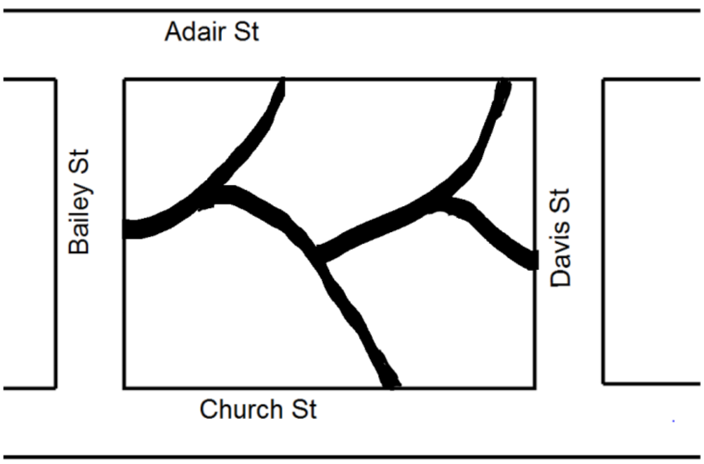
## TASK ONE (19 marks)

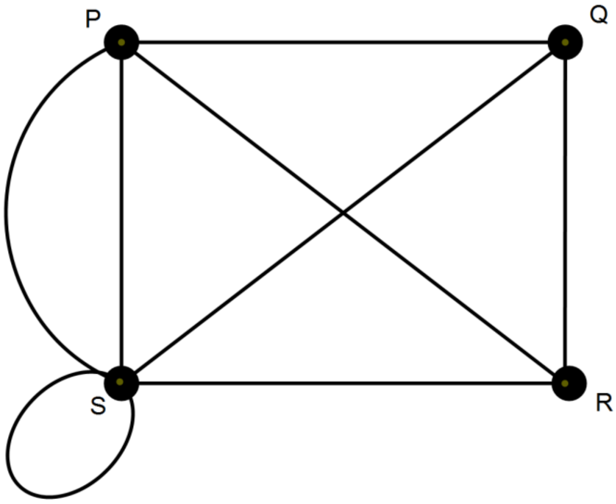
Carolyn has a part time job while she is at University helping to maintain parks. One of the parks that she maintains is bounded by Adair St *(A)* , Bailey St (*B*), Church St (*C*) and Davis St (*D*)as shown below:




Carolyn needs to understand the connections between the different streets, both along the street and through the park.

a) Using the 4 streets as vertices *A*, *B*, *C* and *D* respectively draw a network that shows the direct connections between these streets either along the streets or via the park. (3 marks)

Another park that Carolyn works at has a network as shown below where *P*, *Q*, *R* and *S* are surrounding streets:



b) Redraw the network for this park so that it shows that this is a planar network. (1 mark)

c) Explain why this network could not be described as a simple network. (2 marks)

Carolyn's boss wants to understand the direct connections around the park better and so he asks her to construct an adjacency matrix for the network.

d) Construct an adjacency matrix for this network. (2 marks)

e) The faces in the network represent the grassed areas and surrounds of the park. How many faces are there in this network? (1 mark)

f) Confirm the relationship between vertices, edges and regions using Euler's rule for this network. (2 marks)

The edges in this network represent footpaths between the grassed areas. Carolyn wants to check each of the footpaths for rubbish. Carolyn wants to start and finish at one of the grassed areas and travel along each footpath exactly once.

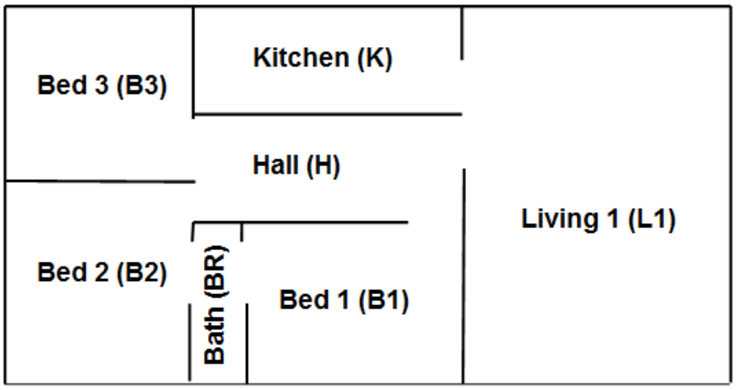
g) Give the name of the type of route that Carolyn would like to take. (2 marks)

