Unit 4 Further Mathematics: Matrices Revision 'Post'

TASK ONE

A local post office charges different amount for express post parcels depending on their weight. Small (S) parcels cost $9.55, medium ($M$) parcels cost $12.80, while large ($L$) parcels cost $13.70.

(a) Write this information in a fully labelled 3×1 matrix, matrix P.

(b) After the price increase a particular company sends 5 small parcels, 8 medium parcels and 3 large parcels in one particular week paying cash for each parcel.
Write the number of each parcel in a labelled 1×3 matrix, N.

(c) Using the matrix obtained in part (c) write and evaluate a matrix product that contains the total cash cost to the company for sending these parcels.
Include the working used in the matrix multiplication in your answer.

TASK TWO

Will is responsible for stocktaking at the post office shop. Every month he orders the new stock using the matrix

$$D = \begin{array}{c} \begin{array}{ccc} S & M & L \end{array} \\ \left[\begin{array}{ccc} 1500 & 2000 & 1000 \\ 50 & 100 & 120 \\ 250 & 380 & 100 \end{array} \right] \begin{array}{l} envelopes \\ padded\ post\ bags \\ express\ post\ satchels \end{array} \end{array}$$

The sales from one particular month are given by the matrix:

$$E = \begin{array}{c} \begin{array}{ccc} S & M & L \end{array} \\ \left[\begin{array}{ccc} 768 & 1820 & 950 \\ 47 & 98 & 102 \\ 189 & 379 & 77 \end{array} \right] \begin{array}{l} envelopes \\ padded\ post\ bags \\ express\ post\ satchels \end{array} \end{array}$$

(a)   Calculate $F = D - E$.

(b)   Explain fully in real terms what is represented by the element f_{23} .

(c)   Will needs to know the total number of envelopes, padded post bags and express post satchels sold in the particular month. He achieves this by multiplying $E \times H$, where H is a 3×1 summing matrix. Write down the matrix H that would achieve this result.

(d)   Calculate $G = E \times H$. Write down matrix G and explain what it represents.

TASK THREE

Will loves playing around with anagrams – letter combinations that can generate a number of different words. For example, the letters A, E, M and T can form the words *MATE*, *MEAT*, *TAME* or *TEAM*.

He also knows that permutation matrices can be used to rearrange the letters in a word.

(a) If matrix $W = \begin{bmatrix} P \\ O \\ S \\ T \end{bmatrix}$ and matrix $P = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, then what word is formed by the

matrix product $P \times W$?

(b) In the matrix provided below, fill in the element values for matrix Q so that the matrix product $Q \times W$ gives the word OPTS.

$$\begin{bmatrix} \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix} \times \begin{bmatrix} P \\ O \\ S \\ T \end{bmatrix} = \begin{bmatrix} O \\ P \\ T \\ S \end{bmatrix}$$

(c) Explain why the matrix product $P^3 \times W$ gives the matrix $\begin{bmatrix} P \\ O \\ S \\ T \end{bmatrix}$.

TASK FOUR
Sarah is an accounts clerk in a large company. She receives an Australia Post account each month for postage of letters in envelopes of various sizes. The sizes of the envelopes that Sarah's company uses are known as C5, C4 and B4 sized envelopes.

In March, 150 C5 letters, 55 C4 letters and 22 B4 letters were sent for a total cost of $459.30.
In April, 145 C5 letters, 45 C4 letters and 17 B4 letters were sent for a total cost of $405.80.
In May, 95 C5 letters, 75 C4 letters and 25 B4 letters were sent for a total cost of $445.00.

(a)   Write this information as a set of simultaneous equations, using x for C5 letters, y for C4 letters and z for B4 letters.

(b)   Write the simultaneous equations as a matrix equation that Sarah could use to calculate the cost of posting each size letter.

(c)   Use matrix arithmetic, showing all working, to solve the matrix equation and hence state the cost of posting each size letter.

Sarah's friend Josie decides to use the same method to work out the price of the Express Post Platinum parcels that her company uses. The company send 10 small satchels and 30 large satchels in one month costing a total of $706.50. The next month her company sends 8 small satchels and 24 large satchels costing a total of $565.20.

The matrix equation that Josie wants to use to determine the price of each type of parcel, using S for the small parcels and L for the large parcels is:

$$\begin{bmatrix} 10 & 30 \\ 8 & 24 \end{bmatrix} \times \begin{bmatrix} S \\ L \end{bmatrix} = \begin{bmatrix} 706.50 \\ 565.20 \end{bmatrix}$$

(d)   Explain **fully why** this matrix equation could not be used in determining the cost of each parcel, ensuring that your explanation includes reference to appropriate matrix terminology.

TASK FIVE

Karla works behind the counter in the post office. She advises people who come into the post office about types of mail. Some people are concerned that their letters may need to be tracked, so she advises them to use either registered mail (R) or express post (E), both of which have tracking numbers that can be traced.

Initially 90% of the people who want their mail tracked use registered mail, while only 10% of people use the express post service. This can be expressed in an initial state matrix,

$$S_0 = \begin{bmatrix} 90 \\ 10 \end{bmatrix} \begin{matrix} R \\ E \end{matrix}$$

Karla is encouraging people to use the express post service and so she records the usage of clients who use either registered mail or express post every week. She finds that 15% of the people who use the registered post service one week will switch to the express post service next week. 80% of the people who use the express post service will use it again next week.

(a) Write the fully labelled transition matrix, T, that would correctly represent this situation.

