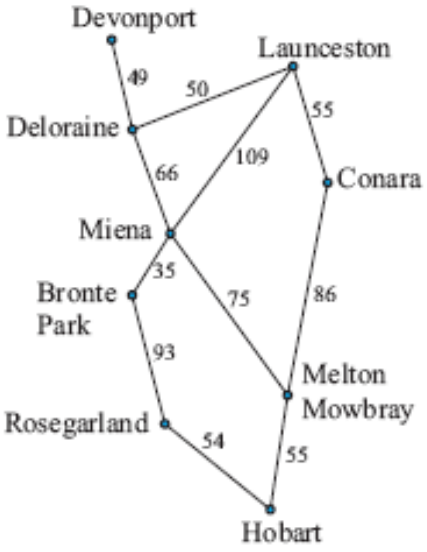
SAC4 REVISION - SOLUTIONS

Only your own handwritten solutions can be bound into your book!

QUESTION 1



a) State the degree of Devonport and Miena.
Devonport = 1 and Miena = 4

b) You plan to travel around to the towns. Where possible identify a route that would satisfy the following criteria.

i. A walk

Many possible answer

ii. A cycle

Many possible answers, no repeated vertices or edges, start and finish the same

iii. An Eulerian trail

Not possible as there are more than 2 vertices that have an odd degree.

c) Find the shortest distance between:

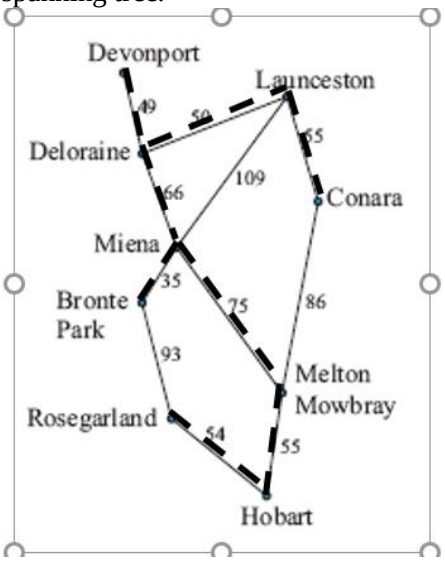
Bronte Park – Melton Mowbray – 35+75 = 110 km
Hobart – Launceston – 55+86+55 = 196 km
Miena – Conara – 75+86 = 161 km

d) Apply Dijkstra's algorithm to find the shortest distance from Devonport to Hobart

	De	L	M	C	BP	MM	R	H
D	49	X	X	X	X	X	X	X
De	49	99	115	X	X	X	X	X
L	49	99	115	154	X	X	X	X
M	49	99	115	154	150	190	X	X
BP	49	99	115	154	150	190	243	X
C	49	99	115	154	150	190	243	X
MM	49	99	115	154	150	190	243	245
R	49	99	115	154	150	190	243	245

The shortest path is Devonport – Deloraine – Miena – Melton Mowbray – Hobart
Length = **49+66+75+55 = 245 km**

e) Find the minimal length spanning tree for this network. Highlight this on the diagram. State the value of this spanning tree.



Total length = 49+50+55+66+35+75+54+55 = 439 km

f) A tourist wishes to travel to Tasmania and visit each of the towns on the diagram. Describe the route the tourist should take to minimise the distance travelled. The tourist must start and end their tour in Devonport.

Devonport-Deloraine-Miena-Bronte park-Rosegarland-Hobart-Melton Mowbray-Conara-Launceston-Deloraine-Devonport Total length = 592km

g) A second tourist is more interested in the scenery between all of the towns. Is it possible for him to do this without travelling on each road more than once? Explain.

No, as this would require an Euler Circuit and this is only possible if all vertices have even degree. This is not the case.

h) Describe the route the tourist should take if he wishes to travel on each road without travelling an unnecessary distance. NB: he may need to travel on each road more than once.
Devonport-Deloraine-Launceston-Conara-Melton Mowbray-Hobart-Rosegarland-Bronte Park-Miena-Melton Mowbray-Conara-Launceston-Miena-Deloraine-Devonport
Total Distance = 1032km

If not starting at Devonport:

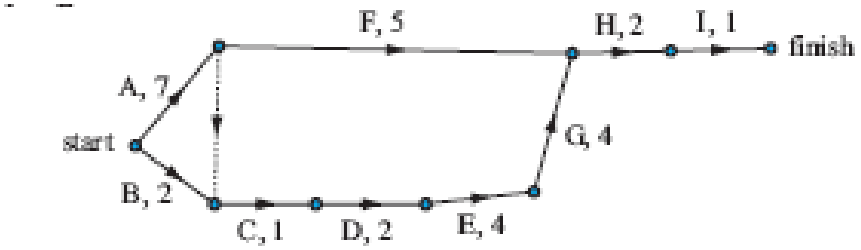
Launceston – Conara – Melton – Hobart –Rosegarland – Bronte – Miena – Deloraine – Devonport – Deloraine – Launceston – Miena – Melton = 776 km.

QUESTION 2

The table below gives the activities involved in landscaping a garden:

Activity	Description	Duration(days)	Predecessor	EST	LST	Float
A	Design garden	7	-	**0**	**0**	**0**
B	Clear ground area	2	-	**0**	**5**	**5**
C	Peg out design	1	A B	**7**	**7**	**0**
D	Complete heavy digging	2	C	**8**	**8**	**0**
E	Install watering system	4	D	**10**	**10**	**0**
F	Buy plants	5	A	**7**	**13**	**6**
G	Pave pathways	4	E	**14**	**14**	**0**
H	Plant trees and shrubs	2	F G	**18**	**18**	**0**
I	Plant lawn	1	H	**20**	**20**	**0**

a) Construct a diagram to represent the project:



b) Find the critical path and its length. Explain why this path is the longest path.

A-C-D-E-G-H-I 21 days

c) Fill in the columns in the table above.

d) State the float/slack time for activity F and explain its meaning.

Float time = 6 days This means that activity F can be delayed up to 6 days without affecting the overall length of the project

e) There has been a delay in the activity B. It is now going to take 6 days longer. ie it will now take 8 days. What affect will this have on the completion time for the project.

This will delay the project by 1 day. That is the delay will use up the float of 5 days and then add an extra day. This will mean the project will now take 22 days

f) An alternative company can help to complete activity B, but it will cost $100/day. What would be the best strategy for the project manager?

The project manager should hire the extra help for one day only at a cost of $100.
Remind students that they want to complete the project on time!

QUESTION 3

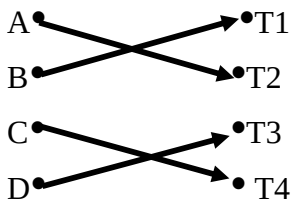
Operators A, B, C and D are to be allocated each of four tasks T1, T2, T3 and T4. They have estimated the time in days that each of these tasks will take them. These are summarised in the matrix. How should the tasks be assigned so that the total time for the four tasks is a minimum?