h) Can Carolyn follow this route in this park? Explain your answer. (3 marks)

i) Carolyn does need to check every footpath for rubbish. What is the most efficient way for her to do this? Fully explain your answer including where she should start and finish, a possible route she could take and the mathematical term for the route that she should take. (3 marks)

**TASK TWO (5 marks)**

George also has a part time job while he is at university vacuuming houses. A map of one of these houses is shown below:



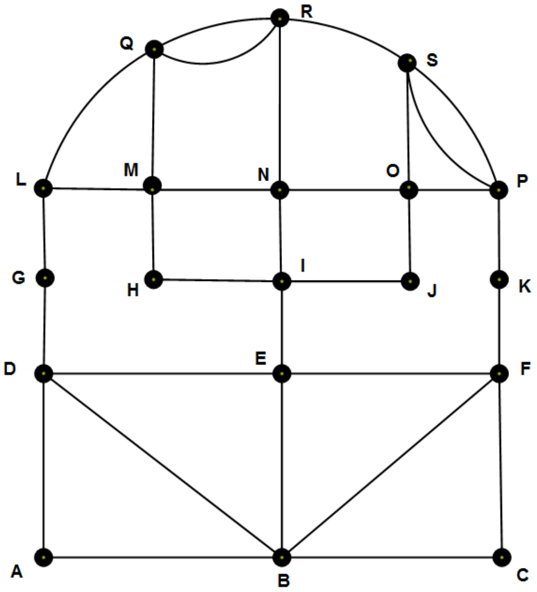

Because George must go from room to room with the vacuum, he is interested in which rooms have connecting doors.

a) Construct a network showing the relationship “has a connecting door with” where the vertices are the rooms and the edges are the relationship. (2 marks)

b) Can George vacuum each room without passing through any room more than once? Explain your answer in network terms including the name of the route that George would like to take. (3 marks)

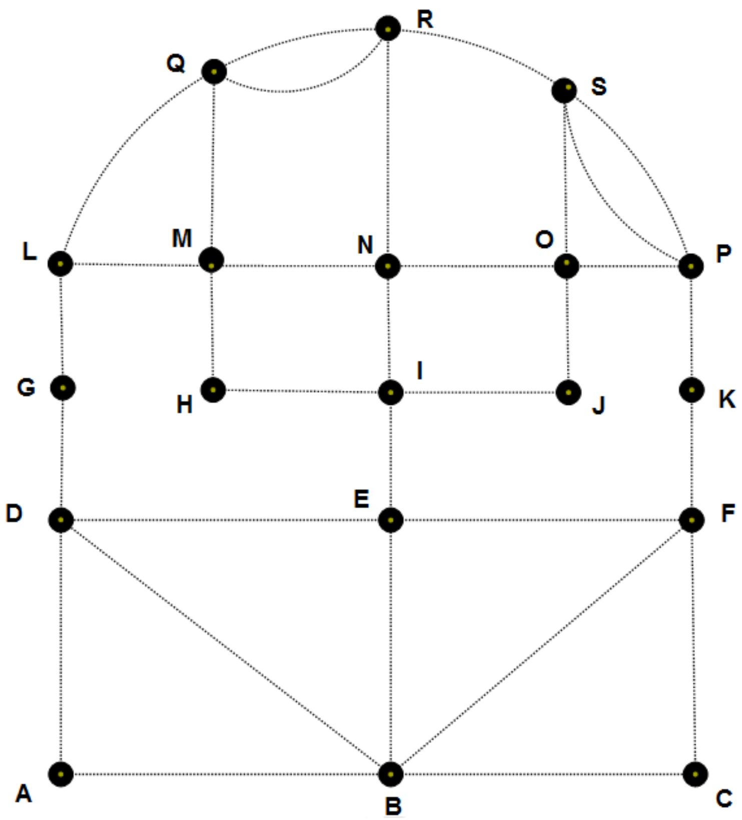
**TASK THREE (16 marks)**

Sophie has a part time job helping to guide people around the university. The university that she attends has been drawn as a network below. The vertices represent different buildings within the university and the edges are the footpaths between them.




