Camberwell Girls Grammar School
An Anglican School - Educating Tomorrow's Woman

STUDENT NUMBER

										Letter
Figures										
Words										

Student Name				
Teacher	Ms. Lobo	Ms. Kinnane	Ms. Bergamin	Mr. Naudi

MATHEMATICAL METHODS
Application Task – Part B

Wednesday 1st May 2016

Reading time: 5 min

Writing time: 55 min

Modelling Task

Number of questions	*Number of questions to be answered*	*Number of marks*
5	**5**	**38**

- Students are permitted to bring into the Assessment room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are not permitted to bring into the Assessment room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 8 pages.
- Working space is provided throughout the book.

Instructions
- Write your name in the space provided above on this page.
- All responses must be written in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room. Students must not disclose the contents of the task; to do so will be a breach of VCE guidelines and will be dealt with according to VCAA regulations.

Instructions for Assessment Task

Answer **all** questions in the spaces provided.
Unless otherwise specified an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

The "Splash" roller coaster uses carriages to carry the passengers around the track. The carriages are a **cuboid shape** without a top (open cuboid) as shown in Figure 1 below.

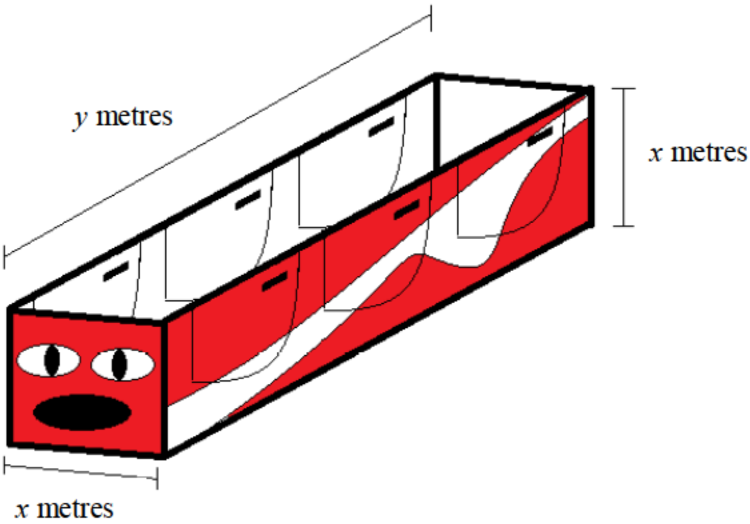


Figure 1: Roller coaster carriage

Question 1 **(1+1+(1+1)+4 = 8 Marks)**

a. Give the external **surface area,** $S(x)$, of the carriage in terms of $\boldsymbol{x}$ and $\boldsymbol{y}$.

b. Give the **volume**, $V(x)$, of the carriage in terms of $\boldsymbol{x}$ and $\boldsymbol{y}$.

c. The volume of a carriage is limited to 4.6 m^3.

i) Give y in terms of x.

ii) Determine $S(x)$ in terms of x.

To construct the carriages at a **minimum** cost, the surface area must kept to a minimum. Design requirements state the carriages must be at least 1.5 metres wide.

d. Determine the values of x and y which allow $S(x)$ to be a minimum, giving your answers correct to three decimal places. Hence, state whether the width requirements are met.

Question 2 **(2+4+2+2 = 10 marks)**

The **seats** for the carriages are made in separate sections, the **headrest** and the **backrest**, as shown in Figure 2 below.
The **backrest** is modelled by two different function, $U(x)$ and $L(x)$.

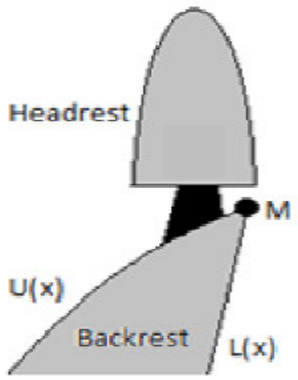


Figure 2: Seats

The **backrest** of the seats are designed using two equations. The **upper** component of the seat, $U(x)$, is modelled using the equation,

$$U(x) = 20\log_e(x+4), \qquad x \in [0, A]$$

The **lower** component of the seat, $L(x)$, is modelled using the equation,

$$L(x) = 2^{x-1} - 16, \qquad x \in [5, A]$$

The point **M** in Figure 2 is the point of intersection for the upper and lower sections of the seat.

a. Determine the value of $\boldsymbol{A}$ (the x-coordinate of **M**), correct to the nearest whole centimetre. Hence, find the height of the backrest, giving your answer correct to the nearest whole centimetre.

b. Sketch $U(x)$ and $L(x)$ on the axes below, labelling all x-intercepts, y-intercepts and points of intersection in coordinate form.

c. Give the equation of the tangent for $U(x)$ at the point where $x = 6$.

d. Give the equation of the tangent for $L(x)$ at the point where $x = 6$.