	T1	**T2**	**T3**	**T4**
A	4	9	8	13
B	5	11	10	16
C	3	12	7	12
D	6	8	6	15

a) Determine the most effective allocation of operators to these tasks.

$$\begin{bmatrix} 0 & 5 & 4 & 9 \\ 0 & 6 & 5 & 11 \\ 0 & 9 & 4 & 9 \\ 0 & 2 & 0 & 9 \end{bmatrix} \quad \begin{bmatrix} 0 & 3 & 4 & 0 \\ 0 & 4 & 5 & 2 \\ 0 & 7 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 4 & 1 & 0 \\ 3 & 0 & 0 & 3 \end{bmatrix}$$

Not required



A - T2
B – T1
C – T4
D – T3 **Total time = 32 days**

b) How would you assign the task if the numbers now represent savings?

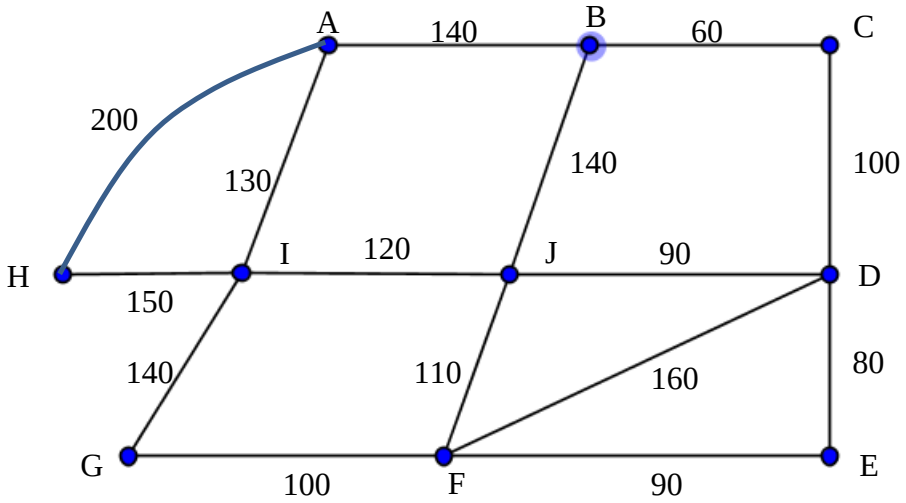
$$\begin{bmatrix} 12 & 7 & 8 & 3 \\ 11 & 5 & 6 & 0 \\ 13 & 4 & 9 & 4 \\ 10 & 8 & 10 & 1 \end{bmatrix} \quad \begin{bmatrix} 9 & 4 & 5 & 0 \\ 11 & 5 & 6 & 0 \\ 9 & 0 & 5 & 0 \\ 9 & 7 & 9 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 4 & 0 & 0 \\ 2 & 5 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 7 & 4 & 0 \end{bmatrix}$$

A – T3
B – T4
C – T2
D – T1 **Total savings = 8+16+12+6 = 42**

Depending on the method taught values in the steps may vary but the final allocation and time will be the same.

Question 4

Recyclable materials are to be collected from households in a part of a particular suburb. The network below represents this area, where the dots are street intersections and the edges are the streets. The numbers on each edge indicate the length, in metres, of the streets between each intersection.



a) Does this represent a planar graph? Explain.

Yes because all edges are drawn without crossing over each other.

b) Verify Euler's formula for this graph.

Euler's formula **V + F = E + 2**

10 + 7 = 15 + 2 **True.**

c) The collectors wish to travel along each street once only, in order to keep travel distance to a minimum. What is the mathematical term used to describe this route? Determine this route.

As there are two vertices of odd degree an Euler trail is possible (start different to finish)

Euler's trail. A – B – C – D – E – F – D – J – F – G – I - H – A – I – J - B

d) As well as travelling along each street once, it is decided that they would like to start and finish at the same intersection. What sort of route is this? If this is possible, determine the route. If it is not, explain what would need to happen for it to be possible? Describe the route that makes this possible and its total length.

Euler circuit – this is not possible as all vertices must be of even degree, which they are not. Both vertex A and B are of odd degree.

The collectors would need to repeat an edge linking the two vertices of odd degree. Best to repeat A-B.

Euler's circuit. A – B – C – D – E – F – D – J – F – G – I - H – A – I – J – B – A

Total length is 1950 metres (Note this is all the edges added together with an extra 140.

e) The Transport Department has placed traffic density monitors at each intersection. A departmental officer wishes to collect all the monitors by visiting each intersection once only. What type of path is required? Describe a path that she could take in order for her to collect all the monitors and minimize the distance. It may be more efficient to revisit an intersection but remember an overall minimum distance is required.

A Hamilton path:

J-D-E-F-G-I-H-A-B-C **Total length = 1050m** **(this is not the minimum)**

Revisiting a vertex

G-F-E-D-J-B-C-B-A-I-H **Total length = 1040m** **(this is not a Hamiltonian path)**

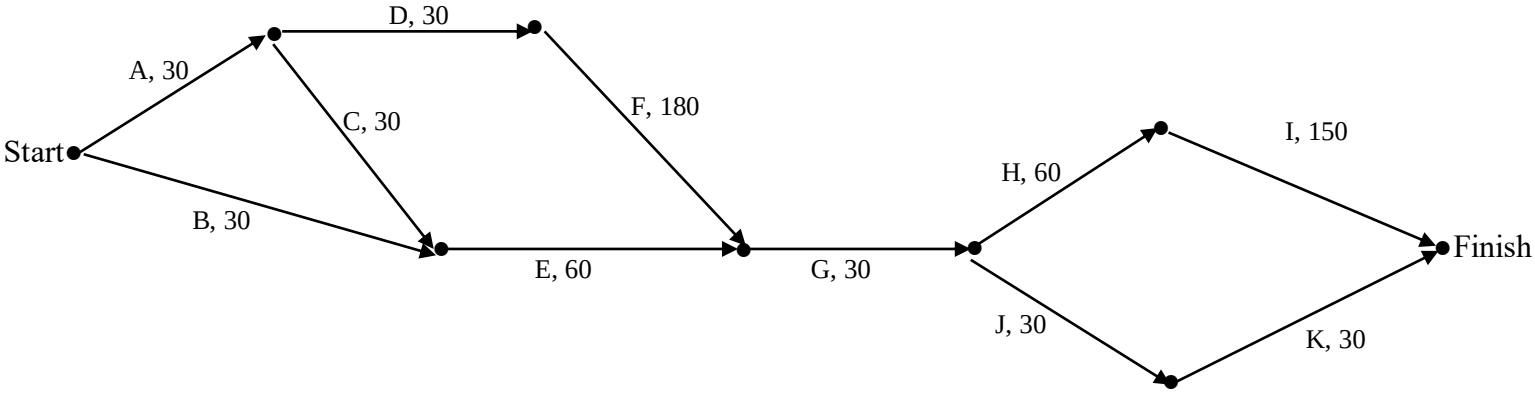
H-I-A-B-C-D-J-D-E-F-G **Total length = 1030m** **(this is not a Hamiltonian path)**

QUESTION 5

The day before a bike ride Carli and Natasha are scheduling some last minute preparation. The activities they hope to get through are listed in the table below:

Activity	Time (minutes)	Predecessor	EST	LST
A	30	-	0	0
B	30	-	0	150
C	30	A	30	150
D	30	A	30	30
E	60	B, C	60	180
F	180	D	60	60
G	30	E, F	240	240
H	60	G	270	270
I	150	H	330	330
J	30	G	270	420
K	30	J	300	450

a) Draw a network diagram for the schedule above.



b) Determine the earliest and latest starting times for each activity. You could add extra columns to the table.

c) Identify the critical path and the shortest time it will take to complete the day's preparation.

