SAC 3 – Problem Solving Task Revision

Question 1 (2011 Kilbaha)

In a round robin five tennis teams, the Anchors, Bull Dogs, Condors, Dragons and Emus played each other. The results were as follows

The Anchors defeated the Bull Dogs and Emus.
The Bull Dogs defeated the Emus
The Condors defeated the Anchors, Bull Dogs and Dragons.
The Dragons defeated the Anchors and Bull Dogs.
The Emus defeated the Condors and Dragons

Construct a diagram which represents these results and use one-step and two-step dominance matrices to determine the ranking and hence state the winning team.

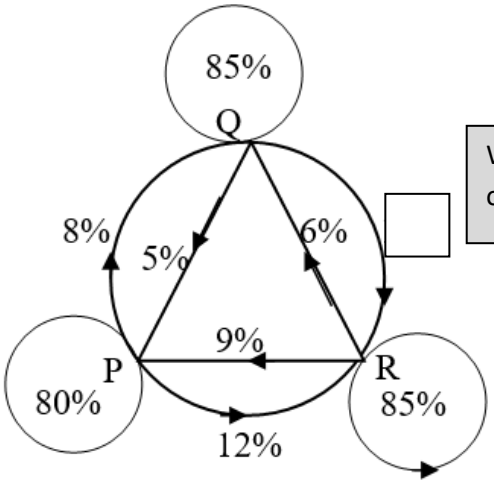
7 marks

Question 2 (2011 Kilbaha)

There are three tennis centres in a certain town, Physical, Quality and Rigour. Physical had 50% of the memberships at the end of the first year while Quality and Rigour had 25% each.

Notice that each tennis centre has 3 edges, for example P to P(80%), P to R(12%) and P to Q(8%). Notice the percentages add to 100%.

2 edges are missing arrows, remember all edges must have arrows



What percentage should go on this edge?

The above diagram shows the movement of members from one tennis centre to another each year.

a. What percentage of members remain with Physical each year?

_____

_____

1 mark

b. What percentage of members move from Rigour to Physical each year?

_____

_____

1 mark

c. Complete the transition matrix that can be used to represent the information in the diagram.

Remember that each column must add up to 1

$$\begin{array}{cc} & \begin{array}{ccc} From & & \\ P & Q & R \end{array} \\ \begin{array}{c} P \\ To\ Q \\ R \end{array} & \begin{bmatrix} & 0.05 & \\ 0.08 & & \\ & & 0.85 \end{bmatrix} \end{array}$$

1 mark

d. If there are 800 members altogether in the three tennis centres, what is the column matrix, N_1, that shows the number of members at the end of the first year.

1 mark

e. Investigate the memberships by finding the state matrices for the first several years and describe the pattern in the memberships.

Several years means 3 or more not including the initial values.

4 mark

f. How many more members would Rigour have at the end of the second year compared to the end of the first year?

_____

_____

1 mark

g. In the long term which club will have the smallest number of members?

Long term = steady state

Remember two consecutive answers are required to show that a steady state has been reached.

Use rounding to the nearest whole number

2 marks

Question 3 (2011 Heffernan)

A gym runs cardio, jump and zumba classes at each of three levels, introductory, mainstream and advanced. Members can bring along a friend to these classes by purchasing a guest pass. The number of guest passes sold last month in each of these classes is shown below.

	cardio	jump	zumba
introductory	7	8	10
mainstream	4	5	3
advanced	2	3	1

a. How many guest passes were sold last month for mainstream zumba classes?

_____

_____

1 mark

The revenue earned by the gym for guest passes to classes last month in cardio, jump and zumba was \$440, \$209 and \$103 respectively.
The cost of a guest pass to a cardio class is \$$x$, to a jump class is \$$y$ and to a zumba class is \$$z$.
The values x, y and z can be found by solving the matrix equation below.

$$\begin{bmatrix} 7 & 8 & 10 \\ 4 & 5 & 3 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 440 \\ 209 \\ 103 \end{bmatrix}$$

b. Using the matrix equation above write out the 3 simultaneous equations it represents.

2 marks

c. Solve the matrix equation above and hence find the cost of a guest pass to a cardio, jump and zumba class.

