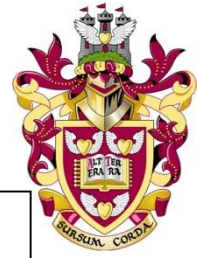


NAME: \_\_\_\_\_

# Unit 4 Further Mathematics

## Term 2 – Holiday Revision



Total marks: 87

Attached you will find revision material for:

1. Core: Statistics

- ❖ Displaying, summarising and describing **univariate** data
- ❖ Displaying, summarising and describing relationships in **bivariate** data
- ❖ Introduction to **regression** analysis

2. Core: Recursion and Financial Modelling

- ❖ Recursion relations and rules for simple interest, compound interest and depreciation
- ❖ Loans and investments

3. Module 1: Matrices

- ❖ Matrix arithmetic
- ❖ Applications of Matrices
- ❖ Directed graphs (Dominance)

**HOLIDAY TASK:**

**DUE:** \_\_\_\_\_

Answer each question in the space provided.

**CORE: STATISTICS (29 marks)**

**Question 1 (10 marks)**

A family wishing to sell their home, investigates the number of houses sold by two real estate firms, M. T. Hooker and U. R. Goode over a certain number of months. The data they collected is shown in the following back-to-back stem and leaf.

M. T. HOOKER										U. R. GOODE													
									9	6	8	9											
								7	7	6	6	7	0	4	7								
7	6	5	4	4	4	0	0	0					1	1	1	3	3	3	4	4	5	7	
									9														9

- a. How many months were investigated by the family?  
1 mark
  
- b. Describe the shape of the distribution of the data for U. R. Goode.  
1 mark
  
- c. What is the median number of houses sold each month by U. R. Goode over the time of the investigation?  
1 mark
  
- d. What is the interquartile range for the M. T. Hooker data?  
1 mark
  
- e. What is the mean number of houses sold each month by M. T. Hooker?  
1 mark
  
- f. Which of the two firms has an outlier? What is this outlier?  
3 marks

g. Construct a boxplot to display the information in the back-to-back stem plot

2 marks

**Question 2** (9 marks)

The I-Help-U Bank wishes to determine the relationship between interest rates and the number of new houses commenced. The data collected by the bank over the last ten years is given in the table below.

Year	Interest Rate(%)	Number of new houses commenced ('000)
1995	17	11
1996	15	13
1997	12	14
1998	10	18
1999	12	15
2000	13	14
2001	10	17
2002	8	21
2003	7	23
2004	6	25

- a. Of the two variables being compared, interest rate and number of new houses commenced, which one is the explanatory variable?

1 mark

- b. Complete the least squares regression equation for the interest rate and the number of new houses commenced. Give your answers to one decimal place.

$$\text{Number of new houses commenced} = \boxed{\phantom{000}} + \boxed{\phantom{000}} \times \text{interest rate.}$$

2 marks

- c. What would you expect the number of new houses commenced to be if the interest rate in a particular year was 11%?

2 marks

- d. Find Pearson's correlation coefficient for this data. Give your answer to 2 decimal places.

1 mark

- e. Explain what this correlation coefficient tells you about interest rates and the number of new houses commenced.

1 mark

- f. If the regression equation for the interest rate and the number of thousands of new houses commenced in another ten year period was:

$$\text{number of new houses ('000s)} = 50 - 2 \times \text{interest rate(\%)}$$

then complete the following sentence.

As interest rate increases by one percent then the number of new houses commenced

Increases/decreases

by

2 marks

**Question 3** (10 marks)

In one study, 380 Year 12 students were asked how often they were engaged in any sporting activity outside school. Students were also asked to classify their stress level in relation to their VCE studies. The results below were obtained.

Level of stress	Engaged in sporting activity outside school		
	Regularly	Sometimes	Never
Low	16	32	36
Medium	12	40	56
High	6	52	130

- a. In this study, which variable is the explanatory variable?

1 mark

- b. In this study how many students reported a high level of stress?

1 mark

- c. What percentage of students were engaged in a sporting activity outside of school? Give your answer to one decimal place.

1 mark

- d. Represent the data in a two-way percentage frequency table (with values correct to the nearest percent) and **appropriate graphical display**.

2 + 3 = 5 marks

- e. Comment on any relationship between the stress level and the amount of sporting activity for this group of Year 12 students.