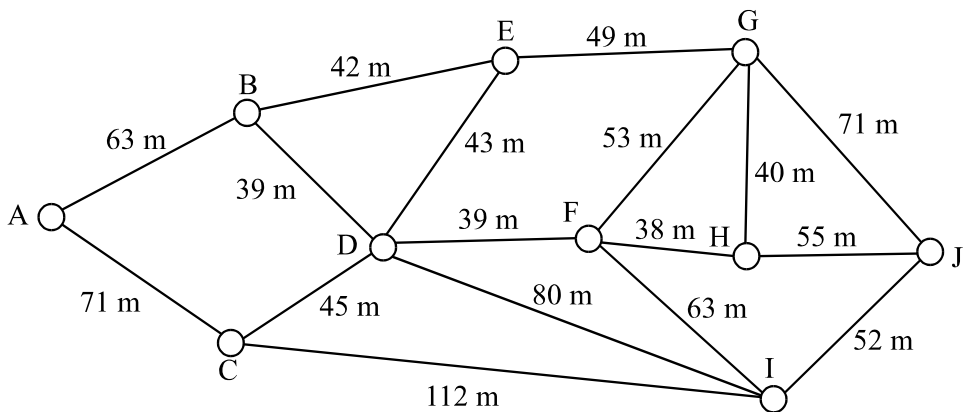
A – D – F – G – H – I 480 minutes

d) Determine the activity that can be delayed the longest without the overall time for the preparation being delayed.

The activities that have the longest float times are activities B, J and K. They all have a float of 150 minutes which tells us that they can be delayed up to 150 minutes without delaying the overall completion time.

QUESTION 6

A new housing estate is being developed and the network below shows possible routes for underground cabling between house blocks in a section of the estate.



The distance in metres of each of the possible routes is shown in the diagram.

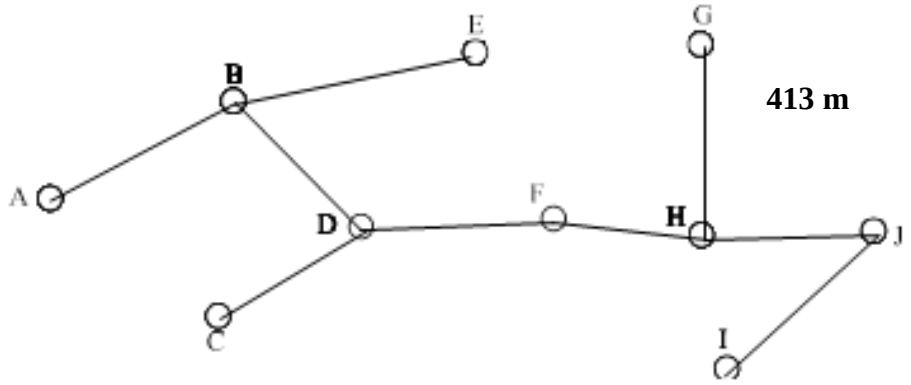
a) Apply Dijkstra's algorithm to find the shortest distance between the house block at A and the one at J.

	B	C	D	E	F	G	H	I	J
A	63	71	X	X	X	X	X	X	X
B	63	71	102	105	X	X	X	X	X
C	63	71	102	105	X	X	X	183	X
D	63	71	102	105	141	X	X	182	X
E	63	71	102	105	141	154	X	182	X
F	63	71	102	105	141	154	179	182	X
G	63	71	102	105	141	154	179	182	225
H	63	71	102	105	141	154	179	182	225
I	63	71	102	105	141	154	179	182	225

A – B – E – G – J 225 metres.

Can also be done on diagram

b) Draw the minimal-length spanning tree which will show the shortest length of underground cabling required so that all the house blocks are connected to one another.



c) If the cable costs $3/metre and the connectors used per house block are $5 each, what is the total cost of the cabling for this estate?

Total cost = 413 x 3 + 5 x 10 = $1289

QUESTION 7

For this housing project, the development company set up a special team in their accounting department. The supervisor of the team estimates the time, in hours, it would take each member to make a report on four different aspects of the project. His estimations are shown in Table 1 below.

	Costing	Finance	Cash-flow	Planning
Kate	6	7	8	6
Monica	8	8	5	7
Gareth	4	4	7	5
Ed	5	4	3	3

Table 1

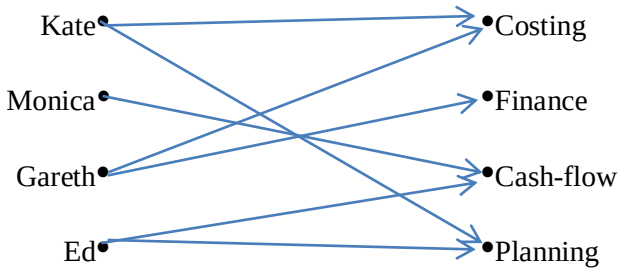
a) Using the Hungarian algorithm, find the allocation of people to writing particular reports and state the total time to complete all reports so that the time taken is minimised.

Only row allocation required:

Kate	**Costing**	**6**
Monica	**Cash-flow**	**5**
Gareth	**Finance**	**4**
Ed	**Planning**	**3**

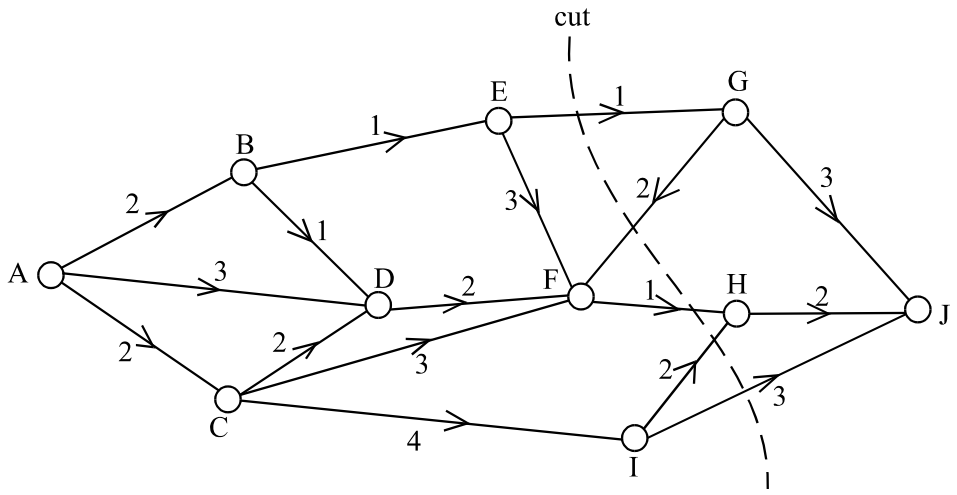
Total time required is 18 hours.

b) Draw a bipartite graph that represents all possible report allocations for each person



QUESTION 8

This section of the new housing estate covers some steep areas. The network below shows a proposed drainage system with the amount (in megalitres, ML) of water each section of the drain can cope with in an hour.
Because of the slope of the land the drainage system flows from A to J. A cut has been made in the network.



