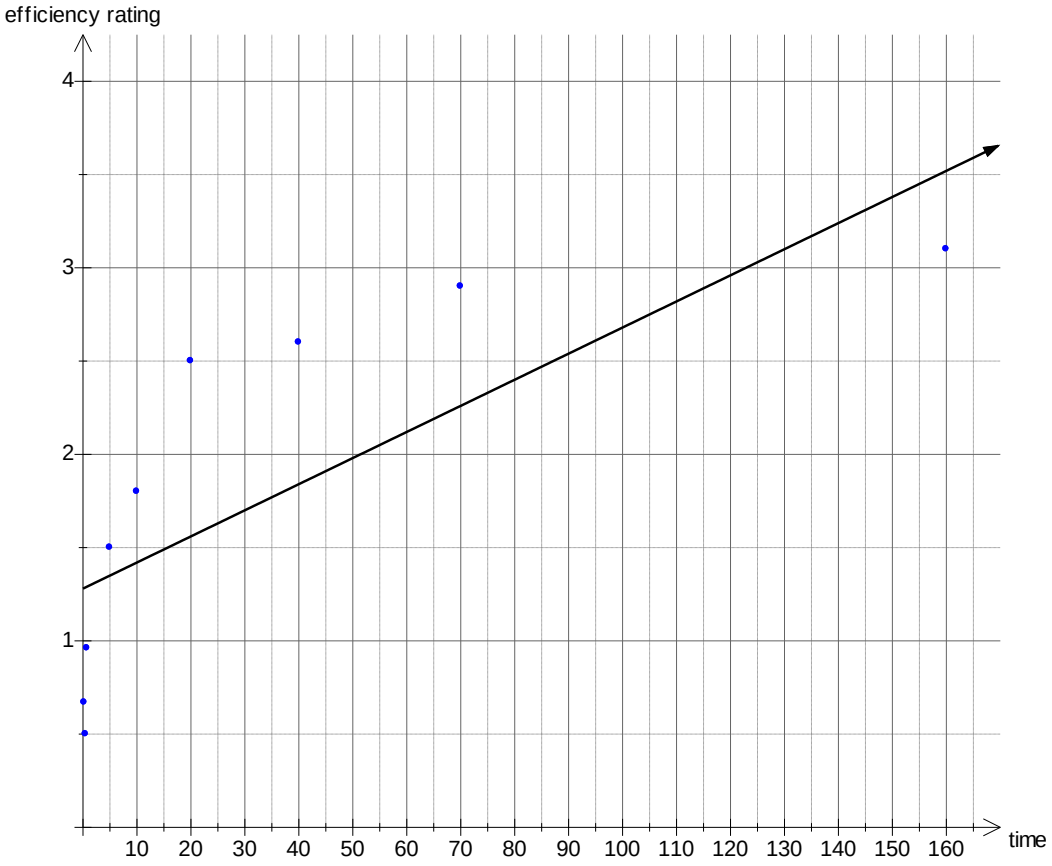
FURTHER MATHEMATICS

SAC 1 REVISION - TRANSFORMATIONS (SOLUTIONS)

The time spent studying in preparation for a SAC and the efficiency rating during the SAC for a number of students has been recorded in the table below:

Time	0.25	0.5	0.75	5.0	10	20	40	70	160
Efficiency rating	0.67	0.5	0.96	1.5	1.8	2.5	2.6	2.9	3.1

a) Construct a scatterplot on the axes below. Don't forget to label the axes.



b) Calculate the coefficient of determination (to the nearest whole percentage) and interpret your results in terms of the variables **efficiency rating** and **time**.

58% of the variation in efficiency rating can be explained by the variation in time studying.