Sophie meets a group at the main gate (vertex *B*) and she takes them to every building on the map except the administration building (vertex *E*). At the end of the tour the group return to the main gate (vertex B)

a) Draw a route that Sophie could take on the copy of the network below. (2 marks)

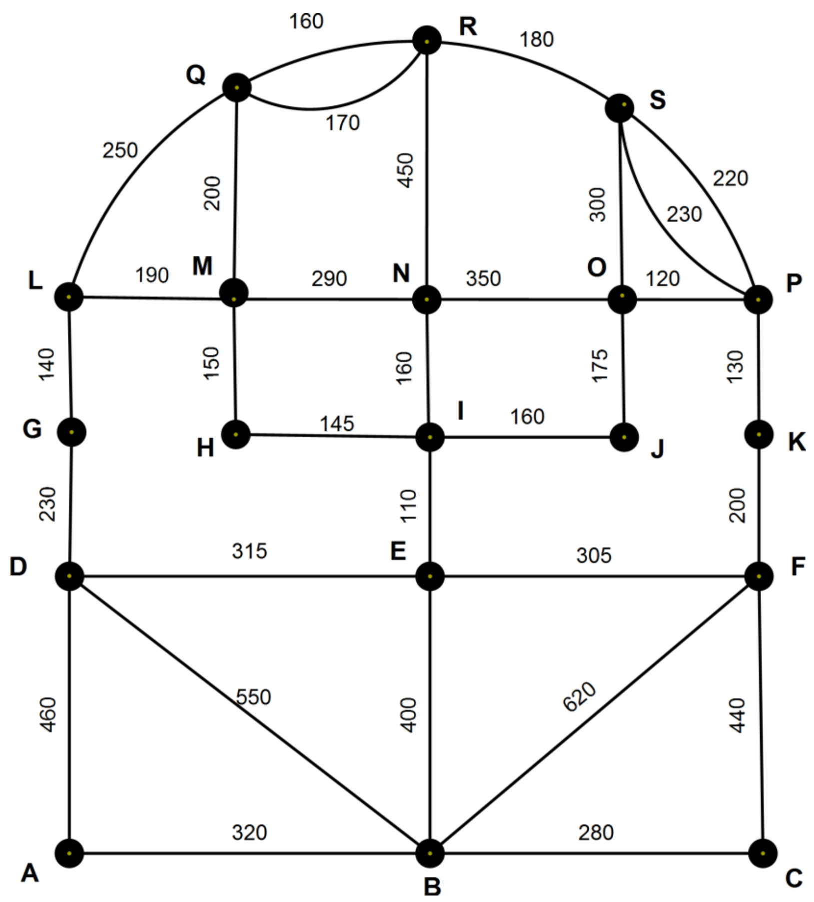


b) Another group Sophie takes want to walk along every path to ensure that they don't miss anything. If Sophie meets them at the main gate (vertex *B*) where would they finish their tour? (1 mark)

c) Explain fully in network terms why it is possible for Sophie to take the group along every footpath exactly once. Include in your answer the name of the route taken. (3 mark)

On open day every building in the university must be connected by computers that are connected by cables. The wifi system cannot be used because of security concerns with so many unregistered users. Because of her expansive knowledge of the university Sophie is asked to plan the connections using the shortest possible length of wire.

