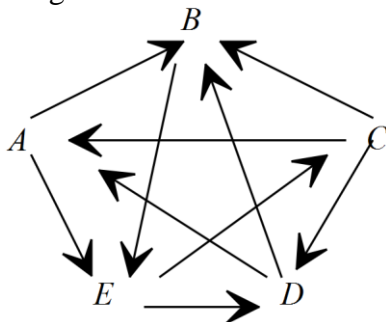


# SAC 3 Revision Solutions

## Question 1

Diagram



The one-step dominance score for each team does not give a clear ranking.

$$D = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}, D^2 = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 2 \\ 2 & 2 & 0 & 1 & 0 \end{bmatrix}$$

$$T = D + D^2 = \begin{bmatrix} 0 & 1 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 & 1 \\ 2 & 3 & 0 & 1 & 2 \\ 1 & 2 & 0 & 0 & 2 \\ 2 & 2 & 1 & 2 & 0 \end{bmatrix} \begin{matrix} 5 \\ 3 \\ 8 \\ 5 \\ 7 \end{matrix}$$

place	
1 <sup>st</sup>	Condors
2 <sup>nd</sup>	Emus
3 <sup>rd</sup>	Anchors, Dragons
5 <sup>th</sup>	Bull Dogs

So the winning team were the Condors.

**Diagram 2 marks,  $D$  1 mark,  $D^2$  1 mark,  $T$  1 mark, places 1 mark, winner 1 mark**

## Question 2

- 80%
- 9%

c.

	From		
	$P$	$Q$	$R$
To $P$	0.8	0.05	0.09
To $Q$	0.08	0.85	0.06
To $R$	0.12	0.1	0.85

- 50% of 800 = 400  
25% of 800 = 200

$$N_1 = \begin{bmatrix} 400 \\ 200 \\ 200 \end{bmatrix} \begin{matrix} P \\ Q \\ R \end{matrix}$$

- $N_2 = T \times N_1$

$$\begin{bmatrix} 0.8 & 0.05 & 0.09 \\ 0.08 & 0.85 & 0.06 \\ 0.12 & 0.1 & 0.85 \end{bmatrix} \begin{bmatrix} 400 \\ 200 \\ 200 \end{bmatrix} = \begin{bmatrix} 348 \\ 214 \\ 238 \end{bmatrix}$$

Therefore, Physical has 348 members at the end of the second year whereas Quality has 214 and Rigour 238.

$$N_3 = T^2 \times N_1 = \begin{bmatrix} 310.52 \\ 224.02 \\ 265.46 \end{bmatrix}$$

Therefore, Physical has 311 members at the end of the third year whereas Quality has 224 and Rigour 265.

$$N_4 = T^3 \times N_1 = \begin{bmatrix} 283.5084 \\ 231.1862 \\ 285.3054 \end{bmatrix}$$

Therefore, Physical has 284 members at the end of the fourth year whereas Quality has 231 and Rigour 285.

The memberships for Physical are decreasing over the first 4 years  
 The memberships for Quality are increasing over the first 4 years  
 The memberships for Rigour are also increasing over the first 4 years  
**1 mark for each state matrix, 1 mark for discussion**

f. end of second year = 238 (from e)  
 end of first year = 200  
 So Rigour has 38 more.

g.

$$\begin{bmatrix} 0.8 & 0.05 & 0.09 \\ 0.08 & 0.85 & 0.06 \\ 0.12 & 0.1 & 0.85 \end{bmatrix}^{50} \begin{bmatrix} 400 \\ 200 \\ 200 \end{bmatrix} = \begin{bmatrix} 214 \\ 249 \\ 337 \end{bmatrix}$$

$$\begin{bmatrix} 0.8 & 0.05 & 0.09 \\ 0.08 & 0.85 & 0.06 \\ 0.12 & 0.1 & 0.85 \end{bmatrix}^{51} \begin{bmatrix} 400 \\ 200 \\ 200 \end{bmatrix} = \begin{bmatrix} 214 \\ 249 \\ 337 \end{bmatrix}$$

In the long run the Physical tennis centre has the lowest member numbers, 214.