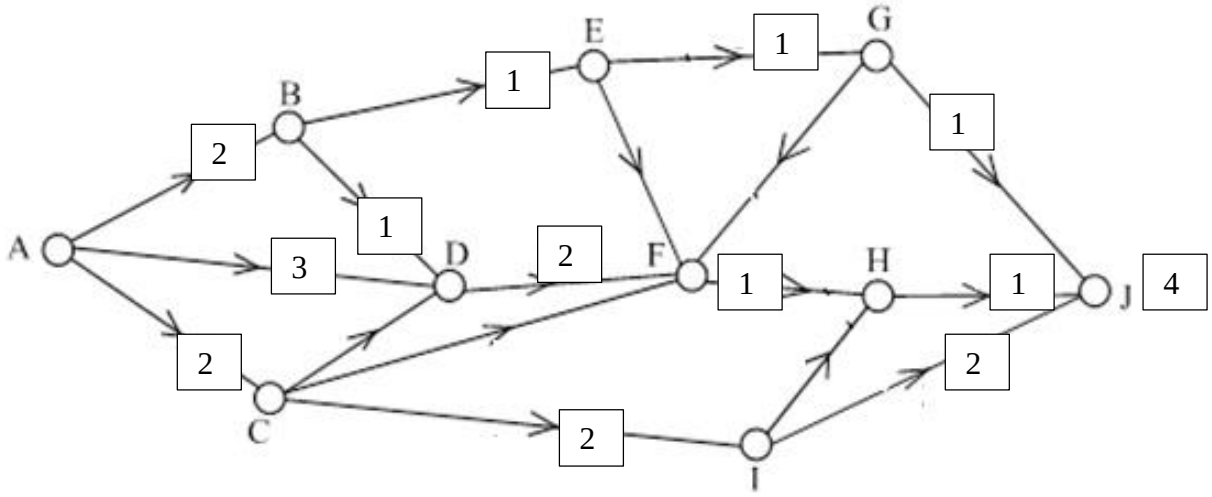
a) What is the capacity of the cut shown?

7 megalitres

b) What is the maximum amount of water, in ML, that this drainage system can cope with in an hour?

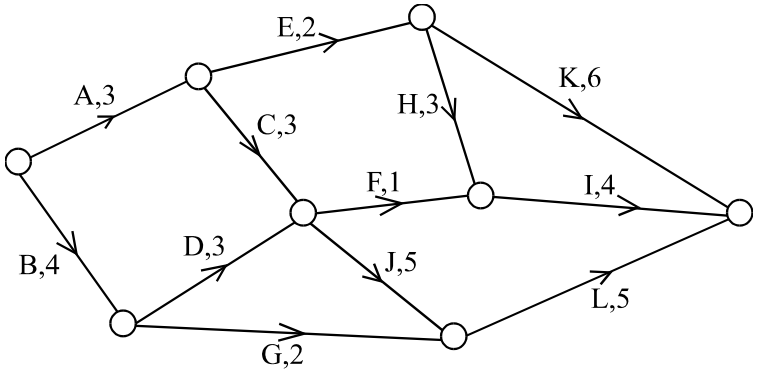
4 megalitres

c) On the diagram below show how this maximal flow passes from the source to the sink. Take out for next year



QUESTION 9

The construction company has identified the major activities and the time they will take to finish in weeks in order to complete the project.
The network below shows the activities and the time they will take, in weeks, to finish.



The table below shows the activity, together with the immediate predecessor(s) of each activity and the earliest and latest start time for each activity.

Activity	Immediate predecessor(s)	Earliest start time	Latest start time
A	-	0	1
B	-	0	**0**
C	A	**3**	**4**
D	B	**4**	**4**
E	A	**3**	**8**
F	C,D	**7**	**12**
G	B	**4**	**10**
H	E	**5**	**10**
I	**F, H**	**8**	**13**
J	C,D	**7**	**7**
K	E	**5**	**11**
L	G,J	**12**	**12**

a) What are the immediate predecessors for Activity I? Place you answer in the table above.
F and H

b) Use the network to fill in the EST and LST in the table above
See table

c) Write down the critical path for the project and its duration.

B – D – J – L 17 weeks

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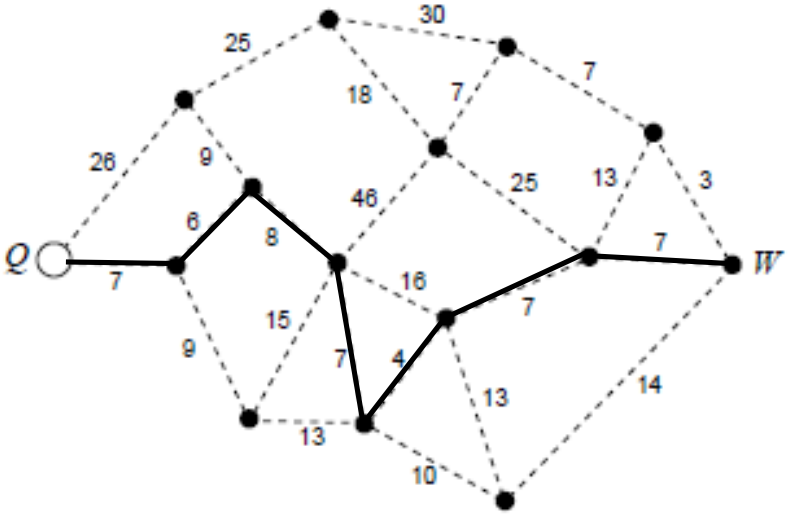
d) By comparing the start times of activities *G* and *H*, explain which will be the first to delay the overall project should both activities experience long-term delays.

H would delay the project first. H can be delayed 5 weeks before affecting the project whilst G can be delayed 6 weeks before the project is delayed, as evidenced by their float times.

QUESTION 10

The network below shows the distance, in kilometres, along a series of roads that connect a quarry, *Q*, with worksites shown as nodes.