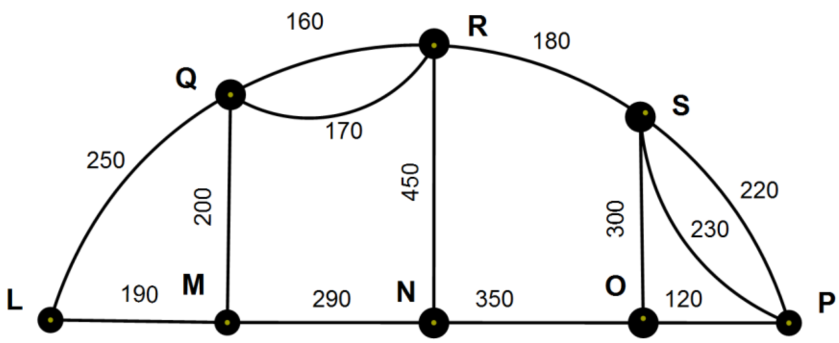
d) The minimal length of wire is represented by a minimal spanning tree. Use Prim's algorithm to draw the minimum spanning tree on the weighted diagram below, where the weights on the edges are the distances in metres represented by that edge. Start your minimal spanning tree at vertex *B.* (2 marks)



e) State the minimum length of cable required to connect the buildings. (1 mark)

f) State the last 3 edges in order that should be connected given that you are starting at vertex *B* and using Prim's Algorithm. (3 marks)

Sophie has been allocated the top section of this network on open day. She must stay within the region shown below:



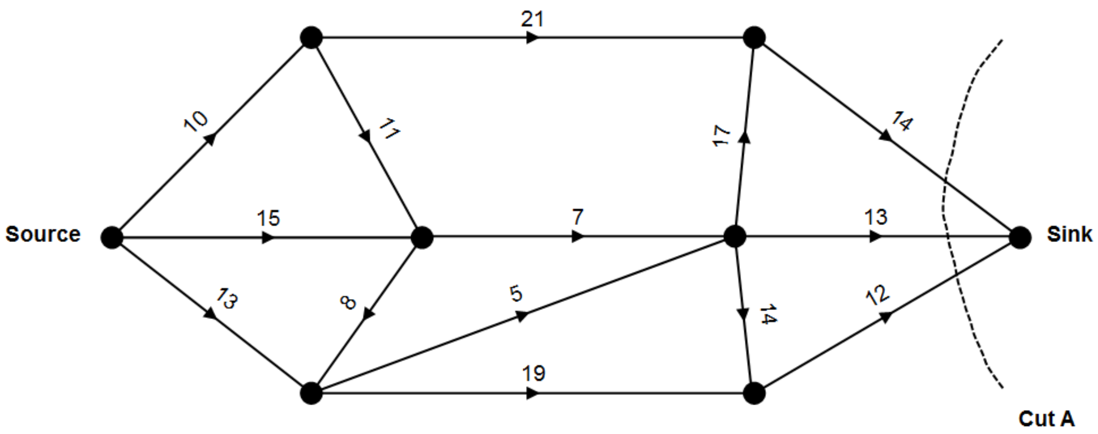

Sophie needs to direct some people from vertex *L* to vertex *P* via the shortest possible path.

g) Use Dijkstra's Algorithm to determine the shortest path from *L* to *P*. Include the shortest path and the length of the path in your answer. (4 marks)

**TASK FOUR (8 marks)**

Brad has been working for a plumber part time while he is at university. He goes to one job where water flow through a system of pipes is not sufficient.

The network shown represents the flow through the series of water pipes from the source to the sink. The weight of each edge represents the maximum capacity of the pipe in litres per minute.