**1 mark for answer, 1 mark 2 consecutive calculations**

### Question 3

a. 3 guest passes were sold last month for mainstream zumba classes.

b.  $7x + 8y + 10z = 440$   
 $4x + 5y + 3z = 209$   
 $2x + 3y + z = 103$

**1 mark for one correct equation, 2 marks for all correct**

c. 
$$\begin{bmatrix} 7 & 8 & 10 \\ 4 & 5 & 3 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 440 \\ 209 \\ 103 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 & 8 & 10 \\ 4 & 5 & 3 \\ 2 & 3 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 440 \\ 209 \\ 103 \end{bmatrix}$$

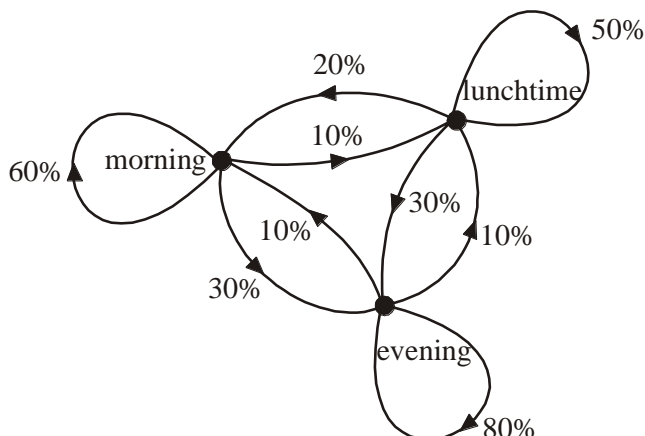
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 20 \\ 15 \\ 18 \end{bmatrix}$$

The cost of a guest pass to a cardio class is \$20, to a jump class is \$15 and to a zumba class is \$18.

**1 mark for use of inverse, 1 mark for sentence answer**

**Question 4**

- a. 30% of people change each week from a lunchtime class to an evening class.  
 b.



**1 mark arrows, 1 mark %, 1 mark all correct**  
 this week

c.

$M$	$L$	$E$	
$T = \begin{bmatrix} 0.6 & 0.2 & 0.1 \\ 0.1 & 0.5 & 0.1 \\ 0.3 & 0.3 & 0.8 \end{bmatrix}$			$M$
			$L$ next week
			$E$

**1 mark for 3 values correct, 1 mark for all correct**

d.

$$S_2 = T \times S_1$$

$$= \begin{bmatrix} 0.6 & 0.2 & 0.1 \\ 0.1 & 0.5 & 0.1 \\ 0.3 & 0.3 & 0.8 \end{bmatrix} \begin{bmatrix} 30 \\ 20 \\ 40 \end{bmatrix}$$

$$= \begin{bmatrix} 26 \\ 17 \\ 47 \end{bmatrix}$$

There will be 47 people in the evening yoga class in week 2.

e.

$$S_4 = T^3 \times S_1$$

$$= \begin{bmatrix} 22.43 \\ 15.32 \\ 52.25 \end{bmatrix}$$

There will be 15 people (to the nearest whole number) in the lunchtime class in week 4.

- f. Looking at the long term so we see if we can find a steady state to see if any classes will be discontinued.

$$S_{15} = T^{14} \times S_1$$

$$= \begin{bmatrix} 21 \\ 15 \\ 54 \end{bmatrix}$$

Over the long term, the minimum class size is 15 so lunchtime classes will be discontinued.

$$S_{16} = T^{15} \times S_1$$

$$= \begin{bmatrix} 21 \\ 15 \\ 54 \end{bmatrix}$$

To investigate when it will be discontinued use trial and error and the fact that week 2 was 17 and week 4 was 15.

Now,  $S_3 = T^2 \times S_1$

$$= \begin{bmatrix} 24 \\ 16 \\ 50^* \end{bmatrix}$$

After the week 3 class it is still 16 so it would be cancelled after week 4 as this is the first time (using whole numbers) that the lunchtime class is below 16.

\*50 was rounded down rather than up as if all 3 values were rounded up there would be 91 people rather than 90.

**1 mark for lunchtime discontinued, 1 mark for after week 4**

### Question 5

- a. B matrix means that 6 additional people will join the classes each week, 4 in Barry's and 2 in Tina's.  
b.

$$N_2 = AN_1 + B$$

$$= \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 32 \\ 24 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 20 \\ 26 \end{bmatrix}$$

- c.

$$N_{n+1} = AN_n + B$$

So,  $N_3 = AN_2 + B$

$$= \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 20 \\ 26 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 14 \\ 28 \end{bmatrix}$$

$N_1, N_2$  have already been given or calculated

$$N_4 = AN_3 + B$$

$$= \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 14 \\ 28 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 11 \\ 30 \end{bmatrix}$$

Over the 4-week period Brian's classes decrease in size and Tina's classes increase in size.