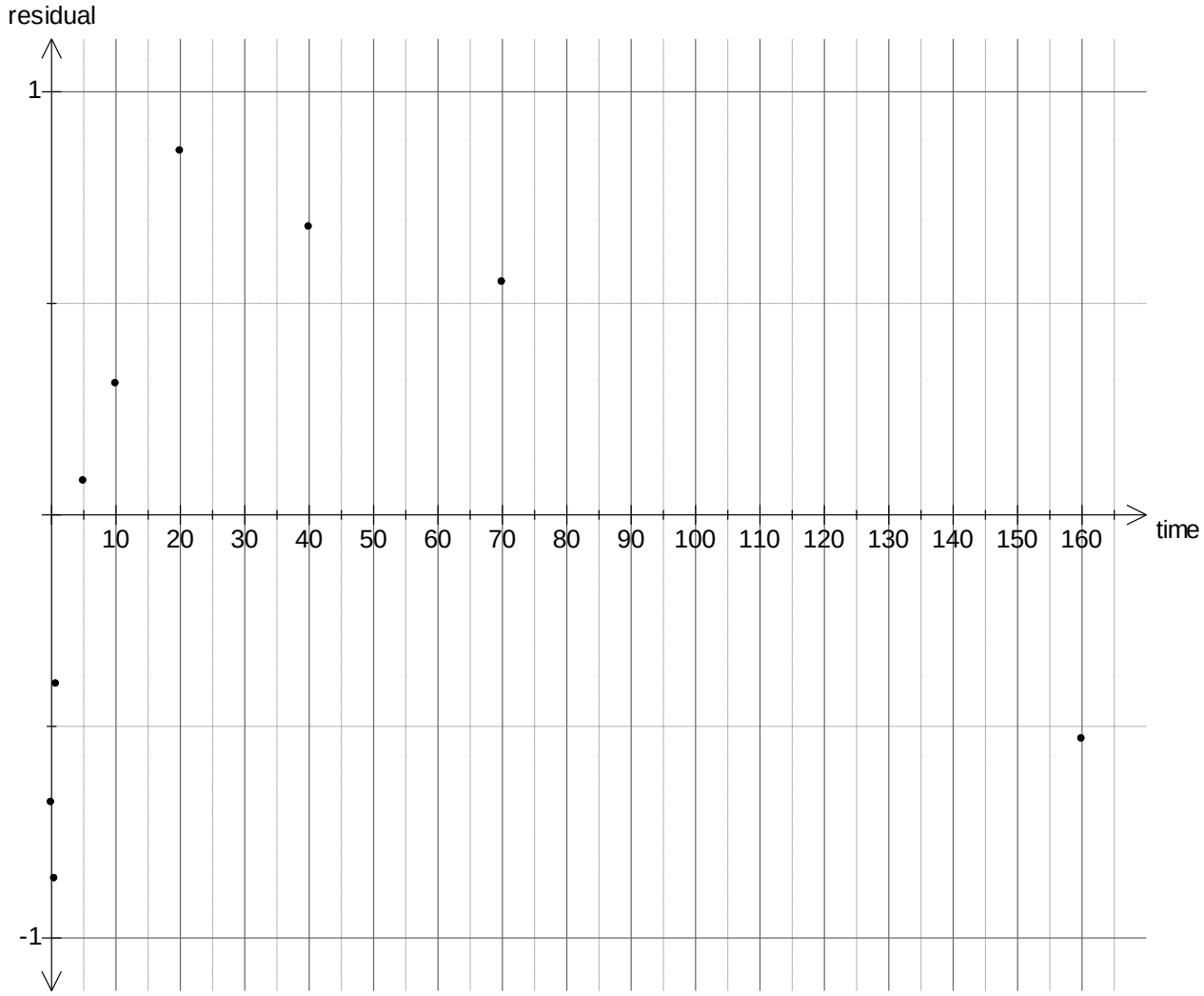
c) Use your calculator to determine the equation of the least squares regression line. (make sure you write it in terms of the given variables) State coefficients correct to three decimal places.

$efficiency\ rating = 1.351 + 0.014 \times time$

d) Use the model found in **partc)** to draw this line on the scatterplot.