2 marks

**CORE: RECURSION AND FINANCIAL MODELLING (25 marks)**

**Question 1**

The recurrence relation that generates the sequence 12, 21, 30, 39, 48, ... is

- A  $V_0 = 12, V_{n+1} = 1 + 0.09V_n$
- B  $V_0 = 12, V_{n+1} = (1 + 0.09)V_n$
- C  $V_0 = 12, V_{n+1} = 9V_n$
- D  $V_0 = 12, V_{n+1} = V_n + 9$
- E  $V_0 = 12, V_{n+1} = V_n - 9$

**Question 2**

The value of the simple interest investment after  $n$  years,  $V_n$  is modelled by the recurrence relation

$$V_0 = 3600, V_{n+1} = V_n + 260$$

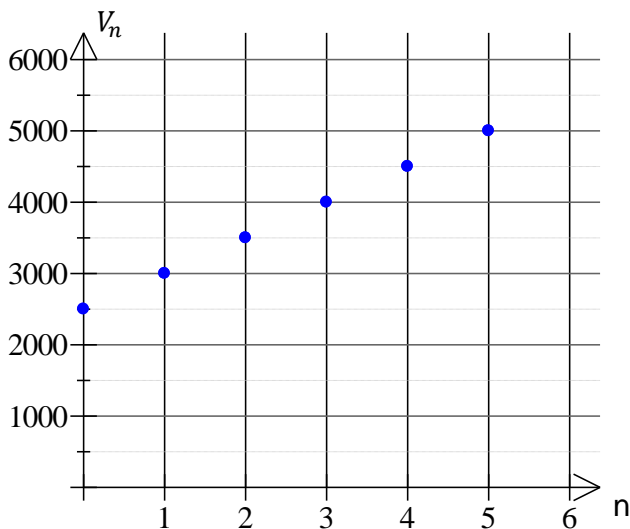
After how many years will the value of the investment first exceed \$6000?

- A 7
- B 8
- C 9
- D 10
- E 11

**Question 3**

The graph below shows the value of a simple interest loan over a period of 5 years. In the graph,  $V_n$  is the value of the loan after  $n$  years.

A rule for the value of the loan after  $n$  years is



- A  $V_n = 2500 + n$
- B  $V_n = 2500 + 500n$
- C  $V_n = 2500n + 500$
- D  $V_n = 2500 - n$
- E  $V_n = 2500 - 500n$

**Question 4**

An amount of money is deposited into an account with Bank Aus.

BankAus will pay simple interest at the rate of 6.5% per annum.

Which of the following investment conditions, offered by BankNZ will earn less interest in one year than the account with BankAus?

- A 6.55% per annum simple interest
- B 6.45% per annum, compounding quarterly
- C 6.4% per annum, compounding monthly
- D 6.35% per annum, compounding fortnightly
- E 6.3% per annum, compounding weekly

**Question 5**

A reducing-balance loan of \$8000 will be repaid with quarterly payments. The amortisation table for this loan is shown below

<i>Payment number</i>	<i>Payment</i>	<i>Interest</i>	<i>Principal reduction</i>	<i>Balance of loan</i>
0	0	0.00	0.00	8000
1	2043.94	70.00	1973.94	6026.06
2	2043.94	52.73	1991.21	4034.85
3	2043.94	35.30	2008.64	2026.21
4	2043.94	17.73	2026.21	0.00

The annual percentage interest rate on this loan is

- A 1.2%
- B 3.42%
- C 3.5%
- D 8.75%
- E 10.5

**Question 6**

Leasa borrows \$150 000 to purchase a flat. She will pay interest at the rate of 6.25% per annum, compounding fortnightly and will make fortnightly payments so that the loan is fully repaid after 15 years.

Leasa's fortnightly repayments will be closest to

- A \$593
- B \$900
- C \$1286
- D \$1308
- E \$2350



**Question 10** (5 marks)

Justin is aged 38 and is planning to retire at 60 years of age. He estimates that he needs \$680 000 to provide for his retirement. His current superannuation fund has a balance of \$40 000 and is delivering 5% p.a. compounded monthly.

a. Find the monthly contributions needed to meet the retirement lump sum target.

1 mark

b. If in the final ten years before retirement, Justin doubles his monthly contribution calculated from a), find the new lump sum amount target for his retirement, to the nearest 10 dollars.

2 marks

c. How much extra could Justin expect if the interest rate from part b) is increased to 8% p.a. compounded monthly (for the final 10 years)? Round the answer to the nearest \$1000.