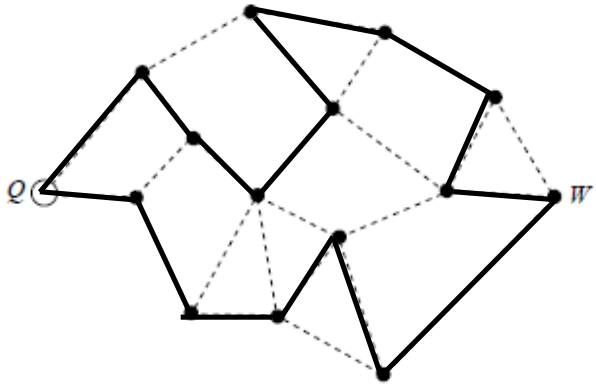


a) One of these worksites is labelled as *W*.

i. On the diagram above, clearly draw in the shortest path from the quarry to *W*.

ii. Determine the length, in kilometres, of the shortest path between the quarry *Q* and the worksite *W*.
46 km

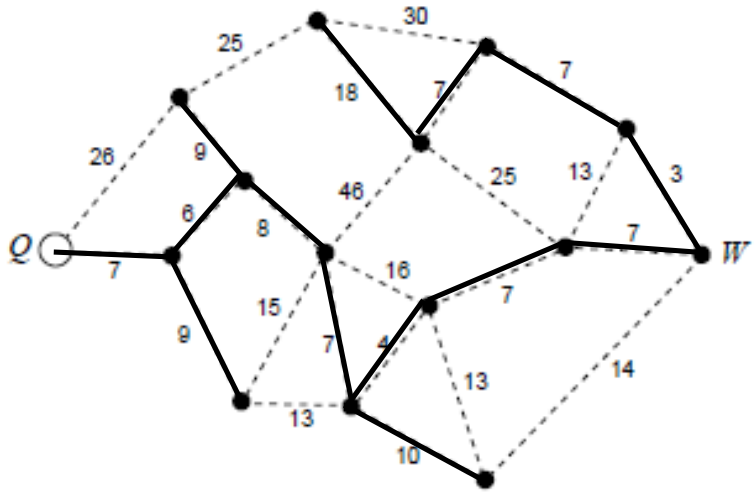
b) The engineer at the quarry wants to visit all worksites in the network. Beginning at *Q*, he wants to pass through each worksite only once before returning to the quarry.



i. What term describes the route the engineer wants to take?
Hamilton Cycle

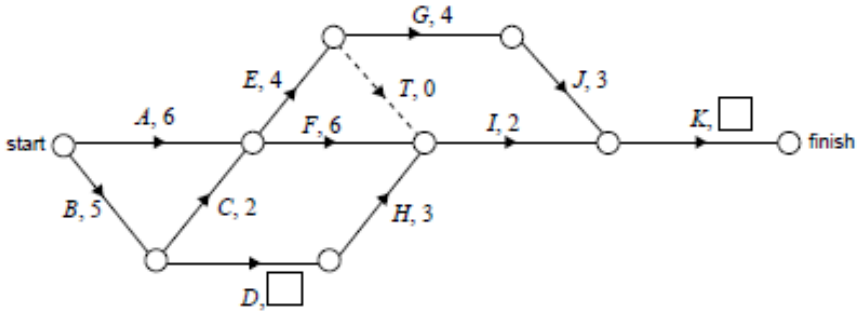
ii. On the diagram below, clearly draw in a complete route that the engineer could take to visit each worksite only once before returning to the quarry. A minimal distance is not required. But can you find it?

c) For safety each of the worksites must be connected with a phone line. Determine the number of connections that need to be made and their minimal total length. Highlight this on the blank template.



14 connections
Total length = 109 km

All the activities and their duration (in hours) in a project at the quarry are shown in the network diagram below. The latest time required to complete the project is 30 hours.



d) For each activity in the project, Table 1 shows the completion time, the earliest starting time and the latest starting time.

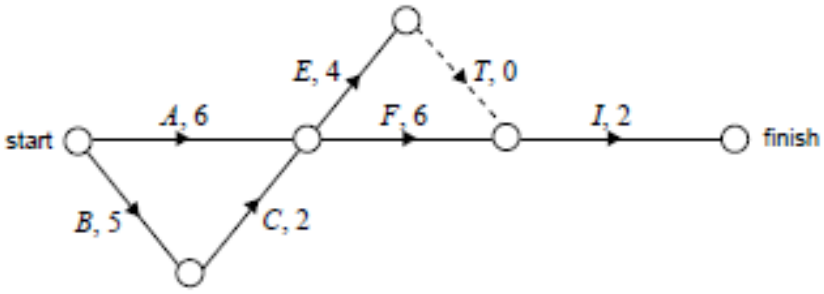
Activity	Completion time	EST (hrs)	LST (hrs)
A	6	0	1
B	5	0	0
C	2	5	5
D	4	5	9
E	4	7	7
F	6	7	10
G	4	11	11
H	3	9	13
I	2	13	16
J	3	15	15
K	12	18	18

i. Complete the missing items in the table.
LST(A) = 1
Completion time (D) = 4
LST (F) = 10
Completion time(K) = 12

ii. Write down the critical path for this project.
B – C – E – G – J - K

iii. A worker says that if activity H was reduced in length the whole project will be finished sooner. Explain whether this is in fact true.
As activity H has a float of 4 hours reducing this activity will not necessarily reduce the overall project length. To reduce the length of the project the activities on the critical path would first need to be reduced.

To speed up the project, several activities can be dropped from the project. The diagram below shows the activities that must remain in this modified version of the project and their usual completion times.



e) Determine the shortest time in which this modified project can be completed.
15 hours

The completion of some of the activities in the **modified project** can be reduced at a cost. The table shows the reduced times (least possible time to complete an activity after maximum reduction of time). The cost of this reduction, per hour, is also shown.

Activity	Usual Completion time	Available reduction in hours	Cost of reduction ($)
A	6	3	50
B	5	1	100
C	2	0	-
E	4	2	20
F	6	2	50
I	2	0	-

For this modified project determine:

i. the activities that should be reduced in time in order to minimise the completion time of the project.
Best method to find the answer is to list all possible paths and their lengths then look at reductions.

A E I 12
A F I 14 12
B C F I 15 13 12
B C E I 13 12

- **Try reducing the longest path first by reducing F by 2 days. This will decrease 2 of the paths**
- **2 paths at 13 can be reduced to 12 by reducing B by 1**
- **As all paths are the same length the only way to reduce further is if they all have a common activity, which they do (I) but I cannot be reduced**

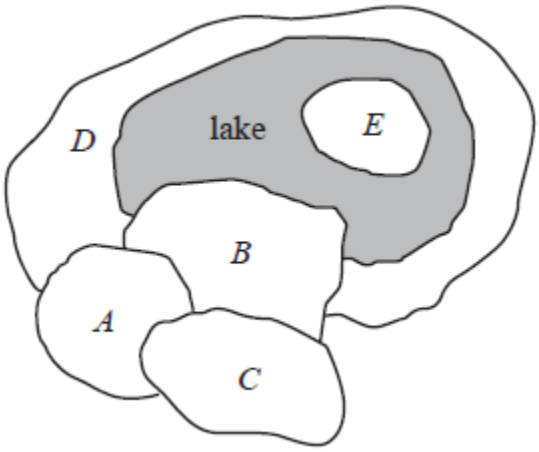
Overall B(1 hour) and F(2 hours) No need to reduce A or E as this does not result in a reduction of the longest path.

ii. the maximum time, in hours, that can be saved by this reduction.
3 hours

iii. the minimum cost to achieve this time saving.
$200

QUESTION 11

The city of Robville is divided into five suburbs labelled as A to E on the **map** below. A lake which is situated in the city is shaded on the map.