**2 marks for state matrices, 1 mark for explanation**

### Question 6

Need to first find the cost of each item at each outlet shop.

You could write out the 3 simultaneous equations first

#### Walker outlet

$$\begin{bmatrix} 5 & 25 & 11 \\ 9 & 19 & 7 \\ 3 & 8 & 5 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 671.50 \\ 874.50 \\ 348.00 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 5 & 25 & 11 \\ 9 & 19 & 7 \\ 3 & 8 & 5 \end{bmatrix}^{-1} \begin{bmatrix} 671.50 \\ 874.50 \\ 348.00 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 75 \\ 3.5 \\ 19 \end{bmatrix}$$

So at Walker outlet racquets are \$75, Hats are \$3.50 and Balls are \$19.

Cost of 1 racquet, 5 balls and 2 hats at Walker outlet is \$130.50

#### Fitness outlet

$$\begin{bmatrix} 6 & 13 & 9 \\ 10 & 22 & 3 \\ 10 & 25 & 12 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 676.50 \\ 876.00 \\ 1069.50 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 6 & 13 & 9 \\ 10 & 22 & 3 \\ 10 & 25 & 12 \end{bmatrix}^{-1} \begin{bmatrix} 676.50 \\ 876.00 \\ 1069.50 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 78 \\ 1.5 \\ 21 \end{bmatrix}$$

So at the Fitness outlet racquets are \$78, Hats are \$1.50 and Balls are \$21.

Cost of 1 racquet, 5 balls and 2 hats at Fitness outlet is \$127.50

Therefore Fitness outlet is cheaper to buy 1 racquet, 5 balls and 2 hats.

**1 mark for correct set up of matrix equations, 1 mark for use of an inverse, 1 mark for each outlets sentence answer, 1 mark for Walker outlet is \$130.50, 1 mark for at Fitness outlet is \$127.50, 1 mark Fitness cheaper**

### Question 7

- $f_{2,3} = 1$
- Bob and Con communicate directly with each other using a social network.
- Mark and Bob cannot communicate directly with each other using social network.
- Mark and Bob can use the social network to communicate via a third person (Con).
- The diagonal elements of the matrix indicate redundant communication links. Therefore, there are 8 redundant links.

### Question 8

$$\begin{array}{cccc} & H & M & A & K \\ \text{a. } D = & \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

- Not possible. The one-step dominance totals are Hunter 1, Max 2, Archer 1, Knox 2 so there is no clear ranking.
- Knox defeated Hunter who has dominance over one other (Archer) and Knox defeated Archer who has dominance over one other (Max). Therefore Knox has a value of 2 for two-step dominance.

d. First Max, second Knox, Third Archer, Fourth Hunter.

Dominance total  
2  
5  
3  
4

1 mark for ranking, 1 mark row totals

**Question 9**

a. 
$$\begin{bmatrix} 0.8 & 0 & 0 & 0 \\ 0.2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.2 \\ 0 & 0 & 0 & 0.7 \end{bmatrix} \begin{bmatrix} 27 \\ 18 \\ 12 \\ 17 \end{bmatrix} = \begin{bmatrix} 21.6 \\ 23.4 \\ 15.4 \\ 13.6 \end{bmatrix}$$

b. The second and third row show the results. Magnus retained his points but also gained 20% of Jason’s points. Therefore Magnus beat Jason. Similarly, Agnes defeated Beatrice

c. Need to do a matrix multiplication.

$$\begin{bmatrix} 0.90 & 0.10 & 0 & 0.1 \\ 0.05 & 0.80 & 0.05 & 0 \\ 0 & 0.10 & 0.80 & 0.10 \\ 0.05 & 0 & 0.15 & 0.80 \end{bmatrix} \begin{bmatrix} 16 \\ 16 \\ 20 \\ 22 \end{bmatrix} = \begin{bmatrix} 18.2 \\ 14.6 \\ 19.8 \\ 21.4 \end{bmatrix}$$

**Question 10**

- a. Only one 1 in each column and row and only zero’s in the leading diagonal so the chef never chooses the same dishes two days in a row.
- b. Pasta
- c. Risotto
- d. Monday =Fish

Tue	Wed	Thur	Fri	Sat	Sun
Risotto	Chicken	Pasta	Fish	Risotto	Chicken