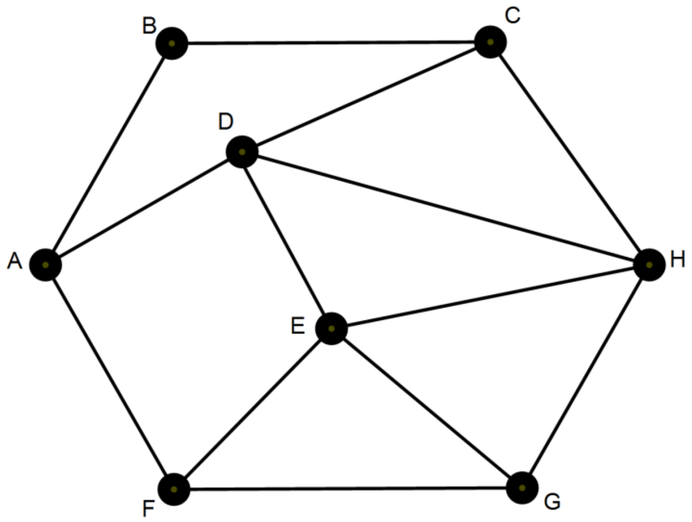
TASK ONE (23 marks)

The coach of a football team has developed a unique fitness regime for his players. On the oval he has set out a number of fitness exercise stations in different parts of the oval. The players during training have to sprint along a track laid out between each station. The exercise plan is laid out below along with an explanation of the exercises at each station:



A stretching
B sit ups
C planks
D burpees
E push ups
F weights
G standing jumps
H handballs

a) Explain why this network would be considered a planar network. (2 marks)

The coach will have an assistant standing in each of the spaces between the tracks and the coach himself will move around the outside of the network.

b) What is the mathematical term for the spaces between the tracks as well as the outside of the network? (1 mark)

c) How many people will be needed for this role including the coach himself? (1 mark)

d) Verify the relationship between the number of faces, edges and vertices using Euler’s rule for planar graphs. (3 marks)

The coach wants each player to move around the network starting and finishing at *A*.

e) Is it possible for each player to go to each station exactly once? If so state all possible paths through the network starting and finishing at *A* that uses every station exactly once. (4 marks)

f) Give the mathematical name of the route that would be taken by a player who started and finished at *A* and went to each station exactly once. (2 marks)

The coach wants to be sure that each player is running enough during training as well as changing direction. He decides to use the same plan of stations, but he wants each player to use every track between the stations on the network exactly once starting and finishing at *A*.

g) Explain fully why it is not possible for a player to use every track exactly once in this network starting and finishing at *A*. Include in your answer the mathematical name given to the required route. (3 marks)

One of the assistant coaches suggests adding another track between 2 stations would allow every track to be used exactly once, but the player would need to start and finish at different places in the network.

h) Explain where the track could be added and where a player would start and finish. Include two possible options in your answer. (4 marks)

i) If the coach wanted the players to start and finish at the same station and travel along every track exactly once, explain how this could be done. Include in your answer the mathematical name given to the route taken. (3 marks)

TASK TWO (5 marks)

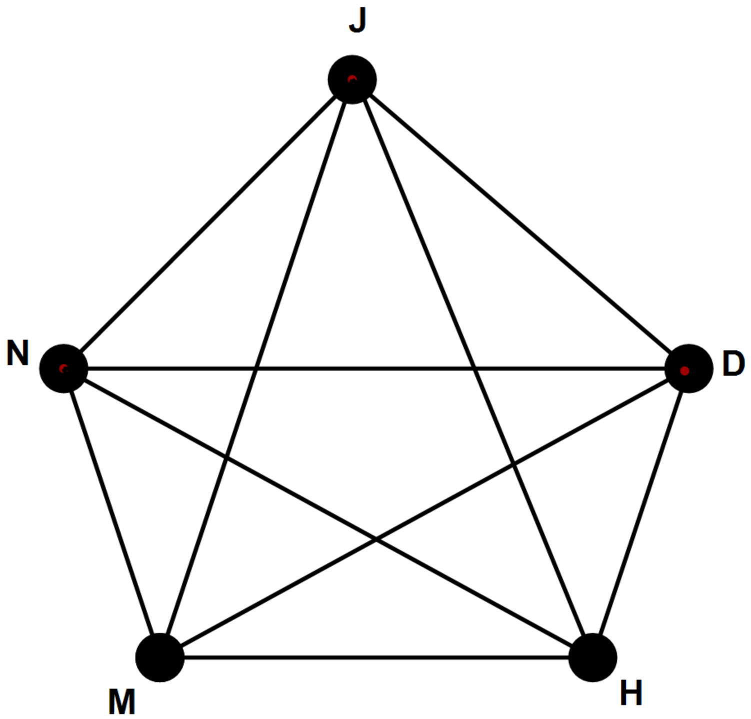
The coach of the football team wants to train the best players for the ruck contest. He runs a training regime with his five potential ruckmen, Nathan, Mark, Josh, Dawson and Hamish, where each player contests the ruck once against every other player. The coach draws the ruck practice in a network where each vertex is attached directly to every other vertex.

a) What is the name given to a network where every vertex is attached directly to every other vertex?