An adjacency matrix is constructed to represent the number of land borders between suburbs. An incomplete matrix is shown below:

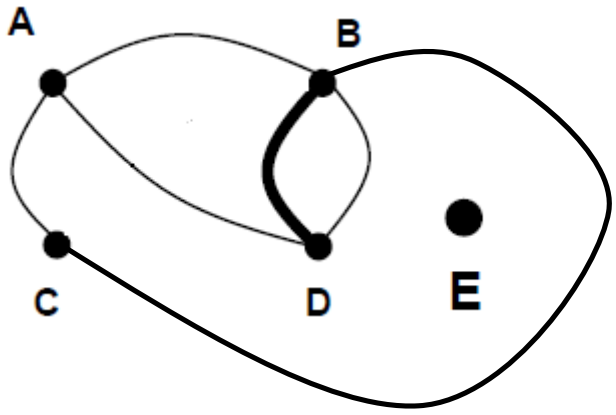
$$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 2 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

a. Complete the entries in the matrix above.

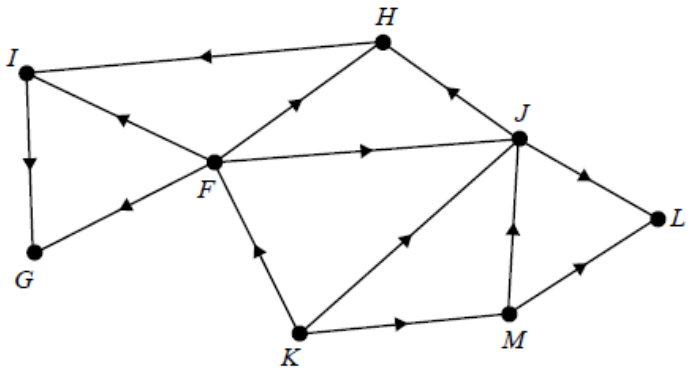
b. Explain why all values in the final row are zero.

There are no land borders between the suburb E and the other suburbs

c. The map above can be represented as an undirected graph. Construct a planar graph where vertices represent suburbs and edges represent land borders between suburbs. Construct the required undirected graph.



One of the landmarks in the city is a hedge maze. The maze contains eight statues. The statues are labelled *F* to *M* on the following network. Walkers within the maze are only allowed to move in the direction of the arrows.

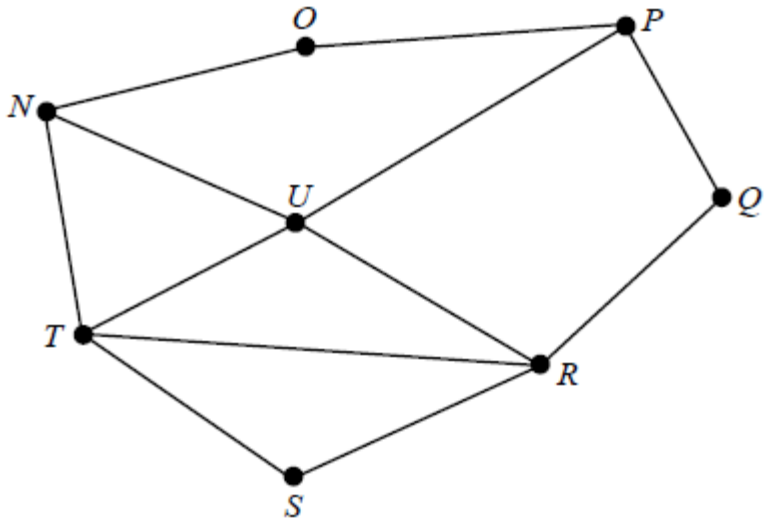


d. Write down the two statues that cannot be reached from statue *M*. ***K and F***

e. One way that statue *H* can be reached from statue is path *KFH*. List the other three ways that statue *H* can be reached from statue K. Identify whether these are walks, trails or paths.

KJH
KFJH **ALL ARE PATHS (no repeated edges or vertices start≠finish)**
KMJH

The city of Robville contains eight landmarks denoted as vertices *N* to *U* on the graph below. The edges on this graph represent the roads that link the eight landmarks.



f. Write down the degree of vertex *U*. **4**

g. Steven wants to visit each landmark, but drive along each road only once. He will begin his journey at landmark *N*.

i. At which landmark must he finish his journey? **P**

ii. Regardless of which route Steven takes, how many of the landmarks (including start and finish) will he see on exactly two occasions. Explain your decision.

5 – N T R P U – These are the vertices that have a degree that is greater than 2.

h. Cathy decides to visit each landmark only once.

i. Suppose she starts at *S*, then visits *R* and finishes at *T*. Write down the order Cathy will visit the landmarks. **(SR) QPONU (T)**

ii. Suppose she starts at *S*, then visits *R* but does not finish at *T*. List three routes that she can visit the landmarks.

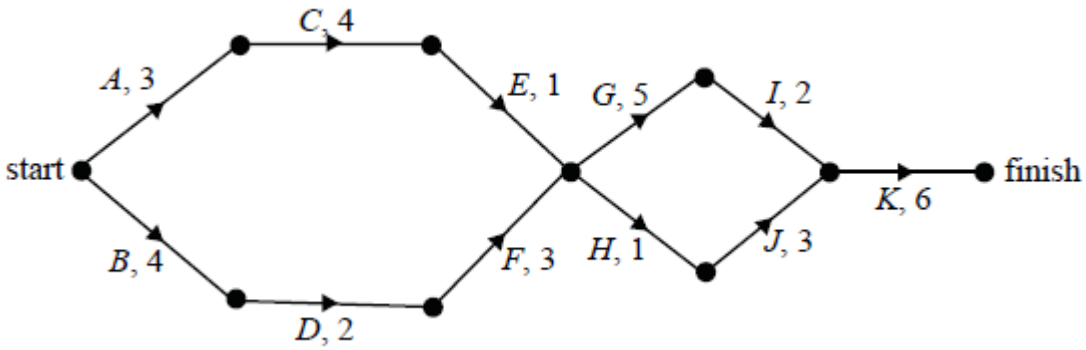
Any three of:

- *(SR)QPUTNO*
- *(SR)QPONTU*
- *(SR)TUNOPQ*
- *(SR)UTNOPQ*.

A walkway is to be built across the lake. Eleven activities must be completed for this building project. The table below shows the tasks, their immediate predecessors and their duration in weeks..

Task	Immediate predecessor	Duration (weeks)	EST	LST	FLOAT
A	-	3	0	1	1
B	-	4	0	0	0
C	A	4	3	4	1
D	B	2	4	4	0
E	C	1	7	8	1
F	D	3	6	6	0
G	E F	5	9	9	0
H	E F	1	9	12	3
I	G	2	14	14	0
J	H	3	10	13	3
K	I J	6	16	16	0

i. Construct a network diagram for this project.



j. Complete the table above by filling in the earliest and latest start times and hence the float times for each activity.