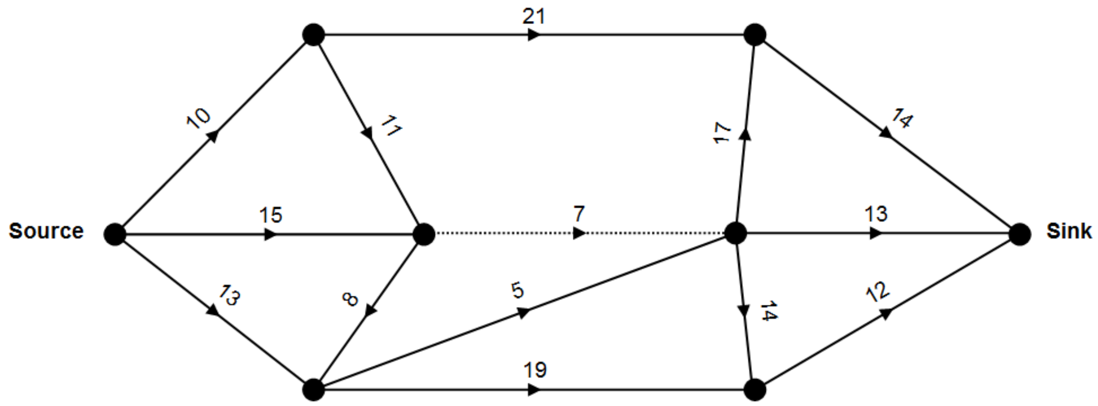

Brad must first determine the current flow through the network.

a) State the capacity of Cut *A* shown on the network. (1 mark)

b) What does Cut *A* tell you about the flow through this network? (1 mark)

c) What is the maximum flow in litres per minute through this network? (3 marks)

The current maximum flow is not sufficient and must be **<u>increased</u>**. Brad determines that he could replace the pipe shown as a dotted line in the diagram below:



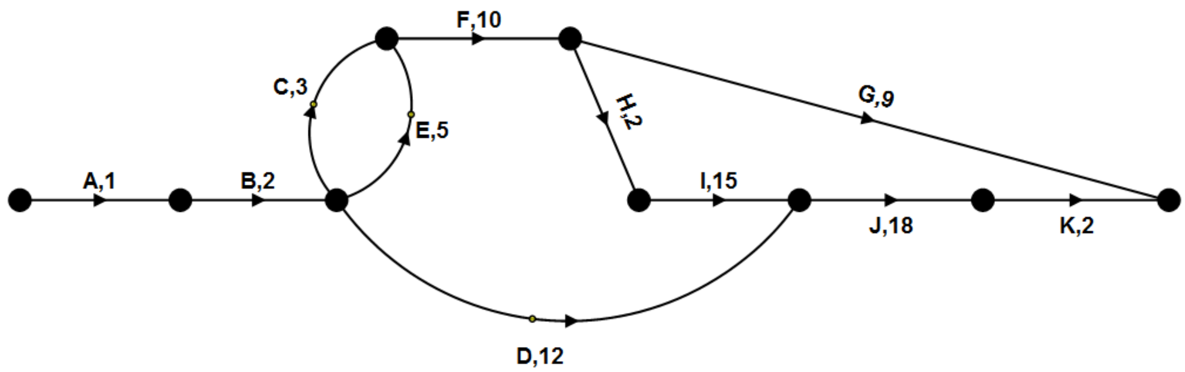

d) Given that the edge shown as a dotted line can be increased as far as required, what would be the new maximum flow through the network? (3 marks)

**TASK FIVE (23 marks)**

The Australian Government have called a State election and a number of university students will work at the polling booths. Because the booths must be set up in public places with other uses and election results must be determined quickly, electoral staff work on a 24 hour basis until the job is completed.

The local Divisional Returning Officer has to organise the local voting and counting of results as a total project.

This project is shown as an activity network below with durations of each activity in hours:




In order to ensure that the polling runs smoothly she lists the activities and the duration of each activity (in hours) in the table below.

a) Complete the predecessors and earliest and latest starting times and the float times for each activity in the table below. (8 marks)

| Activity | Predecessor(s) | EST | LST | Float Time |
|---|---|---|---|---|
| A – Complete temporary staff paperwork | | | | |
| B – Train temporary staff | | | | |
| C - Set up signs & electoral roll desks | | | | |
| D - Count postal votes | | | | |
| E - Erect cardboard polling booths | | | | |
| F – Polling day voting | | | | |
| G - Remove election signs & booths from polling stations | | | | |
| H - Transport voting boxes to counting station | | | | |
| I - Counting votes | | | | |
| J - Recounting votes | | | | |
| K - Reporting results | | | | |

b) What is the critical path for this network? (1 mark)

