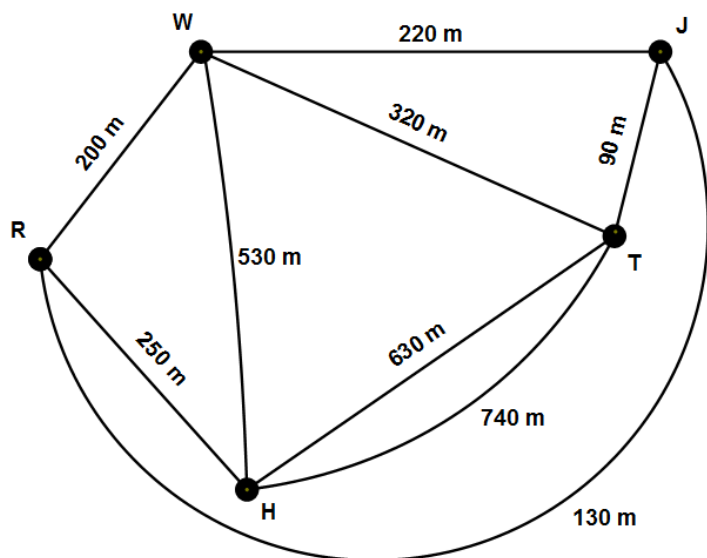


**TASK ONE (28 marks)**

A group of tourists from a bowls club are exploring the Treasury Gardens in Melbourne. There are a number of feature sites in the gardens that the tourists would like to see. They are represented as vertices in the network below, with edges representing the available pathways and their lengths in metres between each of the feature sites. The spaces between the paths are garden beds.



- W* is the Sir William John Clarke Memorial
- J* is the JF Kennedy Memorial
- H* is the Horse Hitching post
- R* is the Robert Burns Memorial Statue
- T* is the public toilets

a) Explain why the network shown is a planar network. Explain this concept in the context of this problem. (2 marks)

b) Use Euler’s rule to show that there are 6 regions in this network. (2 marks)

c) Complete the adjacency matrix,  $M$ , for the network below: (2 marks)

$$M = \begin{matrix} & W & J & H & R & T \\ \begin{matrix} W \\ J \\ H \\ R \\ T \end{matrix} & \left[ \begin{array}{ccccc} & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{array} \right] \end{matrix}$$

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- d) Explain why there are only zeros on the leading diagonal. (1 mark)
- e) Explain why the adjacency matrix is symmetric across the leading diagonal. (1 mark)
- f) Is this network a simple network? Explain your answer. (2 marks)

Some of the tourists would like to make sure that they see every feature site of the gardens exactly once, starting and finishing at the Robert Burns Memorial Statue ( $R$ ).

- g) State the name given to the route that would need to be taken by these tourists. (2 marks)
- h) Write down a route that could be taken by the tourists to make sure that they see all the feature sites exactly once starting and finishing at  $R$ . (1 mark)
- i) Some of the tourists are elderly and so the tourist guide wants to take them to each feature site using the path that has the least total distance. What is the length of the shortest route that takes them to each feature site exactly once starting and finishing at  $R$ ? Include both the length of the route and the actual route taken in your answer. (2 marks)

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- j) Another tour guide suggests that a shorter route that visits every feature site might be found, but that some feature sites would need to be visited twice. The shortest possible route is taken that starts and finishes at  $R$  and visits every feature site at least once. State the length of that route and the actual route taken in your answer. (2 marks)

One of the tour guides is concerned that just visiting the feature sites of the gardens would mean that the tourists would miss the gardens themselves that are in the spaces between the edges in the network. He suggests that it would be better for the group to start at the Robert Burns Memorial Statue ( $R$ ) and travel along every edge exactly once.

- k) Given that the group would want to meet at  $R$  again, explain, using network concepts, why it is not possible for the group to do this in this network without repeating an edge. Include in your answer where the group would finish if they started at  $R$  and used every edge exactly once. (4 marks)

A tourist is at the Horse Hitching Post when she decides that she would like to go to the toilet.