To determine if a linear model is appropriate a residual plot is constructed.

e) Use the following axes to construct a residual plot for the least squares model.

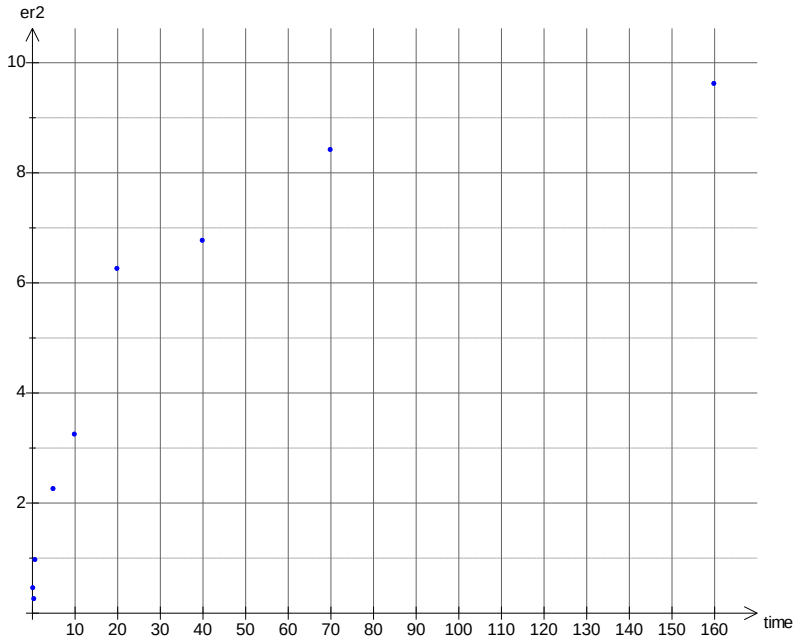


f) Explain why this residual plot supports the suggestion that this model is **not** linear.

Because there is a clear pattern in the residuals it suggests the original data is not linear.

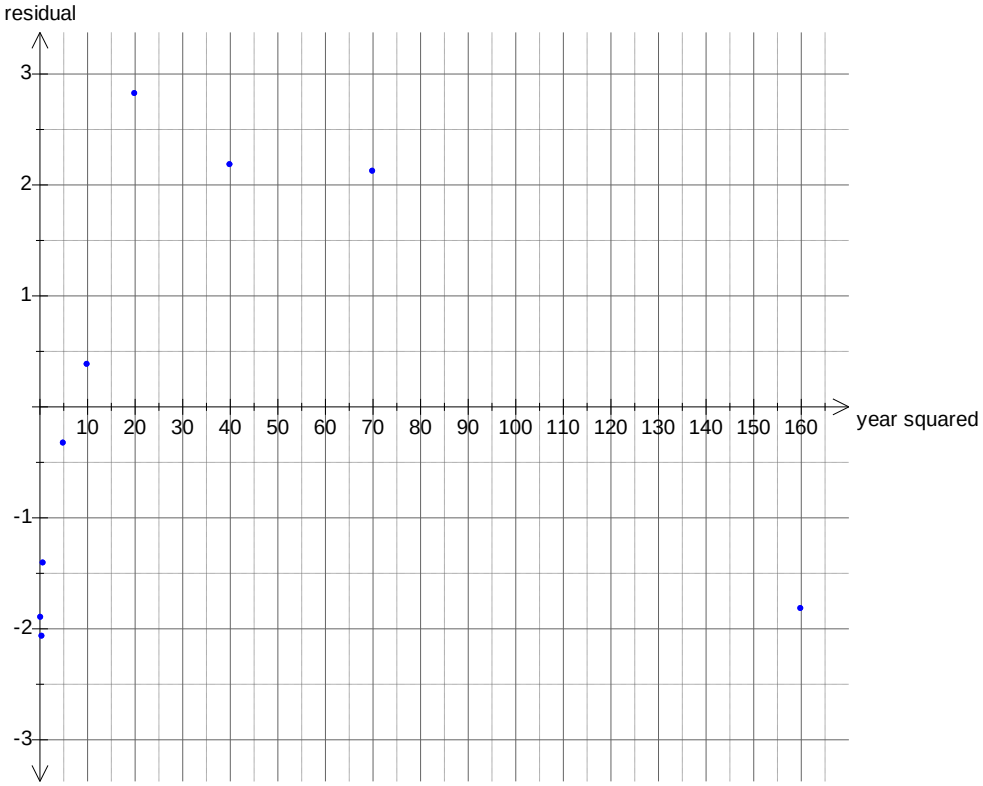
2. To transform this data a number of transformations are possible. To determine whether these will be successful, you are required to complete a y^2 transformation and a $\log_{10}(x)$ transformation. Give all results to three decimal places. Suitable grids have been supplied below:

a) *efficiency rating*^{2} **transformation**

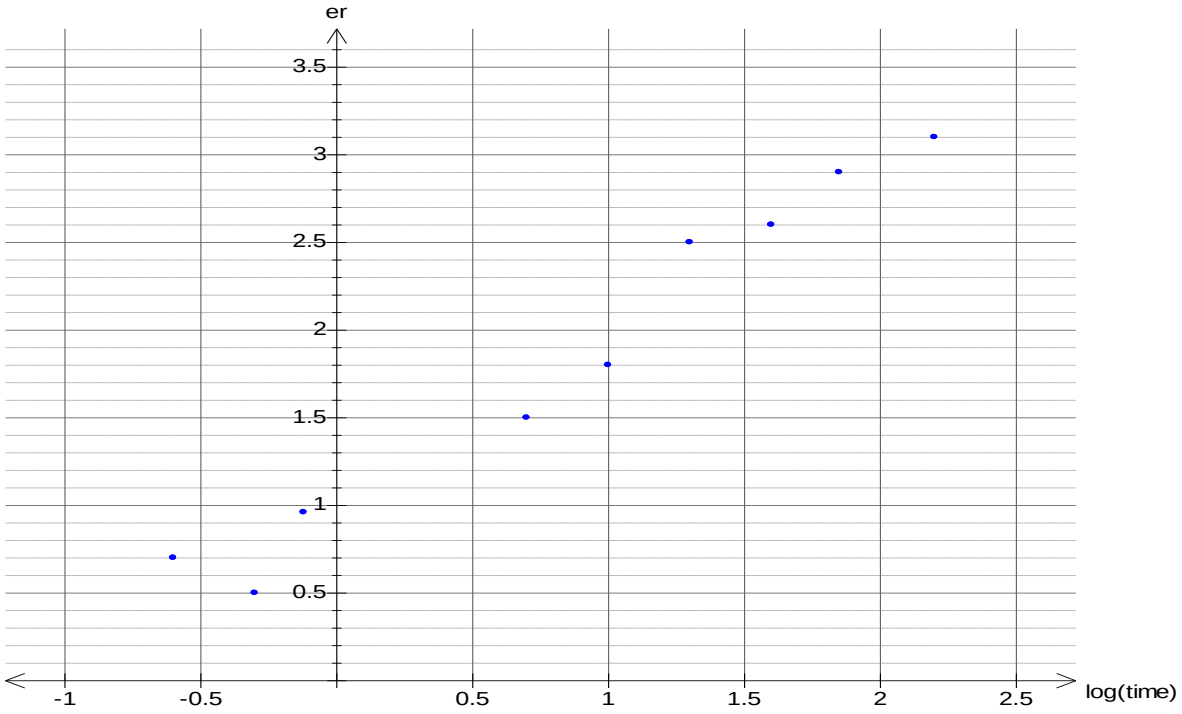


$$efficiency\ rating^2 = 2.293 + 0.057 \times time$$

$$r^2 = 0.704 \approx 70.4\%$$

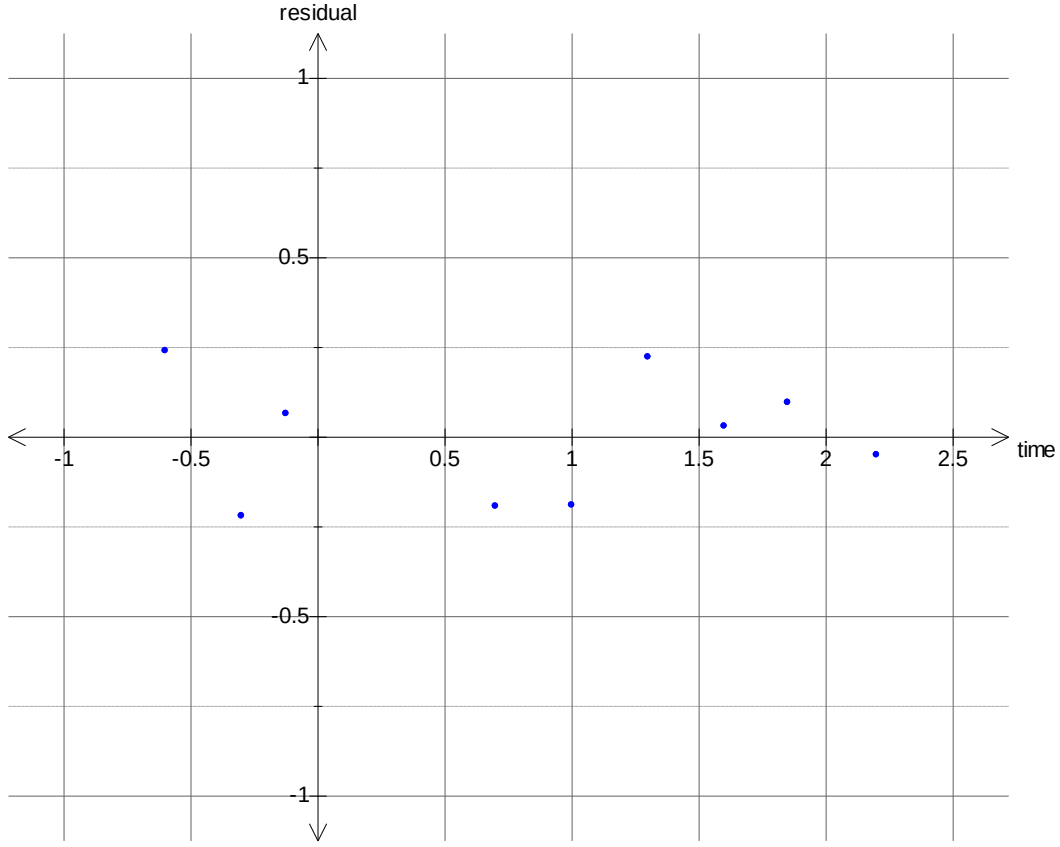


b) $\log_{10}(time)$ **transformation**



$$efficiency\ rating = 1.016 + 0.969 \times \log(time)$$

$$r^2 = 0.968 \approx 96.8\%$$



c) Using the results obtained above choose the best model to predict population from year:

Original Linear Model	(y) squared	log(x) transformation

d) Explain your choice made in part c) using appropriate results from **parts a) and b).**

Given the log(time) transformation yields the highest r^2 value of 96.8% and the residual plot shows no clear pattern it would be the best model. Because both the original and the efficiency rating^2 transformation had patterned residuals they are not suitable.

e) Using the preferred model, predict the efficiency rating, correct to 1 decimal place, (showing all working) of a student who does:

(i) 10 hours of study

$$efficiency\ \ rating = 1.016 + 0.969 \times \log(10)$$
$$efficiency\ \ rating = 2.0$$

(ii) Explain whether you believe this would be a reliable prediction, stating a clear reason.

This would be considered reliable as it is an example of interpolation.