Camberwell Girls Grammar School
An Anglican School - Educating Tomorrow's Woman

STUDENT NUMBER

Figures										Letter
Words										

Student Name				
Teacher	Ms. Lobo	Ms. Kinnane	Mrs. Bergamin	Mr. Naudi

MATHEMATICAL METHODS
Application Task – Part A

Tuesday 31st May 2016
Reading time: 5 minutes
Writing time: 1 hour 55 minutes

Modelling Task

Number of questions	*Number of questions to be answered*	*Number of marks*
7	7	**46**

- Students are permitted to bring into the Assessment room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are **not** permitted to bring into the Assessment room: notes of any kind, a calculator of any kind, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 11 pages.
- Working space is provided throughout the book.

Instructions

- Write your name in the space provided above on this page.
- All responses must be written in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room. Students must not disclose the contents of the task; to do so will be a breach of VCE guidelines and will be dealt with according to VCAA regulations.

Instructions for Assessment Task
Answer **all** questions in the spaces provided.
Unless otherwise specified an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

A roller coaster called ‘Splash’ is in the process of construction and is designed to soak the occupants as part of the ride experience. As such, a trapezoidal pool (as shown in Figure 1 below) will be placed at the end of the roller coaster. The initial plans for the pool include a 6 metres wide pool with a depth of water ranging from 0.5 metres at the shallow end to 1.8 metres at the deep end.

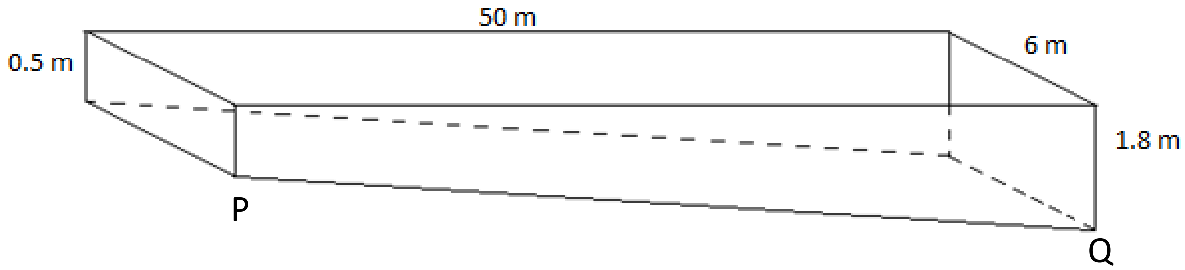


Figure 1: Trapezoidal pool

A hole is being created to fit this trapezoidal pool into the ground, as shown in Figure 2 below.

It is thought that a hole with curved sides as in the diagram below, may be the best space to fit the pool. The width of the hole is 10 metres. Consider the side labelled ABC (front face of the hole).

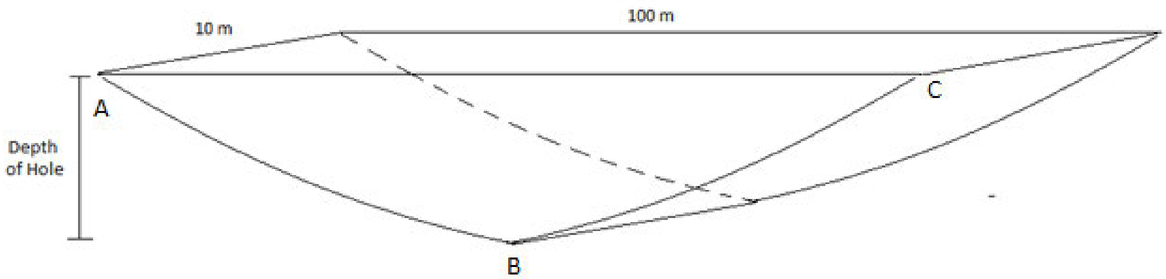
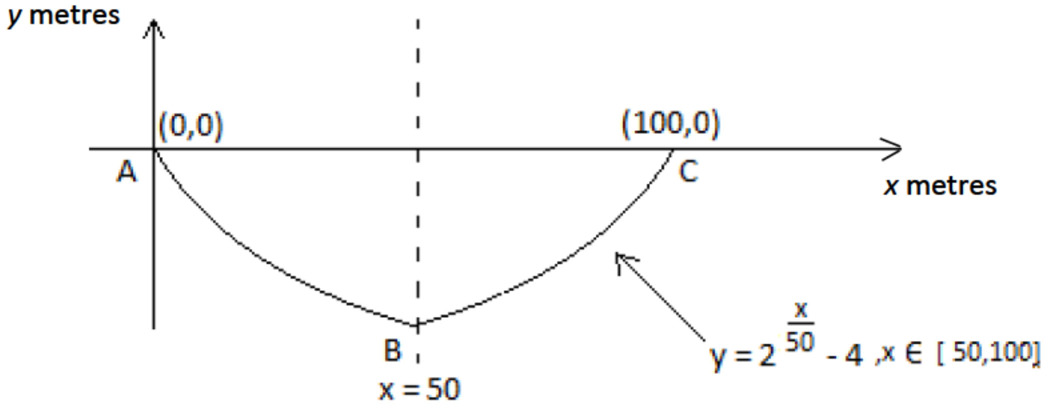


Figure 2: Hole

Two exponential functions are used to model the shape of the hole. Point B to point C **(BC)** is modelled by the equation $y = 2^{\frac{x}{50}} - 4, x \in [50,100]$ and point A to point B **(AB)** is a reflection of BC along the line $x = 50$.