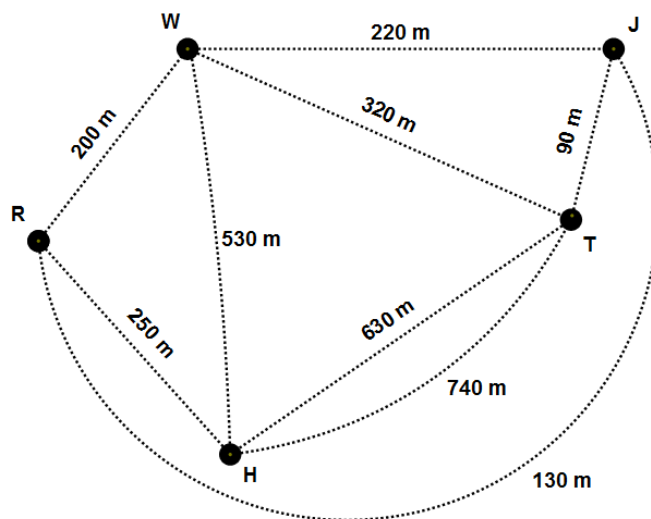
- l) What is the length of the shortest path from  $H$  to  $T$ ? Include the chosen path in your answer. (2 marks)

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The Melbourne City Council want to have the gardens open at night and so they will add security lighting to each of the garden feature sites laying the cables along edges in the network. They wish to explore the minimum length of electrical cable that they could use to connect all of the feature sites to each other using a minimal spanning tree.

m) Add the minimal spanning tree to the network below.

(2 marks)



n) What is the minimum length of cable required to connect all the feature sites?

(1 mark)

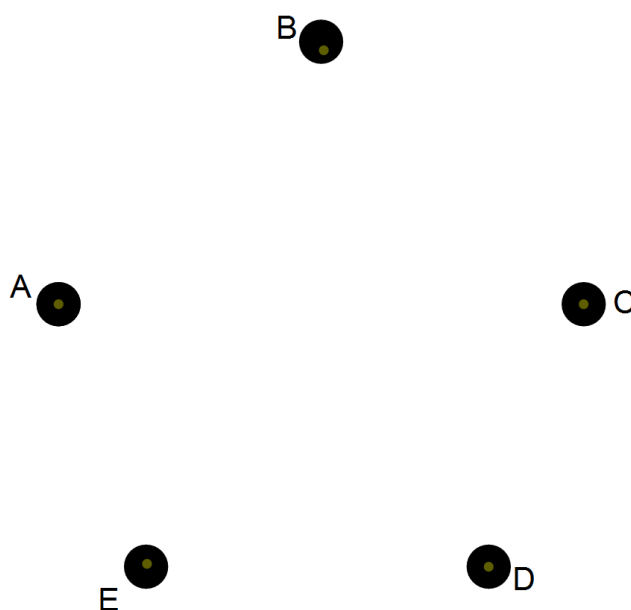
o) Given that the minimum spanning tree must be determined using Prim's algorithm and the starting vertex is *R*, state the order in which the vertices are connected.

(2 marks)

**TASK TWO (7 marks)**

The tourists are all members of five different Bowls Clubs, Ararat (*A*), Ballarat (*B*), Clunes (*C*), Dunolly (*D*) and Enfield (*E*). Before they went on their trip to Melbourne they had a season of bowls games where each club competed once against each other club.

- a) How many games would have been held during the season if each team played every other team exactly once? (2 marks)
- b) What is the name given to a network where every vertex is connected to every other vertex? (1 mark)
- c) Construct the network of games below where the vertices are the teams and the edges are games played between the teams. (2 marks)



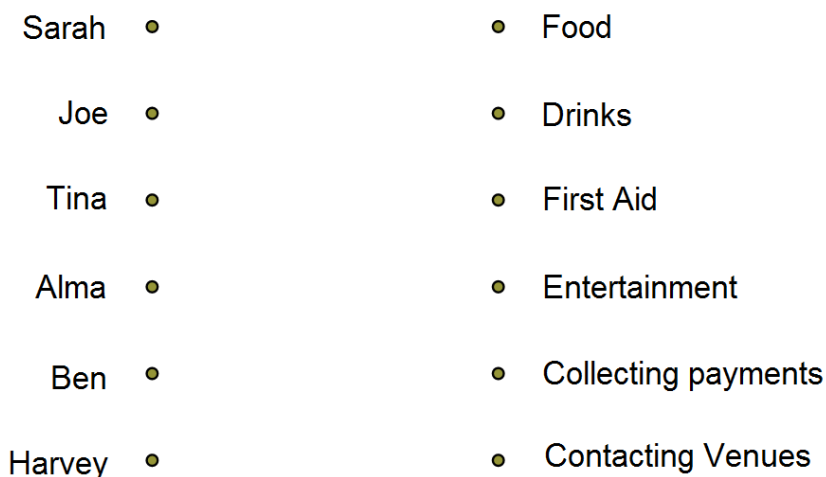
- d) Bowls is proving to be popular and three new teams have joined for the next season. How many extra games will need to be played to ensure that every team plays every other team? Show the working that leads to your answer. (2 marks)