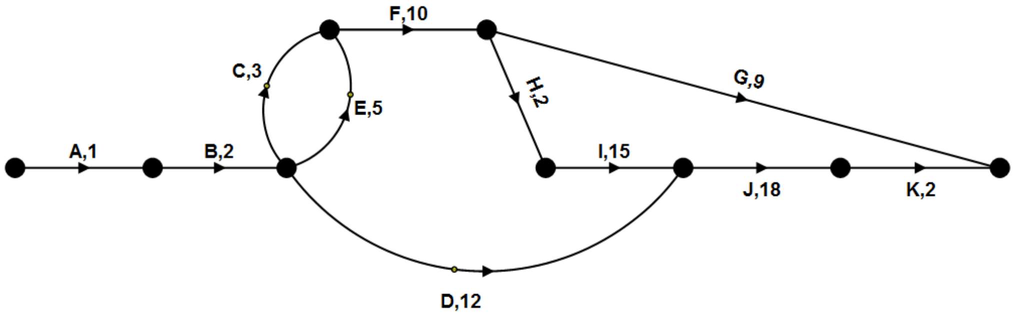
c) What is the shortest time in which the entire project can be completed? (1 mark)

d) Given that polling day voting (activity F) **must** start at 8 am, what is the earliest time of day that counting postal votes (activity D) could be finished? Explain your answer. (3 marks)

A number of activities can be reduced in time by hiring additional staff and streamlining practices. The activities that can be reduced, the maximum number of hours of reduction available and the hourly cost of each reduction is listed below. All reductions could be done partially or completely, but only in complete hour blocks.

| Activity | Cost per hour of reduction | Original duration of activity (in hours) | Maximum number of hours of reduction | Minimum duration after reduction (in hours) |
|---|---|---|---|---|
| C | $200 | 3 | 2 | 1 |
| D | $400 | 12 | 6 | 6 |
| E | $300 | 5 | 3 | 2 |
| G | $400 | 9 | 4 | 5 |
| I | $600 | 15 | 10 | 5 |
| J | $600 | 18 | 12 | 6 |

*Another copy of the network is shown in case it is required:*




e) What is the shortest time in which this project could be completed given these reductions? (1 mark)

f) What activities should be reduced in order to achieve this minimum time? (3 marks)

g) What is the total cost of achieving the minimum time? Show the working that leads to your answer. (3 marks)

h) The Divisional Returning Officer is only allowed to spend a **<u>maximum of $5000</u>** to reduce time. What will be the minimum time for the project given this monetary limit? Explain your answer and include the most cost efficient way of saving this time. (3 marks)

**TASK SIX (9 marks)**

Four university students, Tom, Natalie, Arthur and Gen are all going to work at polling booths during the State election. There are 4 polling locations; Ballarat, Wendouree, Sebastopol and Delacombe. The table below shows how far each of them lives from each of the polling locations in kilometres. Each one of them will work at a different booths.

| | Ballarat | Wendouree | Sebastopol | Delacombe |
|---|---|---|---|---|
| **Tom** | 20 | 12 | 17 | 24 |
| **Natalie** | 18 | 13 | 16 | 28 |
| **Arthur** | 23 | 14 | 19 | 30 |
| **Gen** | 21 | 17 | 18 | 29 |

The electoral commission has to pay employees travelling costs based on their distance travelled to work and so they want to minimise the overall  distance travelled by the four employees. They use the Hungarian Algorithm to determine the minimum allocation.

a) Write down the matrix obtained after a row and column reduction using the Hungarian Algorithm, showing the steps in the process of obtaining this matrix. (3 marks)

b) Explain why the reduced matrix is not yet ready for allocation. (2 marks)

c) Complete the next step of the Hungarian Algorithm that would a produce a matrix ready for allocation and write that matrix below. (2 marks)

d) Complete the table below to show who should go to each polling booth to minimise travelling. (2 marks)

| Polling Booth | Worker |
|---|---|
| Ballarat | |
| Wendouree | |
| Sebastopol | |
| Delacombe | |