Question 1 **(1+2+1 = 4 marks)**

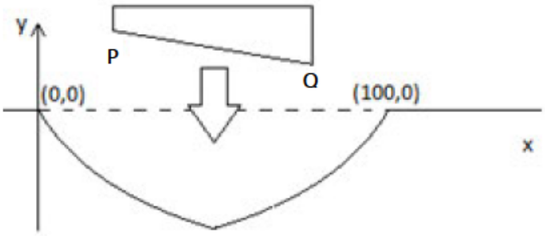
a. The domain of the function representing **BC** is $x \in [50,100]$. Determine the corresponding range.

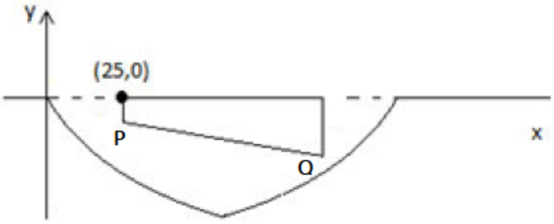
b. Determine the equation representing the **AB**. Include the implied domain.

c. What is the maximum depth of the hole?

Question 2 **(2+1+3 = 6 marks)**

The pool is to be placed so it is centred in a hole.





a. Show that the **equation** of the line that represents the bottom of the pool **PQ** has the equation $P(x) = -\frac{13}{500}x + \frac{3}{20}$

b. Determine the domain for $P(x)$, giving your answer in set notation.

c. Determine whether the hole is deep enough for the pool. If not, how much deeper does the hole need to be? Give your answer to the nearest **centimetre**. (Assume $\sqrt{2} = 1.41$)

Question 3 **(4 marks)**

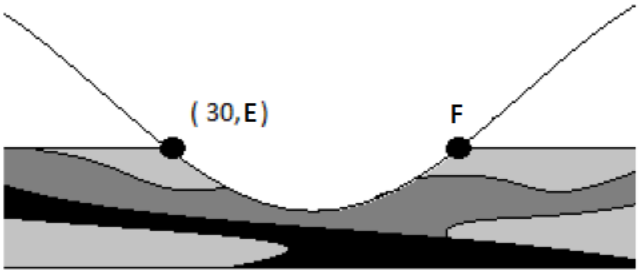
a. The hole is **modified** so that it is represented by the following function.

$$h(x) = \begin{cases} 3\cos\left(\frac{\pi x}{40}\right) - \frac{9}{2}, & x \in [0,80] \\ 0, & x \in (-\infty, 0] \\ 0, & x \in [80, \infty) \end{cases}$$

Sketch $h(x)$ on the axis below. Include endpoints, turning points and intercepts in co-ordinate form.

Question 4 **(2+2+3 = 7 marks)**

When digging the hole a horizontal rock bed is found.



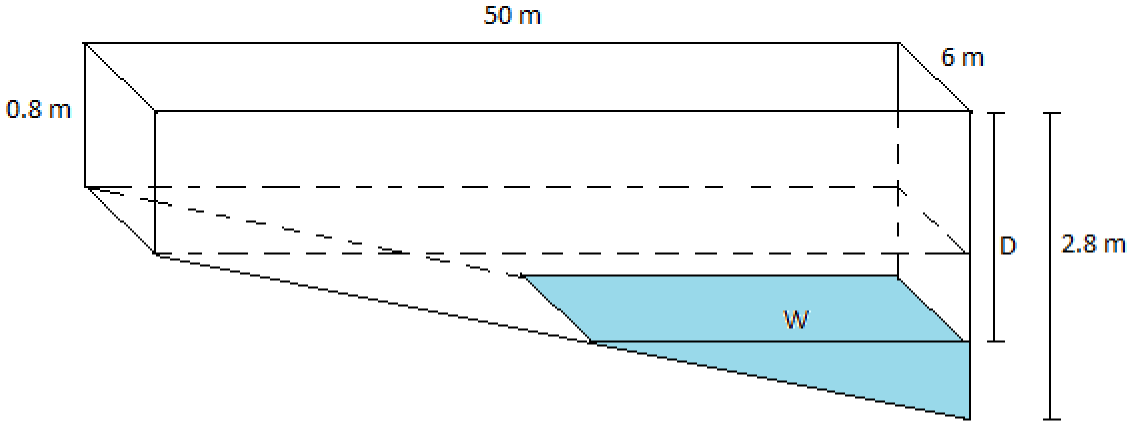
a. The rock layer is hit when $x = 30m$. Find the depth of this rock bed (E).

b. Find the co-ordinates of the point F which represents the end of the digging in the rock bed.

c. If the rock bed is to be avoided, give the new equation of $h(x)$ such that the rock bed is a tangent to the minimum turning point.