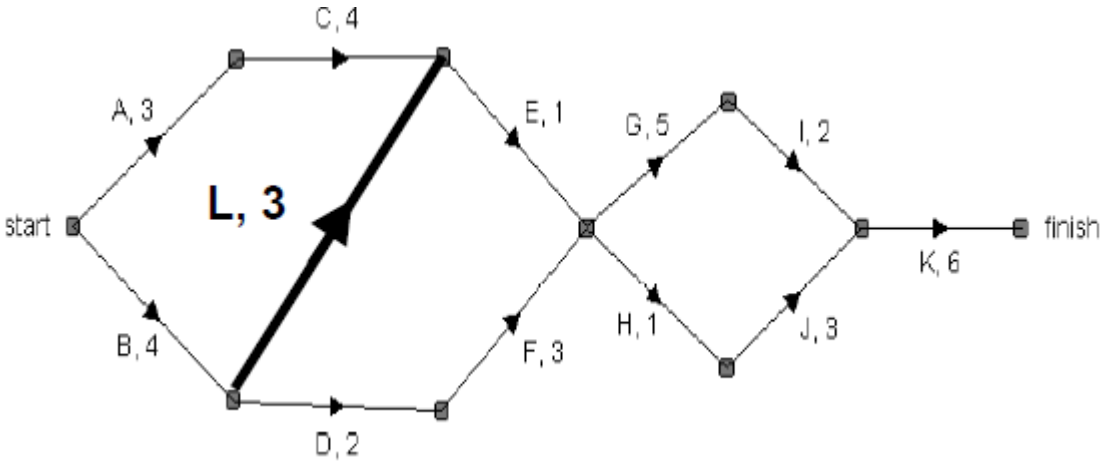
k. Determine the critical path and its length. **B-D-F-G-I-K** **22 weeks**

l. Interpret one of the float times in the context of the project.

eg. **Activity A has a float time of 1 week. This means this activity can be delayed by one week and the project can still be completed in 22 weeks.**

A 12th activity, *L* with duration three weeks is to be added without altering the critical path. Activity L has an earliest start time of four weeks and a latest start time of five weeks.

m. Construct a new network diagram below to include the 12th activity.



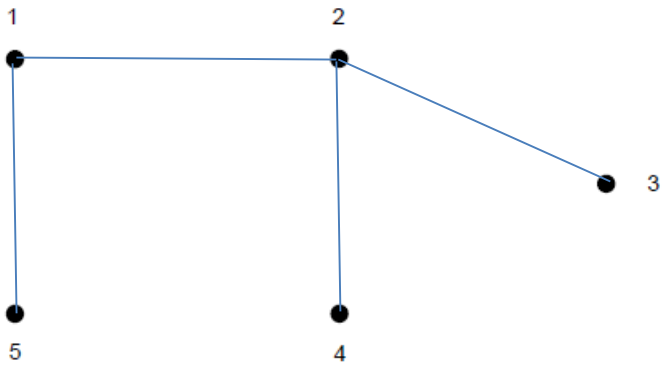
NOTE: This activity must start at the end of B as it has the same EST as activity D. It must finish at a point that has a LST of 8 as it has a duration of 3 weeks. Therefore, join between C and E.

n. Activity *L* starts but then takes four weeks longer than originally planned. Determine the effect this has on the project. Explain what measures could be taken to complete the project in the original time.

The project will now take 25 weeks. A new critical path is created through BLEGIK.
To complete the project in the original time an activity on the critical path would need to be reduced in time. As L has been delayed it may be possible to pay to reduce this time – The longest L can take is 4 weeks (original 3 weeks + 1 week delay).

QUESTION 12

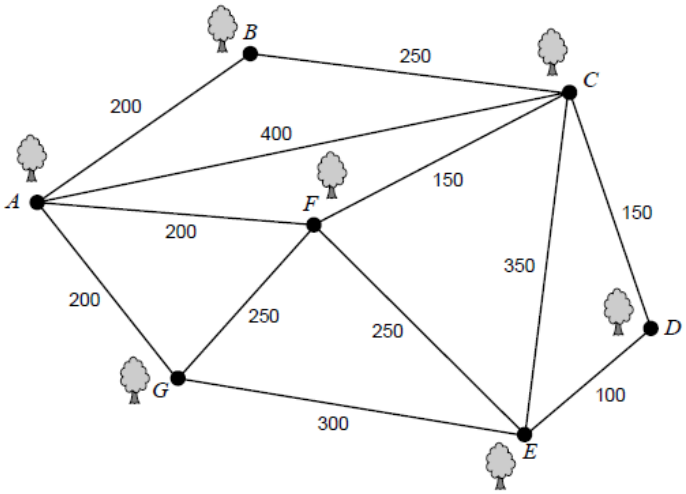
A new housing estate is being developed. There are five houses under construction in one location. These houses are numbered as points 1 – 5 below:



The builders require the five houses to be connected by electrical cables to enable the workers to have a supply of power on each site.

a. Determine the minimum number of edges required and draw a possible set of connections on the diagram above. **4 edges are required (*n*-1 edges for *n* vertices)**

The estate has large open parklands that contain seven large trees. The trees are denoted as vertices A to G on the graph below. Walking paths link the trees as shown. The numbers on the edges represent the lengths of the paths in metres.



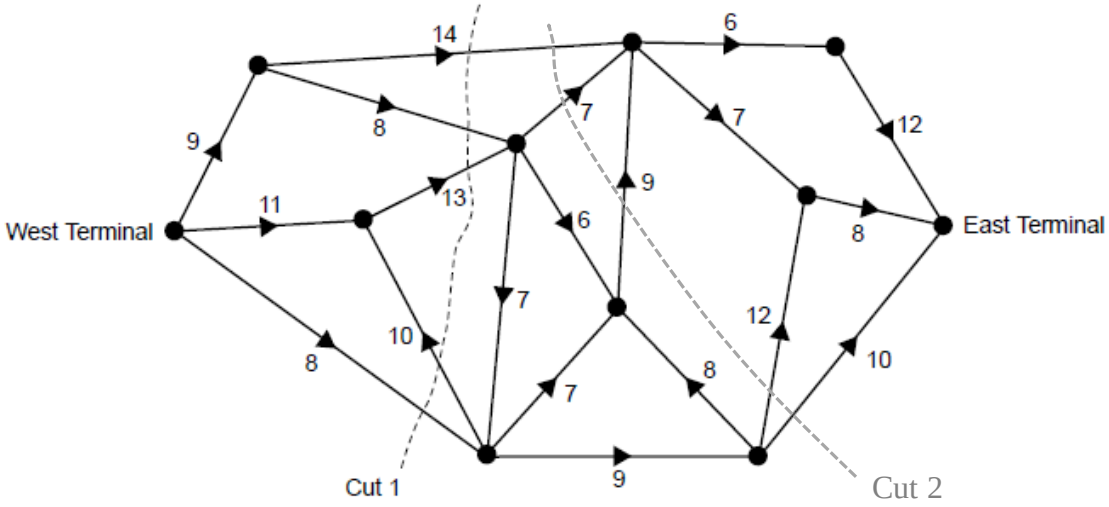
b. One day Jamie decides to go for a walk that will take him along each of the paths between the trees. He wishes to walk the minimum distance. State a vertex at which Jamie could begin. Determine a suitable route and state its length.

