at the rate of $20 per month.
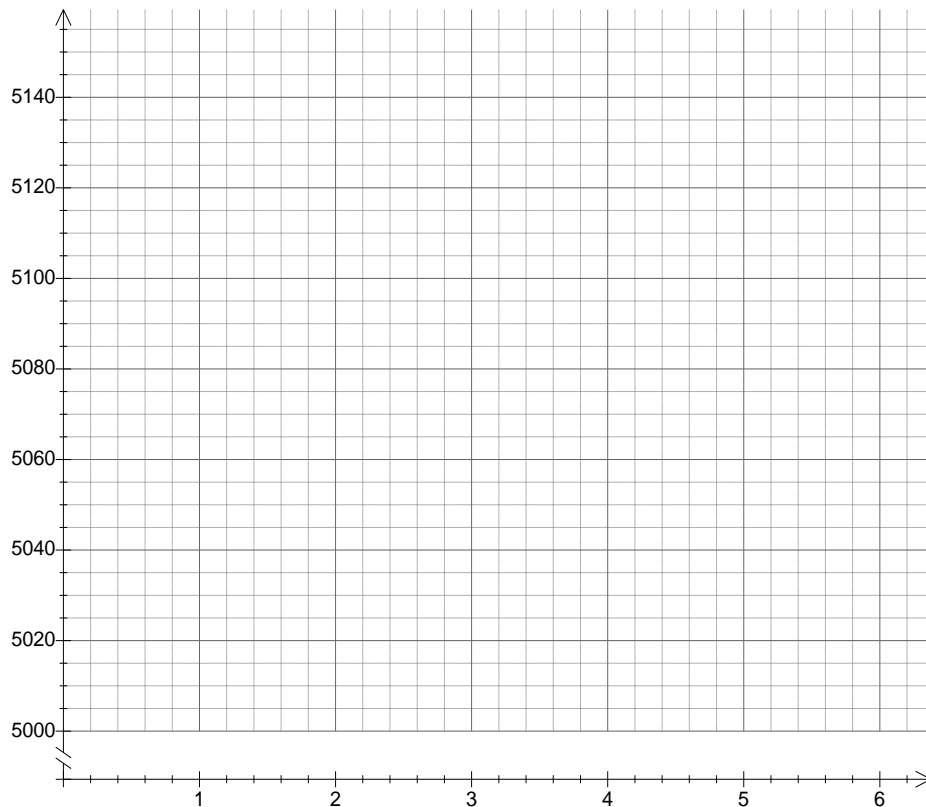
c) How much interest will Andrea be charged in one year?

1 mark

d) Show that the annual percentage rate for this interest is 4.8% per annum.

1 mark

e) Sketch a graph of the value of Andrea's loan, V_n, against the number of months, n, showing a period of six months.

3 marks

f) How much will Andrea pay her parents if she fully repays the loan after 2 years?

1 mark

Question 2

Andrea's car will be depreciated using a reducing-balance method. The value of the car after n years, V_n , can be modelled by the recurrence relation below.

$$V_0 = 5000, \qquad V_{n+1} = 0.92V_n$$

a) Use the recurrence relation to write down the value of V_1, V_2 and V_3.

3 marks

b) By how much is the value of the car depreciated in the third year?

1 mark

c) What is the annual percentage rate of depreciation of the car?

d) i) Write down the rule for the value of the car after *n* years.

iii) How many years will it take for the value of the car to first fall below $2000?

2 +2 = 4 marks

Question 3

At the same time as they lend Andrea the money for her car, Andrea's parents invest another $5000 into an account that earns compound interest. The rule for the value of this investment after *n* years, V_n , is

$$V_n = (1.086)^n \times 5000$$

a) What is the annual percentage interest rate for this investment?

1 mark

b) What is the value of the investment after 2 years?

1 mark

c) Write down the recurrence relation that models the value of the investment if the interest was compounded quarterly.

2 marks

Andrea will sell her car after 4 years, at a price determined by the reducing-balance depreciation from **Question 2**. She will add this to the interest that has been earned from her parents' investment (compounded yearly) over the 4 years and use the money to buy another car.

d) Calculate the maximum cost of her next car, correct to the nearest cent.

2 marks