Question 5 **(1+2+1+2+(1+1+3)= 11 marks)**

The pool is given new dimension as shown in the diagram below. When the project is finished the pool must be completely filled with water. The diagram below shows the pool partly filled with water, where D represents the **top of the water surface up to the ground level** and W represents the horizontal length of the water surface.



a. Show that $W = 25\left(\frac{14}{5} - D\right)$ when expressed in terms of $\boldsymbol{D}$.

b. The volume of a triangular prism is found using the formula $V = \frac{1}{2}$ *x Base x Width x Height.* Show that the volume of water in the bottom of the pool is found using the formula

$$V(D) = 75\left(\frac{196}{25} - \frac{28D}{5} + D^2\right).$$

c. Give the range of D for which the $V(D) = 75(\frac{196}{25} - \frac{28D}{5} + D^2)$ equation is **relevant**.

d. Find the rate at which the volume is changing when $D = 1.8$ metres

e. The pool is being filled with water. The volume of water increases following the rule $V = \frac{1}{10}t^2$, where t is time measured in minutes and V is volume measured in cubic metres.

Find;

i) The volume when $t = 20$ minutes.

ii) The rate at which the volume is changing?

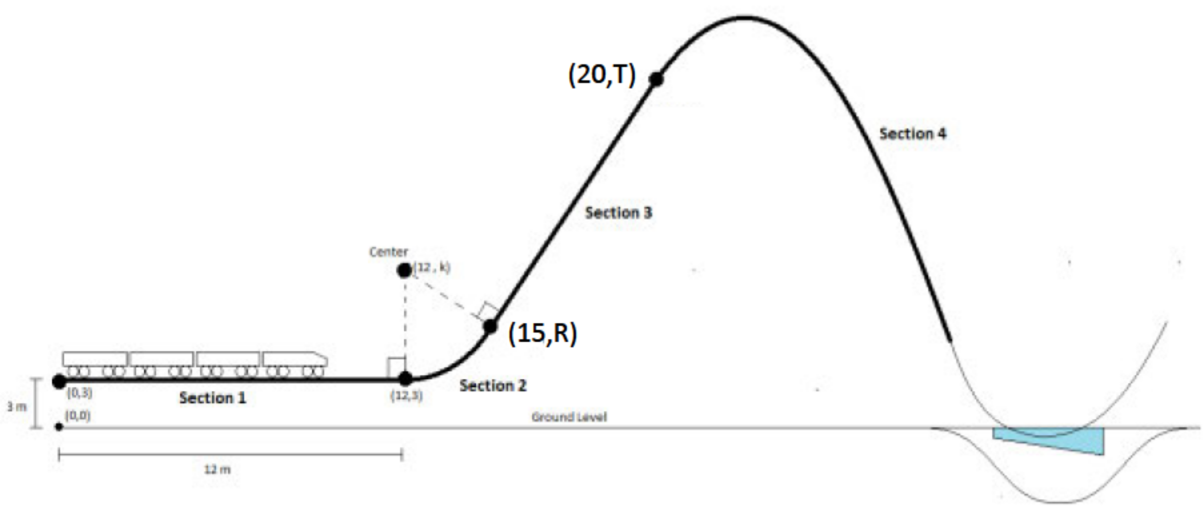
iii) Give the domain for $\frac{dV}{dt}$ (Write your answer in exact form using set notation).

Question 6 **(1+1+3+2+2+1+1= 11 marks)**

The rollercoaster design for the theme park is to be made of a series of different track components. There are four Sections labelled 1, 2, 3, and 4.

- **Section 1** is from (0,3) to (12,3)
- **Section 2** is from (12,3) to (15,R)
- **Section 3** is from (15,R) to (20,T)
- **Section 4** is from (20,T) onwards.

The start of the roller coaster is a level piece of track at a height of 3 metres off the ground. It has a gradient of zero and is 12 metres long.



a. Find the equation of **Section 1** of the track, stating its domain in interval notation.

b. **Section 3** represents a linear function. Find the gradient m of this section, terms of R and T.

c. i) The equation of **Section 2** is a circle with a restricted domain. The point $(12{,}3)$ represents the start of the circle, where the gradient is zero. The **centre** of the circle is $(12, k)$. If the general equation of a circle is $(x-h)^2 + (y-j)^2 = r^2$ show that the equation of **Section 2** is represented by

$$y = -\sqrt{(k-3)^2 - (x-12)^2} + k,\ [12,15]$$

ii) Find the gradient equation for **Section 2** of the track.

iii) **Section 2** and **Section 3** connect smoothly (where the gradient of each section is equal). If $(T-R) = 15$, show that $k = \sqrt{10} + 3$

iv) Determine the value of the radius of the circle used to construct **Section 2** of the track.

v) Give the final equation for **Section 2** in terms of x.

Question 7 **(3 marks)**

Section 4 represents a parabola. The parabola has a turning point when $x = 25$. Find the equation of this section if the gradient is -3 when it passes through the point $(30, \sqrt{10} + 16)$.

END OF PART A