Unit 4 Further Mathematics: Matrices Revision (Shop)

TASK ONE

In a holiday resort, a general store sells sun protection merchandise, and has recorded the following sales for particular items, as well as the total takings for each of the first three weeks of the Christmas holiday season for those items. The prices of each item are different and have remained unchanged over this three–week period.

Week	Hats	T-shirts	SunLotion	Takings
1	35	20	30	$1430
2	30	20	35	$1455
3	30	15	20	$1075

(a) Using the pronumerals H, T and S to represent the price of the one (1) hat, one (1) T-shirt and one (1) tube of sunlotion respectively, write a system of three simultaneous linear equations to represent this situation. (3 marks)
(Do NOT attempt to solve the system of equations)

(b) Write this system of simultaneous equations in matrix form. (1 mark)

(c) For the **square** matrix in (b), find its determinant. (1 mark)

(d) For the **square** matrix in (b), use your calculator to determine its inverse and display the inverse matrix in decimals. (2 marks)

(e) Use your calculator to solve the matrix equation from (b) to find the price of each type of sun protection merchandise. (1 mark)

(f) After the price changes a family purchases 4 hats, 2 T-shirts and 1 tube of sunlotion paying cash for all items.
Write the number of each item purchased in a labelled 1×3 matrix, N. (1 mark)

(g) Using the matrix obtained in part (h), write and evaluate a matrix product that will show the total cash cost to the family for these purchases.
Include the working used in the matrix multiplication in your answer. (3 marks)

TASK TWO

The general store also sells two daily papers – *The Times* and *The Herald*. On the first day of the Christmas–New Year holiday period when only the local residents were in town, 90 copies of *The Herald* and30 copies of *The Times* were sold. On the second day of the holidays, the store owner notices that 70% of the people who bought *The Herald* on the first day buy it again on the second day, and the rest buy *The Times*. In contrast, only 15% of customers who bought *The Times* on the first day change to *The Herald* on the second day. The total number of papers sold each day is the same. The storekeeper notices that this trend continues for the next nine days.

(a) Write the initial state matrix as a column matrix, putting *The Herald* in the first row. (1 mark)

(b) Write down the transition matrix that represents this situation. (1 mark)

(c) On the 10th day of the holidays the store sells a total of 120 newspapers,
Show that 40 copies of *The Herald* and 80 copies of *The Times* are sold on this day. (2 marks)

(d) Calculate the estimated percentage market share, correct to two significant figures, at this store of each newspaper after ten days. (2 marks)

On the first day of a new week, many of the locals left and the tourists starting moving in for their holiday breaks. The store owner now notices that 80 % of the people who bought *The Herald* on the first day buy it again on the second day, and the rest buy *The Times*. In contrast, only 15 % of customers who bought *The Times* on the first day change to *The Herald* on the second day. Each day there are an extra 25 people buying *The Herald* and an extra 12 people buying *The Times*.

(e) Write down the transition matrix that represents this new situation.

(1 mark)

(f) Using the answer to part (c) as the initial state matrix (S_0) for this new week, write a matrix relationship to model the newspaper sales at this store.

(2 marks)

(g) Use the matrix relationship to show the number of papers sold, as whole numbers, for the next three days.

(3 marks)

TASK THREE

Penny, who is working in the store for her holiday job, loves playing around with anagrams – letter combinations that can generate a number of different words. For example, the letters *E*, *I*, *L* and *V* can form the words *EVIL*, *LIVE*, *VEIL* or *VILE*.

She also knows that permutation matrices can be used to rearrange the letters in a word.

(a) If matrix $W = \begin{bmatrix} A \\ R \\ T \\ S \end{bmatrix}$ and matrix $P = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$, then what word is formed by the matrix product $P \times W$? (1 mark)

(b) In the matrix provided below, fill in the element values for matrix Q so that the matrix product $Q \times W$ gives the word TSAR. (3 marks)

$$\begin{bmatrix} \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \end{bmatrix} \times \begin{bmatrix} A \\ R \\ T \\ S \end{bmatrix} = \begin{bmatrix} T \\ S \\ A \\ R \end{bmatrix}$$

(c) Explain why the matrix product $P^4 \times W$ gives the matrix $\begin{bmatrix} A \\ R \\ T \\ S \end{bmatrix}$. (1 mark)

TASK FOUR

Four of the employees of the general store played a round-robin pinball tournament during their lunch breaks, in which every employee played against every other employee once. In each game there was a winner and a loser.

A table of one-step and two-step dominances was prepared to summarise the results.

Employee	one-step dominances	two-step dominances
Penny (P)	2	3
Quinn (Q)	1	1
Roxie (R)	1	2
Sarah (S)	2	2

(a) Use the one-step and two-step dominance values to construct a one-step dominance matrix, clearly explaining the allocation of values for each employee. Label the rows as Winners and the columns as Losers. (4 marks)

(b) Use the one-step and two-step dominance values to construct a two-step dominance matrix, clearly explaining the allocation of values for each employee. Label the rows as Winners and the columns as Losers. (4 marks)

(c) Construct the final dominance matrix for this competition by adding together the one-step and two-step dominance matrices. (1 mark)

(d) What is the finishing order in this competition? (1 mark)

(e) Write in the results of every match in the table below. (3 marks)

Round	Participants	Result
1	Roxie vs Penny	.....
	Quinn vs Sarah	.....
2	Penny vs Quinn	.....
	Roxie vs Sarah	.....
3	Quinn vs Roxie	.....
	Penny vs Sarah	.....

(f) Draw a directed graph which displays the results of this competition, with the arrow pointing towards the loser of each game. (2 marks)