2 marks

Question 4 (2011 Heffernan)

The gym has 90 people who have signed up for weekly yoga classes. They can attend a yoga class either in the morning (M), at lunchtime (L) or in the evening (E).
In the first week of classes, 30 people attend the morning class, 20 people attend the lunchtime class and 40 attend the evening class.
The state matrix S_1, showing the number of people attending morning, lunchtime and evening yoga classes in the first week is shown below.

$$S_1 = \begin{bmatrix} 30 \\ 20 \\ 40 \end{bmatrix} \begin{matrix} M \\ L \\ E \end{matrix}$$

It is also determined that:

- 60% of people who attend a morning (M) yoga class one week will attend a morning (M) yoga class the next week
- 30% of people who attend a morning (M) yoga class one week will attend an evening (E) yoga class the next week
- 20% of people who attend a lunchtime (L) yoga class one week will attend a morning (M) yoga class the next week
- 30% of people who attend a lunchtime (L) yoga class one week will attend an evening (E) yoga class the next week
- 80% of people who attend an evening (E) yoga class one week will attend an evening (E) yoga class the next week
- 10% of people who attend an evening (E) yoga class one week will attend a lunchtime (L) yoga class the next week

a. What percentage of people each week change from a lunchtime yoga class to an evening yoga class?

_____

1 mark

b. Construct a network diagram the represents this information.

3 marks

c. Hence construct the corresponding transition matrix, T

2 marks

d. How many people will be in the evening yoga class in the second week?

1 mark

e. How many people will be in the lunchtime class in the 4^{th} week?

1 mark

f. Any class that drops below 16 people is discontinued.
Explain whether or not any of the classes are discontinued, and in what week that would occur.

To show when it is discontinued you need to show the matrix when this happens and then one on either side.

2 marks

Question 5 (2011 Heffernan)

Two cardio classes are run at the same time on Sundays. One is run by Barry and the other by Tina.
The matrix N_1 below shows the number of people who went to each of the classes the first week that they ran.

$$N_1 = \begin{bmatrix} 32 \\ 24 \end{bmatrix} \begin{matrix} \text{Barry} \\ \text{Tina} \end{matrix}$$

The number of people in the classes in the second week is given by

$$N_2 = AN_1 + B$$

$$\text{where } A = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

a. Explain matrix B in the context of the question

1 mark

b. Find N_2.

1 mark

The number of people in each of the classes can be found using the matrix equation

$$N_{n+1} = AN_n + B$$

where N_n gives the number of people in the classes in the nth week.

c. Explain, using the above equation, how the number of people in each of the classes change over a 4-week period.

Calculations must be shown and a written explanation.

4-week period includes week 1, N_1.

3 marks

Question 6

Two tennis centres have their own outlet shop where they sell racquets, balls and hats.

Let a be the cost of a racquet, b the cost of a ball and c the cost of a hat.

The **Walker** outlet shop recorded the following sales over three days:

Day	Racquets	Balls	Hats	Total Sales $
Day 1	5	25	11	671.50
Day 2	9	19	7	874.50
Day 3	3	8	5	348.00

The **Fitness** outlet shop recorded the following sales over three days:

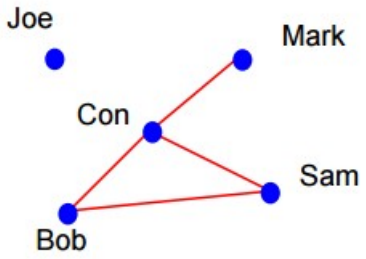
Day	Racquets	Balls	Hats	Total Sales $
Day 1	6	13	9	676.50
Day 2	10	22	3	876.00
Day 3	10	25	12	1069.50

You wish to purchase 1 racquet, 5 balls and 2 hats. Which shop would it be cheaper to purchase these items from and how much would it cost? Make sure you include all relevant working needed to reach your answer.

7 marks

Question 7 (Cambridge Ch 13 Test 2)

The diagram opposite shows the social network links between five boys, Joe, Mark, Con, Bob and Sam.



This network has been represented by the 5×5 communication matrix below using the rules:

- element = 1 if the pair of boys can use the social network to communicate directly
- element = 0 if they cannot use the social network to communicate directly
- All diagonal elements are zero.

$$F = \begin{array}{c} \\ \text{J} \\ \text{M} \\ \text{C} \\ \text{B} \\ \text{S} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{J} & \text{M} & \text{C} & \text{B} & \text{S} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

a. Write down the value of $f_{2,3}$.

1 mark

b. In the matrix F, the element $f_{4,3}=1$. What does this tell us?

1 mark

c. In the matrix F, the element $f_{2,4}=0$. What does this tell us?