(b) Given that S_n represents the proportion of customers using registered mail or express post after week n, calculate the value of S_1 and hence state what percentage of the customers will use the express post service after one week.

(c) How long will it take for more than 40% of clients to use express post assuming that the pattern continues? Show the matrices that lead to your answer.

(d) Assuming that the same pattern continues, will there ever be more people using express post than registered mail? Demonstrate fully how you came to your answer.

TASK SIX

Cameron has also been encouraging clients to change to express post in his post office, but he is also encouraging people who normally use a local parcel delivery service (P) to switch. He discovers that the change in the number of people using each service can be modelled using the transition matrix,

$$\begin{array}{c} this\ week \\ \begin{array}{ccc} R & E & P \end{array} \\ M = \begin{bmatrix} 0.5 & 0.45 & 0.1 \\ 0.5 & 0.55 & 0.1 \\ 0 & 0 & 0.8 \end{bmatrix} \begin{array}{l} R \\ E \\ P \end{array} \end{array} \quad next\ week$$

After 1 week 375 people use registered post (R), 385 people use express post (E) and 240 people are using the local parcel delivery service (P).

(a) Using the information contained in the transition matrix, explain why Cameron's mission to change client preference to Express Post is working.

(b) Calculate the numbers of people who used registered post, express post and local delivery **initially** given the numbers **after one week** are as above.

(c) Sam also has been promoting the use of express post at a number of locations across the state. His marketing campaign is in fact so attractive that he has secured new clients to use the service who have never used either registered post or express post or a local delivery service before. Sam started recording the changes and he has found that the number of people using the service each week has changed according to the matrix equation:

$$U_{n+1} = \begin{bmatrix} 0.45 & 0.35 & 0.1 \\ 0.55 & 0.65 & 0.1 \\ 0 & 0 & 0.8 \end{bmatrix} \times U_n + \begin{bmatrix} 1000 \\ 1000 \\ 0 \end{bmatrix}, U_1 = \begin{bmatrix} 50000 \\ 10000 \\ 20000 \end{bmatrix} \begin{array}{l} registered\ post \\ express\ post \\ local\ delivery \end{array}$$

Determine the number of people who use express post each week for the next three weeks.

TASK SEVEN
A local postal sorting exchange has 30 employees. Because sorting mail can be a tedious job, the staff stop after 55 minutes to do a series of exercises for 5 minutes. The postal staff have a choice of doing neck rolls (N), leg stretches (S) or breathing exercises (B).

Initially an equal number of staff perform each exercise.

(a)   Write an initial state matrix, S_0, to represent this information.

The number of staff doing each exercise change each break according to the transition diagram shown below:
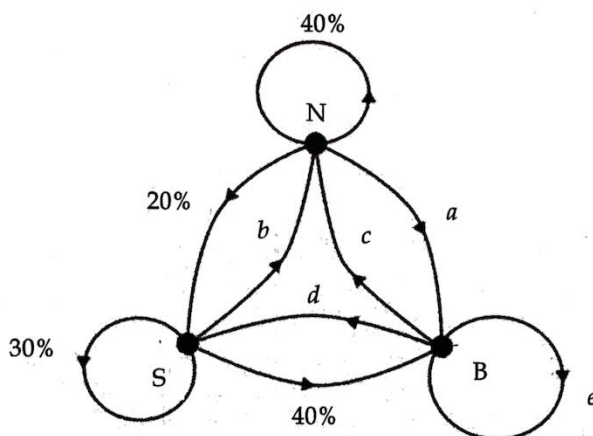

(a)   Add the information in the transition diagram into the transition matrix, T, below.

$$\begin{array}{c} \textit{this time} \\ \begin{array}{ccc} N & S & B \end{array} \\ T = \left[\begin{array}{ccc} \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \end{array} \right] \begin{array}{l} N \\ S \\ B \end{array} \quad \textit{next time} \end{array}$$

(b)   In the second break, 9 of the staff did neck rolls, 8 of the staff did leg stretches and 13 of the staff did breathing exercises. Using this information or otherwise calculate the values of a, b, c, d and e.

TASK EIGHT

The postal exchange employees divided themselves into five teams for a weekly lunchtime darts competition, and the five team captains were Krista, Ling, Manny, Nol and Olive.
Each team played each other team once in the competition, and the results are summarised in the directed graph below. An arrow from K to N signifies that Krista's team (K) defeated Nol's team (N).
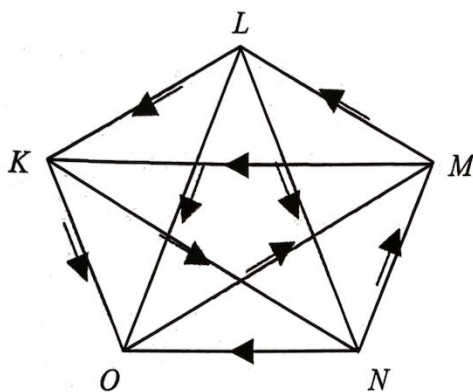
(a)   Construct a one-step dominance matrix for this competition, labelling winners on the rows and losers on the columns.

(b)   Construct a two-step dominance matrix for this competition labelling winners on the rows and losers on the columns.

(c)   Construct a matrix containing the sum of the one-step and two-step dominance matrices, and hence determine the ranking sum for each team. What are the final team placings overall? Note that if two teams finish equal in the ranking sum calculations, the team that won their match will be placed higher.