**TASK THREE (3 marks)**

Before going on their trip the tourists allocate a number of jobs that are required for the trip. The following people have stated that they would be willing to do the following jobs;

- Sarah is willing to do food or drinks
- Joe is willing to do drinks or first aid or organise entertainment on the bus
- Tina is willing to do first aid or contacting venues
- Alma is willing to do food or collect payments
- Ben is willing to collect payments
- Harvey is willing to do entertainment or contact venues

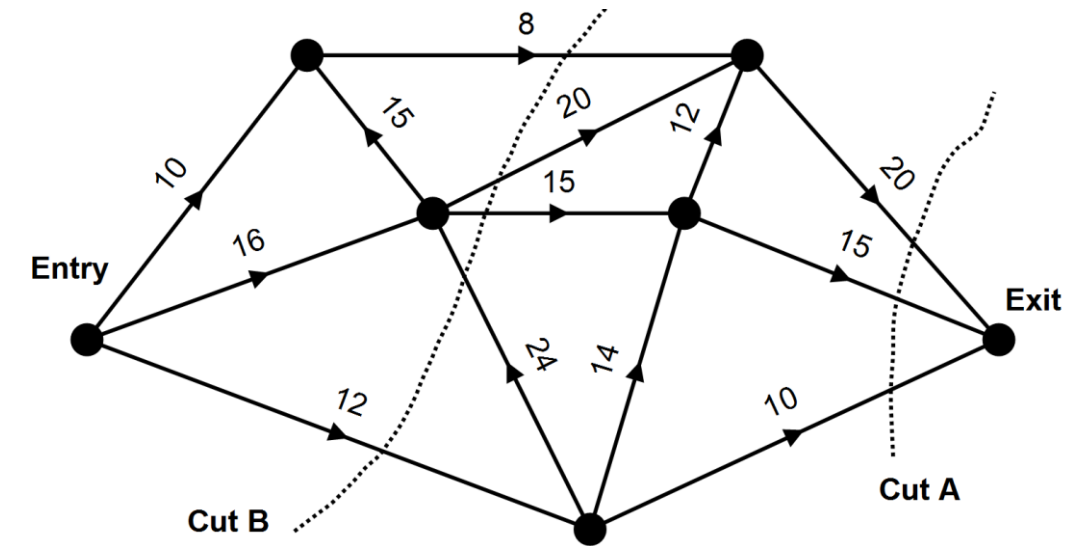
a) Complete the following bipartite graph using the give information. (2 marks)



b) Given that each person does only one job, what jobs should each person be allocated? (1 mark)

**TASK FOUR (11 marks)**

The bowls tourists move on from the Treasury Gardens and go on to visit the Museum. Each section of the museum has guides that move the tourists through different sections, but each guide can only take a certain number of tourists per hour. A network showing the flow through each section, including the maximum number of people who could be guided in each section is shown below.

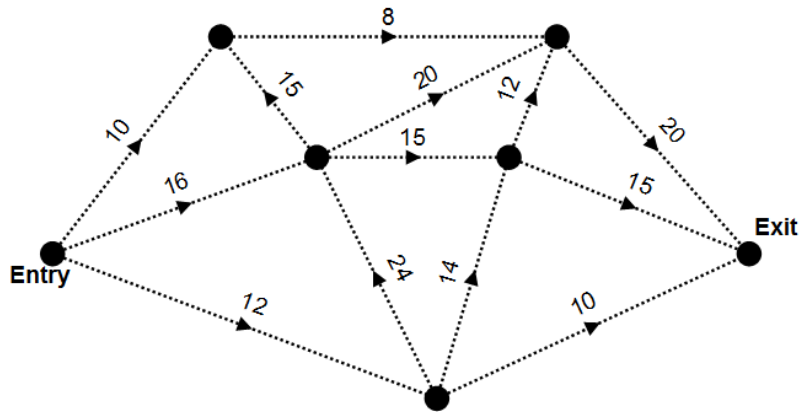


- a) Cut *A* has been shown on the network. What is the capacity of Cut *A*? (1 mark)
  
- b) Cut *B* has also been shown on the network. State the capacity of Cut *B* and show the working that leads to your answer. (2 marks)
  
- c) What do cuts *A* and *B* specifically tell us about the flow through this network from the entry to the exit of the museum? Explain your answer. (2 marks)
  
- d) What is the maximum number of people who can be guided through the museum, from the entry to the exit, per hour? Show and label the cut that you have used to determine this answer on the network above. (2 marks)

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The tourist group has more members than the maximum flow would allow. The museum has offered one extra guide in one section of the museum in order to allow more people through. The guide can take up to 20 extra people.

