

HAILEYBURY FURTHER MATHEMATICS

UNIT **4** - **0** **2**



Name: _____

Total: 33 marks

Multiple Choice

Question 1

For the matrix $X = \begin{bmatrix} 4 & 2 & 6 & -5 \\ -1 & 5 & -3 & 2 \\ 3 & 8 & 7 & 1 \end{bmatrix}$, the product of element $x_{2,3}$ and element $x_{3,1}$ is

- A. -9
- B. -6
- C. 0
- D. 24
- E. 48

Question 2

$2 \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} 5 \\ -3 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is equal to

- A. $\begin{bmatrix} -3 \\ -8 \end{bmatrix}$
- B. $\begin{bmatrix} 13 \\ 10 \end{bmatrix}$
- C. $\begin{bmatrix} -9 \\ -18 \end{bmatrix}$
- D. $\begin{bmatrix} 27 \\ -18 \end{bmatrix}$
- E. $\begin{bmatrix} 27 \\ 54 \end{bmatrix}$

Question 3

If $A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$ then $A^2 - 3A$ is

A. $\begin{bmatrix} 0 & -2 \\ 4 & -2 \end{bmatrix}$

B. $\begin{bmatrix} 4 & 2 \\ 8 & 2 \end{bmatrix}$

C. $\begin{bmatrix} 4 & 2 \\ 0 & -4 \end{bmatrix}$

D. $\begin{bmatrix} 2 & 4 \\ -2 & 0 \end{bmatrix}$

E. not defined.

Question 4

If $A = \begin{bmatrix} 3 & 6 & 8 \\ 2 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 5 \\ 0 \\ 3 \end{bmatrix}$ then the order of the matrix product AB is

A. 1×3

B. 1×2

C. 2×1

D. 3×1

E. not defined

Question 5

If $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ then the matrix product AB is

A. $\begin{bmatrix} -2 & 3 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 3 & -1 \end{bmatrix}$

C. $\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$

D. $\begin{bmatrix} -3 \\ 6 \\ 0 \end{bmatrix}$

E. not defined.

Question 6

The table below shows an inventory sheet for a warehouse that stores tyres.

February	Tyre			Totals
Status	XRT	XRT40	XRT80	
inhouse	242	312	187	741
delivered to dealers	73	105	72	250
on order	150	200	120	470

The matrix that shows the number of XRT, XRT40 and XRT80 tyres delivered to dealers during February is

- A. $\begin{bmatrix} 242 & 312 & 187 \end{bmatrix}$
- B. $\begin{bmatrix} 73 & 105 & 72 \end{bmatrix}$
- C. $\begin{bmatrix} 4 & 2 \\ 0 & -4 \end{bmatrix}$
- D. $\begin{bmatrix} 7 & 4 & 1 \\ 2 & 5 & 0 \\ 4 & 7 & 0 \end{bmatrix}$
- E. $\begin{bmatrix} 73 & 105 & 72 & 250 \end{bmatrix}$

Question 7

Consider the following system of linear equations.

$$x + 3y = 4$$

$$2x + y = -7$$

Using matrices to solve this system, the solution would be obtained by evaluating the product.

- A. $\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ -7 \end{bmatrix}$
- B. $\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} -0.2 & 0.6 \\ 0.4 & -0.2 \end{bmatrix}$
- C. $\begin{bmatrix} -0.2 & 0.6 \\ 0.4 & -0.2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$
- D. $\begin{bmatrix} 4 \\ -7 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$
- E. $\begin{bmatrix} -0.2 & 0.6 \\ 0.4 & -0.2 \end{bmatrix} \begin{bmatrix} 4 \\ -7 \end{bmatrix}$

Question 8

The number of chocolate cakes (c), banana cakes (b) and apple cakes (a) sold at a cake shop each week is described by the matrix

$$\begin{array}{ccc} c & b & a \\ [25 & 15 & 22] \end{array}$$

The profit made on each of the cakes is the same and is \$4.

Which one of the following gives the total profit made in a week by the sale of the chocolate, banana and apple cakes at this shop?

- A. $4[25 \ 15 \ 22]$
- B. $[4][25 \ 15 \ 22]$
- C. $[25 \ 15 \ 22][4]$
- D. $[25 \ 15 \ 22] \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}$
- E. $\begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix} [25 \ 15 \ 22]$

Question 9

For the matrix equation $AX = Y$

where $A = \begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $Y = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$, which one of the following statements is true?