1 mark

To investigate the presence of two-step communication links, the communication matrix F is squared, with the result shown opposite

$$F^2 = \begin{array}{c} \\ \text{J} \\ \text{M} \\ \text{C} \\ \text{B} \\ \text{S} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{J} & \text{M} & \text{C} & \text{B} & \text{S} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 3 & 1 & 1 \\ 0 & 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 & 2 \end{bmatrix} \end{array}$$

d. In the matrix $F^2, f_{2,4}=1$. What does this tell us? The diagram will help here.

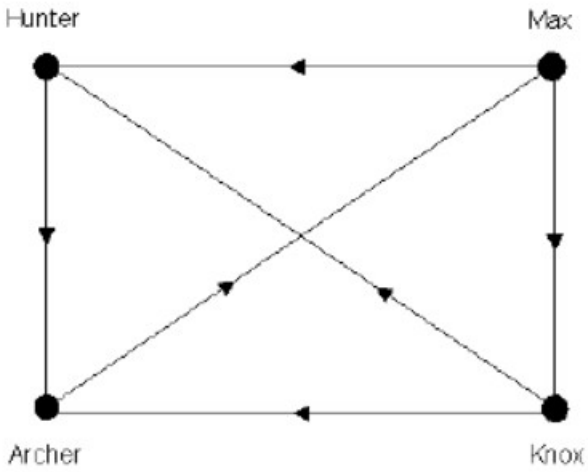
1 mark

e. How many of the two-step communication links are redundant?

1 mark

Question 8 (MAV 2012)

Four builders are all enthusiastic wrestlers. They engage in a friendly round-robin tournament where each builder will compete against each other.
In each contest there is a winner and a loser.
The directed graph illustrates the results where an arrow from Hunter to Archer shows that Hunter defeated Archer.



a. Use the graph to construct a one-step dominance matrix, *D*.

1 mark

b. Explain whether or not it is possible to rank the builders from first to fourth using the one-step dominance matrix.

1 mark

c. Explain why Knox has a value of 2 for two step dominance.

1 mark

d. Use an appropriate method to determine the ranking of the builders from first to fourth. Show clear working.

2 marks

Question 9 (Neap 2012)

In a ping pong tournament each of four players(Jason, Magnus, Agnes and Beatrice) has a rating and the points system works like this:

The loser gives 20% of their points to the winner of a match.

Jason is responsible for the system of tallying points. He decides to determine the new points after a round of matches by pre multiplying the current points, matrix P, by another matrix A.

In round six, matrix A is

$$\begin{matrix} J & M & A & B \end{matrix}$$
$$\begin{bmatrix} 0.8 & 0 & 0 & 0 \\ 0.2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.2 \\ 0 & 0 & 0 & 0.8 \end{bmatrix} \begin{matrix} J \\ M \\ A \\ B \end{matrix}$$

The matrix P, $\begin{bmatrix} 27 \\ 18 \\ 12 \\ 17 \end{bmatrix}$, is the points after round five.

a. Find the product AP.

1 mark

b. Who were the winners in round six and who did they defeat?

2 marks

After eight rounds, the singles matches are complete and the doubles begin.

The points after round 8 are $\begin{bmatrix} 16 \\ 16 \\ 20 \\ 22 \end{bmatrix}$

A new system is instituted for these. Each player is rated individually regardless of the result.

Jason makes a prediction that in each doubles round the points total will be altered according to the recurrence relation $P_{n+1} = A^{¿} \times P_n$, where

$$A^{¿} = \begin{bmatrix} 0.9 & 0.1 & 0 & 0.1 \\ 0.05 & 0.8 & 0.05 & 0 \\ 0 & 0.1 & 0.8 & 0.1 \\ 0.05 & 0 & 0.15 & 0.8 \end{bmatrix}$$

c. Determine the points after round nine (the first doubles round).

1 mark

Question 10

A chef produces one main course "special" each day.
He chooses either a pasta (P), risotto (R), fish (F) or chicken (C) dish for the specials.
The transition matrix T, shows how the chef determines which dish he will prepare from day to day.

$$T = \begin{array}{c} \\ P \\ R \\ F \\ C \end{array} \begin{array}{c} \begin{array}{cccc} P & R & F & C \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{l} \text{one day} \\ \\ \\ \\ \\ \text{next day} \end{array}$$

a. Explain whether or not the chef ever chooses the same dish two days in a row.

1 mark

b. The chicken dish always comes before which dish?

1 mark

c. If the chef chooses pasta one day, then what dish will he choose two days later?

1 mark

d. On Monday the chef chooses fish as the "special" dish. List the "special" dishes for the remainder of the week.

2 marks