Question 3 **(1+3+3 = 7 marks)**

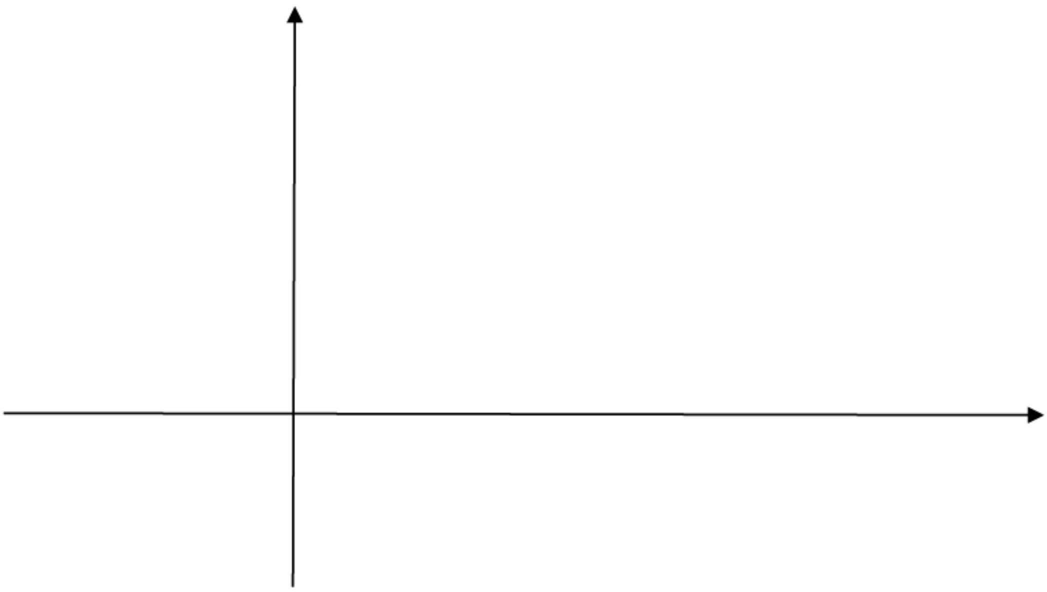
Another roller coaster, "Supreme Scream", is under construction. The track for this ride is modelled by the function,

$$T(x) = x^3 - 6x^2 - x + 30, x \in [-3, 6]$$

a. Show that $(x + 2)$ is a factor of $T(x)$.

b. Fully factorise $T(x)$ using polynomial division.

c. Sketch the graph of $T(x) = x^3 - 6x^2 - x + 30, x \in [-3, 6]$ giving all intercepts and turning points in co-ordinate form.



Question 4 **(3+2+3 = 8 marks)**

Support structures (braces) beneath the "Supreme Scream" track connect to the ground and are modelled on exponential functions. The original design for a brace has the equation

$$B(x) = e^{(4x-2)} + 1, x \in [2,5]$$

a. Show that the inverse equation, $B^{-1}(x) = \frac{1}{4}\log_e(x-1) + \frac{1}{2}$

b. Show that $B\left(B^{-1}(x)\right) = x$

It is thought that a different design of brace may be stronger.

c. Give a new equation, $N(x)$, after applying the following transformations to $B(x) = e^{(4x-2)} + 1$:

- translate the function 2 units down,
- dilate the function by a factor of 3 from the y-axis,
- translate the function 4 units left.

Question 5 **(4+1 = 5 marks)**

For part of the "Supreme Scream" journey, the acceleration of the carriages is modelled on the function

$$F(t) = at^4 + bt^3 - ct^2 + dt$$

Recorded data shows that

- $F(1.3) = 12.9643$ and $F'(1.3) = 25.42$
- $F(2.1) = 53.4755$ and $F'(2.1) = 83.855$

a. Use the **recorded data** to find the values of a, b, c and d, giving your answers correctly to one decimal place.

b. Determine the **largest** possible **domain** required to make the function $F(t)$ a one–to–one function and give the corresponding **range**.

END OF PART B