- A. The matrix equation has a unique solution.
- B. The matrix A^{-1} exists.
- C. $X = \begin{bmatrix} 7 \\ 21 \end{bmatrix}$
- D. The product AX is defined.
- E. The inverse matrix Y^{-1} exists.

Short answer**Question 1** (6 marks)

A local council operates three libraries. The number of books a , b and c , borrowed in December 2005 from the three libraries located respectively at Alplinghurst, Broadlake and Casaville can be determined from the matrix equation below.

$$\begin{bmatrix} 3 & 2 & 4 \\ 1 & 2 & 2 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 8517 \\ 4831 \\ 5803 \end{bmatrix}$$

- a. Express this matrix equation as a system of three simultaneous linear equations.
(Do not attempt to solve the system of equations.)

1 mark

- b. Explain whether or not it is possible to complete the following matrix multiplication.

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} \times \begin{bmatrix} 3 & 2 & 4 \\ 1 & 2 & 2 \\ 2 & 3 & 1 \end{bmatrix}$$

1 mark

- c. For the square matrix $\begin{bmatrix} 3 & 2 & 4 \\ 1 & 2 & 2 \\ 2 & 3 & 1 \end{bmatrix}$ find

- i. its determinant.

- ii. its inverse.

1 + 1 = 2 marks

- d. Find the number of books borrowed in December 2005 from each of the three libraries by solving the matrix equation

$$\begin{bmatrix} 3 & 2 & 4 \\ 1 & 2 & 2 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 8517 \\ 4831 \\ 5803 \end{bmatrix}$$

2 marks

Question 2 (9 marks)

A street gang has five members. A social worker interviews each gang member and, for each pair, assesses which one of the pair has influence over the other.

- Ben has influence over Cal, Jack and Pete
- Cal has influence over Jack, Ken and Pete
- Jack has influence over Ken and Pete
- Ken has influence over Ben and Pete
- Pete has influence over no-one

- a. Draw a directed graph (digraph) to represent this situation.

2 marks

- b. Construct a dominance matrix, D_1 for this situation.

$$\begin{bmatrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix} \begin{matrix} B \\ C \\ J \\ K \\ P \end{matrix}$$

2 marks

c. Using one and two step dominances determine the gang leader. Show all working.

3 marks

d. Explain why Ben has five 2-step dominances.

2 marks

Question 3 (9 marks)

Ratepayers can borrow books from any of the three libraries for up to a month and may return the books to any of the three libraries.

Over the last few years, it has been found that:

- 65% of books borrowed from library A are returned to library A
- 15% of books borrowed from library A are returned to library B
- 20% of books borrowed from library A are returned to library C

- 9% of books borrowed from library B are returned to library A
- 82% of books borrowed from library B are returned to library B
- 9% of books borrowed from library B are returned to library C

- 11% of books borrowed from library C are returned to library A
- 15% of books borrowed from library C are returned to library B
- 74% of books borrowed from library C are returned to library C

where library A refers to the library at Alphinghurst, library B refers to the library at Broadlake and library C refers to the library at Casaville.

- a. Complete the transition matrix T below using the information above and expressing percentages as decimals.

$$T = \begin{array}{c} \text{Returned to} \\ \left[\begin{array}{ccc} 0.65 & & \\ & 0.82 & \\ & & 0.74 \end{array} \right] \begin{array}{l} A \\ B \\ C \end{array} \end{array}$$

Borrowed from
A B C

2 marks

At the start of 2006 the librarians at the three libraries made available for loan 80 new books with large print. They decided that initially, 35 of these books would be located at Alphinghurst, 25 would be located at Broadlake and 20 would be located at Casaville.

Such is the demand for these books, that it can be assumed that they are always out on loan and always for the maximum period of one month.

- b. Construct a matrix of order 3×1 , labelled N_0 , that describes the number of these new books with large print that are initially located at Alphinghurst, Broadlake and Casaville respectively.

$$N_0 =$$

1 mark

- c. Using the matrices T and N_0 , find the number of these large print books that are located at the three libraries one month after they are made available for loan. Express your answers to the nearest whole number.

2 marks

- d. Find the number of these large print books that would be located at each of the three libraries at the start of 2007. Express your answer to the nearest whole number.

2 marks

- e. One of the librarians at Alphinghurst is concerned that there will eventually be only approximately half of these large print books originally allocated to his library, still located there. Assuming that the pattern of borrowing and returning outlined earlier is to continue, explain whether or not the librarian's concern is correct.

2 marks