(1 mark)

b) How many individual ruck contests will there be? Show an appropriate calculation that gives your answer. (2 marks)

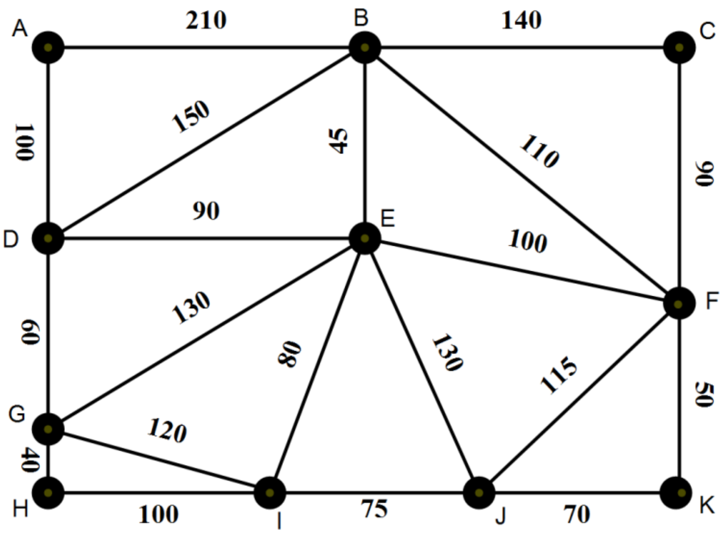
The network below shows the practice runs required.



c) Construct an adjacency matrix for this network. (2 marks)

TASK THREE (12 marks)

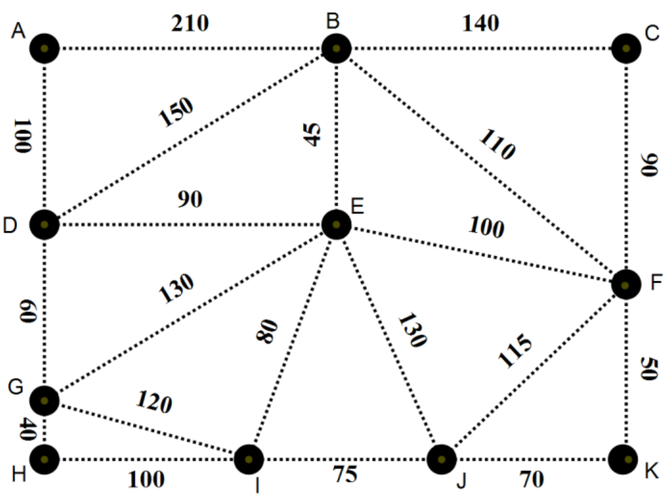
The football coach needs communication between a number of points around the oval. A network showing the wifi extender points that require power and the distances in metres between them is shown below:



The coach wants to connect the wifi extender points with the shortest length of cable.

a) What number of edges would be required to form a spanning tree for this network? Explain your answer. (2 marks)

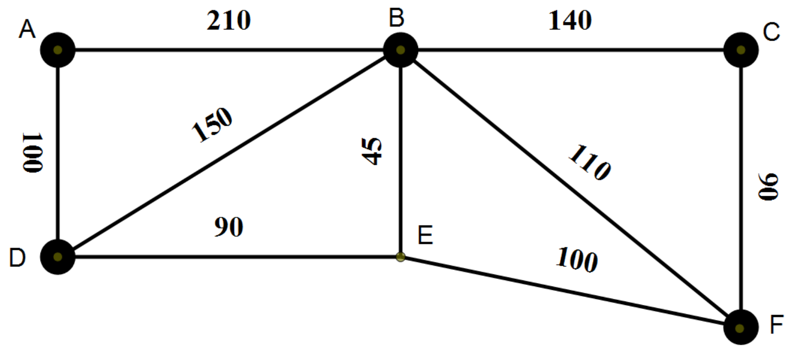
b) Add the minimum spanning tree to the copy of the network below. (2 marks)



c) What is the minimum length of cable required to connect these points? (1 mark)