2 marks



**Question 11** (5 marks)

Rachel repaid a reducing balance loan of \$46 000 over 10 years with fortnightly instalments and interest charged at 9.3% per annum, compounded fortnightly.

Calculate:

**a.** The repayment value

1 mark

**b.** The total amount of interest paid to the nearest 10 dollars

2 marks

**c.** The rate of interest for an equivalent flat rate loan at which all other variables were the same. Give your answer to 2 decimal places

2 marks

**MODULE 1: MATRICES (33 marks)**

**Question 1** (3 marks)

Given  $A = \begin{bmatrix} 2 & -6 \\ 5 & 0 \end{bmatrix}$  and  $B = \begin{bmatrix} -8 & 0 \\ 7 & -3 \end{bmatrix}$ ,

find:

a.  $A + 2B$

b.  $2A - B$

c.  $AB$

1+1+1 = 3 marks

**Question 2** (2 marks)

If  $\begin{bmatrix} a & 2 \\ 6 & -6 \end{bmatrix} + \begin{bmatrix} -3 & b \\ c & 2 \end{bmatrix} = \begin{bmatrix} 6 & -3 \\ -2 & d \end{bmatrix}$ , find the values of  $a$ ,  $b$ ,  $c$  and  $d$ .

2 marks

**Question 3** (3 marks)

Steve buys 4 kg of bananas and 3 kg of apples, for which he pays \$23.75. From the same fruit shop, Donna buys 2.5 kg of bananas and 2 kg of apples, for which she pays \$15.25. Using matrix methods find the cost per kilogram of the bananas and apples.

3 marks

**Question 4** (2 marks)

For the matrix equation  $RS = T$  where  $S = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$  and  $T = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$ , find matrix  $R$ .

2 marks

**Question 5** (2 marks)

Show why there is no unique solution to this system of simultaneous equations.

$$\begin{aligned} 2x - 3y &= 5 \\ -6x + 9y &= -15 \end{aligned}$$

2 marks

**Question 6** (9 marks)

Phil and Peter are property investors. Phil has a portfolio of 2 houses, 3 apartments and 2 commercial properties, while Peter has 1 house, 2 apartments and 3 commercial properties.

- a. Express this information as a  $2 \times 3$  matrix  $P$ .

1 mark

The following table shows the fixed annual costs in dollars, associated with the maintenance of these properties, and includes insurance, water rates, council rates and agents' fees.

	<b>Insurance</b>	<b>Water rates</b>	<b>Council rates</b>	<b>Agents' fees</b>
<b>Houses</b>	350	700	1200	1800
<b>Apartments</b>	230	550	750	1440
<b>Commercial</b>	800	1250	1500	2140

- b. i. Write the information in the table as a  $3 \times 4$  matrix,  $Q$ .

1 mark

- ii. Find  $PQ$  and interpret the meaning of the result.

2 marks

- c. The next year, both Phil and Peter find that some of their costs have changed. Water has decreased by 3% and council rates increased by 7%. Using a scalar matrix and matrix multiplication find the new  $3 \times 4$  costs matrix,  $Q'$ .

2 marks

- d. Find the product of  $P$  and  $Q'$  and interpret the result.

2 marks

- e. From the answer to d) state Peter's total water rates for the year.

**Question 7** (8 marks)

In the local badminton round robin, the results were:

- Diamods(D) beat Heat(H), Ice(I) and Roar(R)
- Heat beat Ice and Lakers(L)
- Ice beat Lakers
- Lakers beat Diamods
- Roar beat Heat, Ice and Lakers

**a.** Construct a directed graph to show the results of the competition.

2 marks

**b.** Construct a dominance matrix,  $D_1$ . Complete the matrix with the order of the columns as D, H, I, R and L.

2 marks

**c.** No team can be declared the winner of the tournament purely on their one-step dominances. Explain why.

1 mark

**d.** Find the first, second and third placegetters of the tournament based on one-step and two-step dominances.

2 marks

**e.** In the following year another three teams joined the competition. Find the number of additional games that need to be played to complete the round robin.

1 mark

**Question 8** (4 marks)

- a. Use matrix multiplication (show working) to construct a matrix that displays row sums of the matrix

$$\begin{bmatrix} 2 & 2 & 5 \\ 3 & 3 & -2 \\ 2 & -3 & 1 \end{bmatrix}$$

2 marks

- b. Hence use matrix multiplication to construct a matrix that displays column sum of the matrix found in part a)

2 marks

**End of Holiday Homework**