- e) If flow can be increased on one edge only, what would the new **maximum** flow be? Fully explain the answer that you have given. Include in your answer where the additional guide should be placed in order to achieve the new maximum flow by labelling the appropriate section of the network. (3 marks)

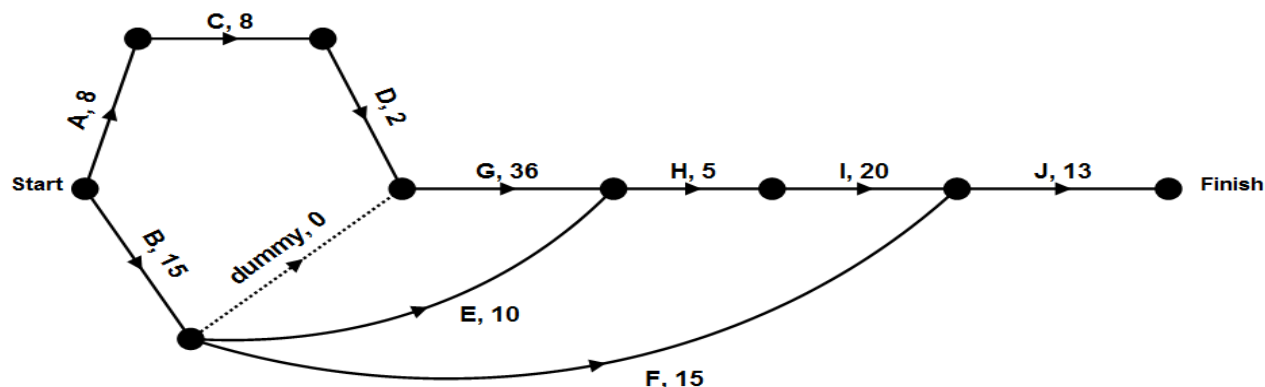


- f) What is the minimum number of people the guide would need to take? (1 mark)



**TASK FIVE (24 marks)**

The Ararat Bowls club are building a sheltered seat for people to watch the bowls competitions. A network of the entire project is shown below.



The activities that make up the project are given in the table below:

Activity	Description	Duration in hours
A	Prepare site	8
B	Purchase materials	15
C	Dig post holes	8
D	Inspection of post holes	2
E	Build roof section on ground	10
F	Build seat section	15
G	Set vertical posts in concrete	36
H	Attach roof section to vertical posts	5
I	Attach panels to frame	20
J	Paint shelter and seat	13

- a) Explain fully why a dummy was used to draw this network. Include in your answer any relevant activities and their predecessor(s). (2 marks)

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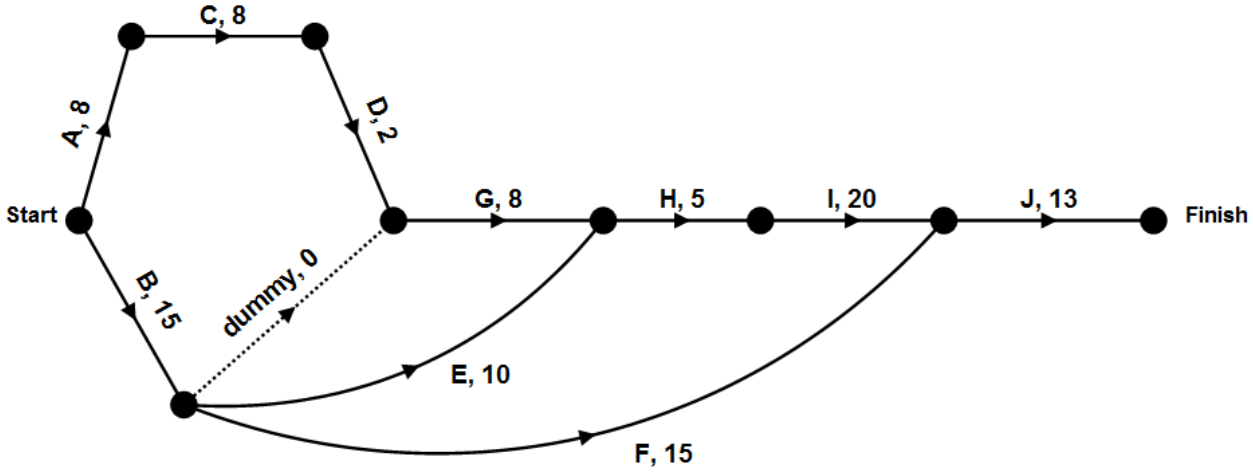
- b) Complete the following table of predecessors, earliest and latest starting times and float times for this network. (8 marks)