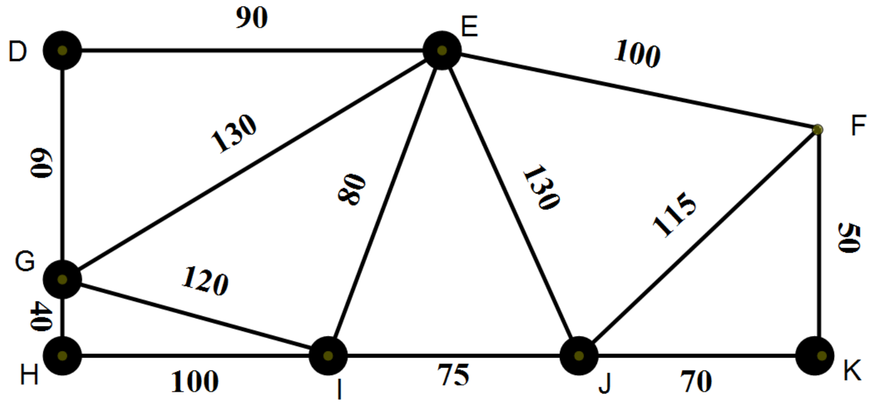
The coach is at point *A* and the assistant coach is at Point *H*. They need to meet at point *F*, but they must each stay in their half of the network.

The coach must find the shortest path from *A* to *F* using the network shown below:



d) Use Dijkstra's algorithm to determine the shortest path from *A* to *F*. Include the actual path taken and the length in your answer. (3 marks)

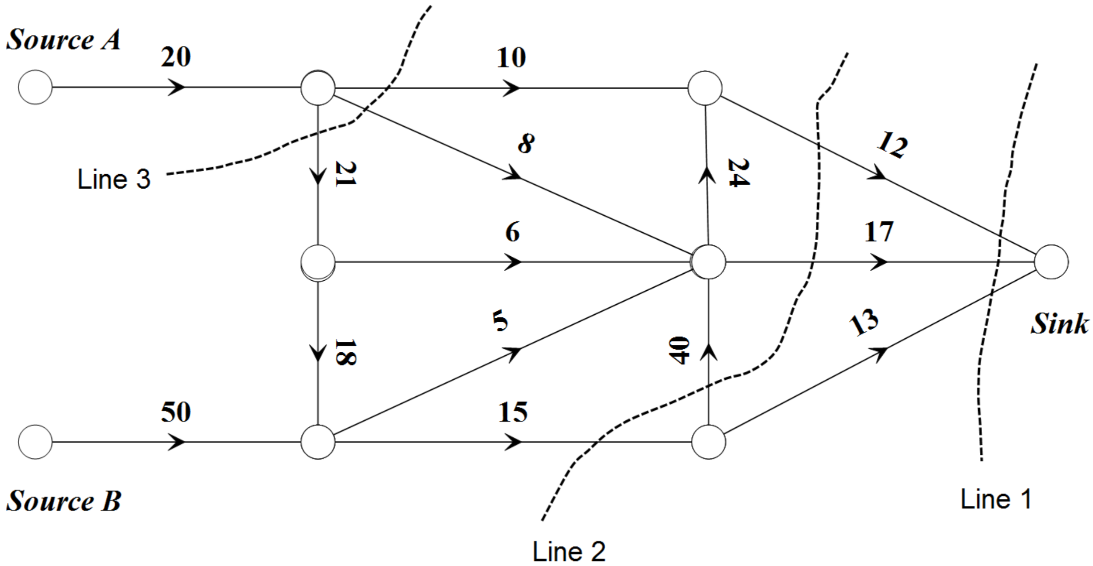
The assistant coach must find the shortest path from *H* to *F* using the network shown below:



e) Use Dijkstra's algorithm to determine the shortest path from *H* to *F*. Include the actual path taken and the length in your answer. (4 marks)

TASK FOUR (12 marks)

The football ground is watered through a series of pipes from two different sources, A and B as shown in the diagram below. Water flows from two different sources *A* and *B* and flows to the sink. The maximum capacity of each pipe in litres per minute is shown on each edge. Also shown are three lines, 1, 2 and 3:



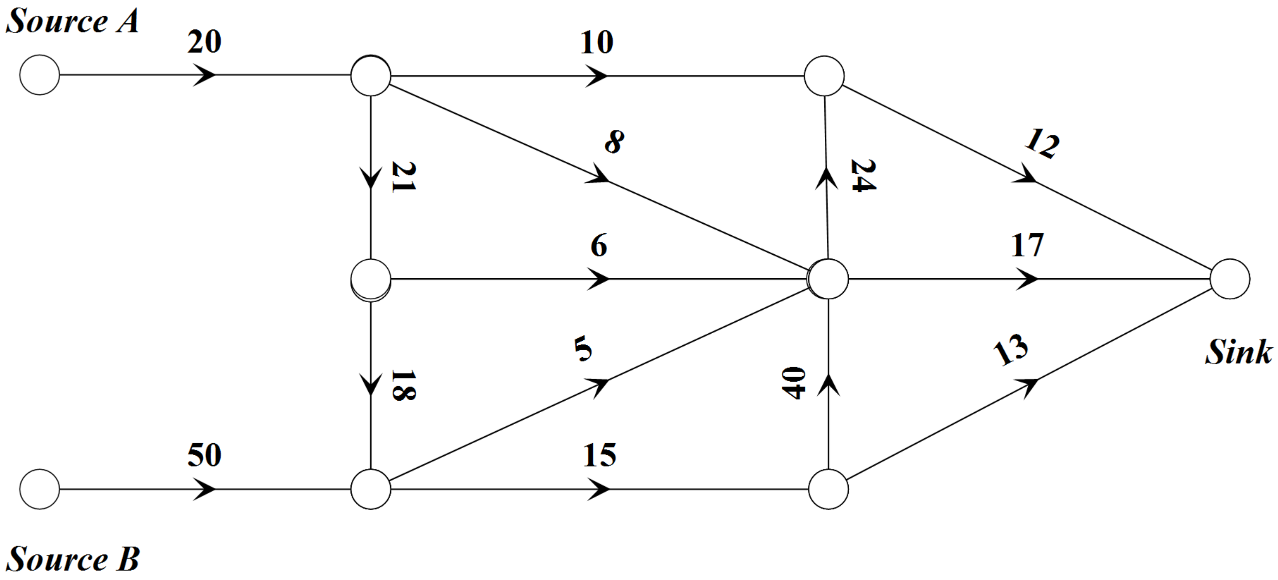
a) Line 1 is a cut through the network. What is the capacity of the cut represented by Line 1? (1 mark)

b) Line 2 is also a cut through the network. What is the capacity of the cut represented by Line 2? Show how you calculated your answer. (2 marks)

c) What do Line 1 and Line 2 specifically tell us about the flow through the network? Ignore any other potential cuts through the network from the two sources to the sink. Explain your answer. (2 marks)

d) One student used Line 3 as a cut. Explain in detail why Line 3 is not a cut in this network. (2 marks)

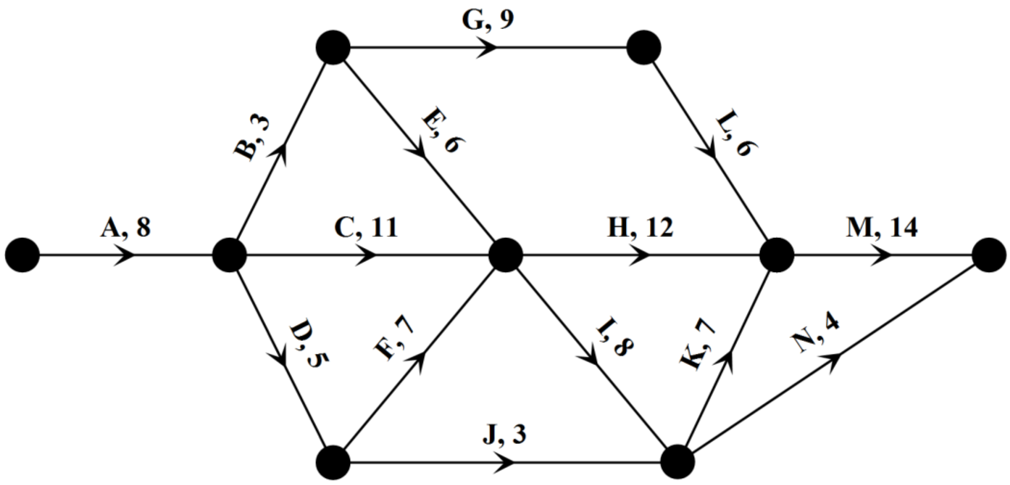
e) What is the maximum flow through this network? (2 marks)



f) The pipe with a capacity of 15 is to be replaced with a pipe of capacity of 40 litres. What is the maximum flow through the network if this is done? Fully explain your answer. (3 marks)

TASK FIVE (16 marks)