As there are two vertices of odd degree an Eulerian trail is possible.
Therefore start at C or G.
A possible trail is C-B-A-C-F-A-G-F-E-C-D-E-G Length 2800m

c. Michelle is currently at *F*. She wishes to follow a route that can be described as the shortest Hamiltonian cycle. Write down a route that Michelle can take.

F-G-A-B-C-D-E-F OR F-E-D-C-B-A-G-F

As an attraction for young children, a miniature railway runs throughout the housing estate. The trains travel through stations that are represented by vertices on the network below. The number of seats available for children, between each pair of vertices is indicated.



Cut 1 and Cut 2 through the network, are shown on the diagram.

d. Determine the capacity of Cut 1 and Cut 2. Explain what this means about the maximum flow for this network.

Cut 1: 43 (8+13+8+14) **Cut 2: 52 (10+12+9+7+14)**
This means that at most the maximal flow would be 43 based on these cuts. There could, however, be another cut that further restricts the number of seats for children.

e. Determine the maximum number of seats available for children for a journey that begins at West Terminal and finishes at East Terminal.

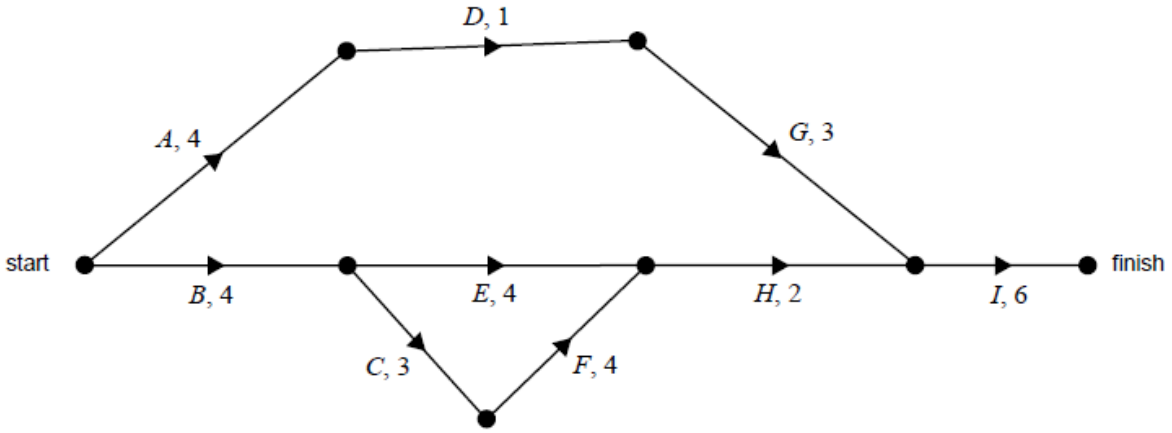
22 seats for children (min cut shown in red on the diagram)

On one particular train 10 children set out from West Terminal. No new passengers board the train on the journey to the East Terminal.

f. Determine the maximum number of children who can arrive at the East Terminal on this train.

7 is the maximum number of seats that can go on a continuous journey between the source and sink.

A community centre is to be built on the new housing estate. Nine activities have been identified for this building project. The directed network below shows the activities and their completion times in weeks.



g. Determine the critical path and its length.
Critical path is B-C-F-H-I Length 19 weeks

h. Determine the float time for activity D and interpret its meaning in the context of the project.
Float time is 5 weeks. This means that this activity can be delayed up to 5 weeks without the project running over time.

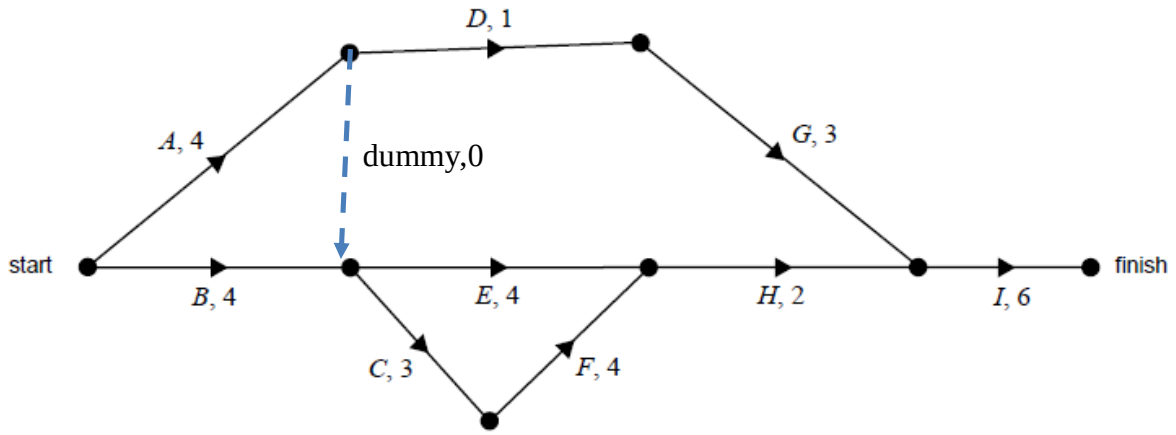
The builders of the project are able to speed up the project. Some of the activities can be reduced at an additional cost. The activities that can be reduced in time are A, C, E, F and G.

i. Which of these activities, if reduced in time individually, would not result in an earlier completion of the project? Explain your choices.
A, E and G (D also has a float time but as it's duration is only one it cannot be reduced)
These activities are not on the critical path so if reduced in time they will only increase the available float time.

The owner of the estate is prepared to pay the additional cost to achieve earlier completion. The cost of reducing the time of each activity is $5000 per week. The maximum reduction in time for each one of the five activities A, C, E, F and G is 2 weeks.

j. Determine a strategy the builders can use to reduce the completion time of the project. Also state the cost of these reductions.
Consider all of the possible reductions:
A-D-G-I 10
B-E-H-I 14
B-C-F-H-I 15
This means that the best possible result is a length of 15 weeks.
This requires a reduction in C and a reduction in F at a cost of 4 x $5000=$20000
To ensure that this path is the longest we also need to reduce activity E by 1 week at a cost of $5000. There is no gain by reducing A or G as A-D-G-I is only 14 without reductions.
Total cost is $25 000.

k. The builders have realised that an extra activity is required for the original project. They have decided that prior to completing activity E and C, activity A and B must be completed. Explain how this will affect the network diagram and whether this will change the critical path. An extra network can be used to make your decision.



A dummy activity is now required as activity D has only one predecessor - activity A to be completed, while activities E and C have two predecessors - activity A and B. As there cannot be two activities called A, a dummy activity(with zero time) is drawn as a form of extension of A to the start of C and E to indicate that A is a predecessor for these two activities as well.

Overall this has not changed the length of the critical path and the project will still take 19 weeks. The main difference is that activity A now has a reduced float time of 3 weeks instead of 5 weeks.