Activity	Duration in hours	Predecessor(s)	EST	LST	Float
A	8				
B	15				
C	8				
D	2				
E	10				
F	15				
G	36				
H	5				
I	20				
J	13				

- c) What is the critical path for this project? (1 mark)
- d) What is the minimum time required to complete the project? (1 mark)
- e) What is the earliest finishing time for activity F? (1 mark)
- f) It has been suggested that using “quickset” concrete would reduce the time taken for activity G, so that it now has a duration of 4 hours at no additional cost. What is the minimum time taken to complete the project if “quickset” concrete is used? Explain your answer. (3 marks)

One of the builders is concerned that “quickset” concrete actually requires 8 full hours to cure, so activity G is now allocated a time of 8 hours as shown on the network below.

**For all questions from now on you should assume that the duration of G is 8 hours.**



A number of other reductions are possible, but each has a cost associated with the time reduction. They are shown in the table below along with the cost associated with each hour of reduction. Note that each of the reductions could be completed partially, so a possible reduction of up to 4 hours would allow a reduction of 0, 1, 2, 3 or 4 hours. No part hour reductions are possible.

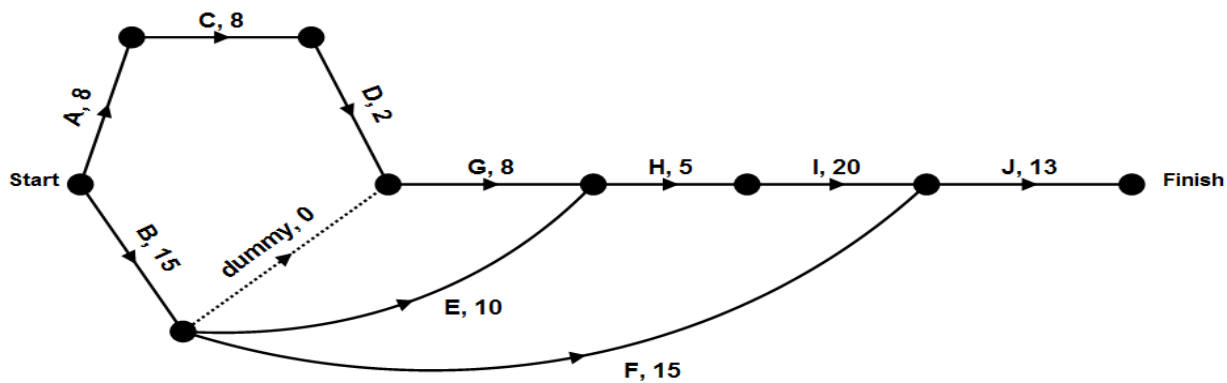
Activity	Original Duration in hours	Available reduction in hours	Minimum duration after reduction in hours	Cost per hour in dollars
A	8	4	4	\$80
C	8	6	2	\$100
E	10	5	5	\$40
F	15	5	10	\$30
I	20	15	5	\$50

- g) What is the new minimum completion time given that activity G has been reduced to 8 hours for no cost and the club has an unlimited budget for time saving? (2 marks)

- h) What is the minimum cost associated with this reduction? Explain your answer including a list of which activities should be reduced and by how much. (4 marks)

- i) If the club only has an additional \$200 to spend on time saving activities (given that G has already been reduced to 8 hours), what would the minimum time to complete the project? Explain your answer by stating how this money would be spent to maximise the reduction. (2 marks)

Below is an additional copy of the network if you require it:



**TASK SIX (5 marks)**

The Clunes Bowls club is also planning to install new seating. They have asked for quotes from a number of businesses and each business has quoted separately for each of 4 components of the job, levelling the site, concreting, building seats and painting. The table below shows the quotations in dollars received:

	Levelling	Concreting	Building Seats	Painting
Business A	800	900	700	650
Business B	700	700	650	600
Business C	700	600	600	400
Business D	650	700	600	500

The bowling club want to support local businesses, so they decide to give one job to each business.

Use the Hungarian Algorithm to determine which business should do each job, showing all steps in your working. (5 marks)