A project for one of the team publicists is preparing for the best and fairest medal dinner. She is using the activity network below that shows the individual activities in the project. The duration of each activity in days is also given:



a) Complete the table of immediate predecessors, earliest and latest starting times and float times below for this network. (8 marks)

Activity	Immediate Predecessor	Earliest Starting Time	Latest Starting Time	Float Time
A				
B				
C				
D				
E				
F				
G				
H				
I				
J				
K				
L				
M				
N				

b) What is the minimum time in which this project could be completed? (1 mark)

c) What is the critical path for this network? (1 mark)

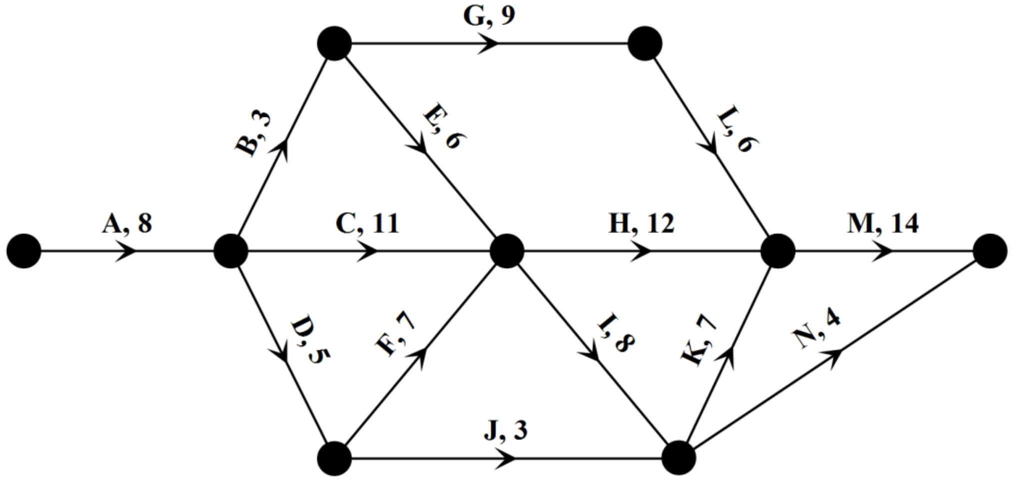
d) The publicist thinks that some of the activities may need to be delayed. Which activity could be delayed for the longest period of time? (1 mark)

e) Activity B is one of the activities that needs to be delayed. When is the latest activity *B* could commence without delaying the project? (1 mark)

f) What is the latest finishing time for activity *E* that will not delay the project? (1 mark)

The publicist has decided that activities *N* and *K* cannot start before activity *G* is completed.

g) Alter the network below to reflect this change. (1 mark)



h) Will this alteration change the completion time of the project? Explain your answer. (2 marks)

TASK SIX (12 marks)

Four of the footballers in the team can each play in different positions on the field. The coach has determined the average number of possessions each player has per game in each of the positions. These are listed in the table below:

	Rover	Wing	Centre	Full Back
Joel	28	25	21	23
Stevie	26	24	18	23
Harry	17	23	22	30
Andrew	19	22	16	26

The coach wants to use this information to allocate players to positions in the team. He wants to place the players so that they will get the maximum number of possessions.

a) Explain why the given values could not be used in the Hungarian Algorithm to determine the allocation. (2 marks)

The coach subtracts all values in the table from 30 and forms the matrix given below that he can use in the Hungarian Algorithm to allocate positions to the players:

$$\begin{array}{c} \\ J \\ S \\ H \\ A \end{array} \begin{array}{c} \begin{array}{cccc} R & W & C & F \end{array} \\ \begin{bmatrix} 2 & 5 & 9 & 7 \\ 4 & 6 & 12 & 7 \\ 13 & 7 & 8 & 0 \\ 11 & 8 & 14 & 4 \end{bmatrix} \end{array}$$

The coach performs a row reduction of the matrix, followed by a column reduction to obtain a reduced matrix.

b) Complete the entries in the blank matrices using a row reduction followed by a column reduction. (4 marks)

$$\begin{bmatrix} 2 & 5 & 9 & 7 \\ 4 & 6 & 12 & 7 \\ 13 & 7 & 8 & 0 \\ 11 & 8 & 14 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \end{bmatrix} \rightarrow \begin{bmatrix} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

c) Explain why the Hungarian Algorithm does not allow an allocation from the reduced matrix found in 3a) above. (2 marks)

d) Complete the matrix that would be required for allocation (the next step in the Hungarian Algorithm). (2 marks)

e) Complete the table below showing the players that the coach should put in each position, given that he allocated using this method. (2 marks)

Position	Player
Rover	
Wing	
Centre	
Full Back	