Camberwell Girls Grammar School
An Anglican School - Educating Tomorrow's Woman

STUDENT NUMBER

Figures									Letter
Words									

Student Name				
Teacher	Ms. Lobo	Ms. Kinnane	Mrs. Bergamin	Mr. Naudi

MATHEMATICAL METHODS
Modelling Task – Calculus

Monday 1st August 2016
Reading time: 5 minutes
Writing time: 1 hour 55 minutes

Modelling Task

Number of questions	*Number of questions to be answered*	*Number of marks*
6	6	**63**

- Students are permitted to bring into the Assessment room: pens, pencils, highlighters, erasers, sharpeners, rulers, one approved bound reference, one CAS and/or one scientific calculator.

Materials supplied

- Question and answer book of 12 pages.
- Working space is provided throughout the book.

Instructions

- Write your name in the space provided above on this page.
- All responses must be written in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room. Students must not disclose the contents of the task; to do so will be a breach of VCE guidelines and will be dealt with according to VCAA regulations.

A park in County Cove is due for an upgrade and the town planners have decided to add some new features to the park, including a pedestrian paths and a pond and a fountain.

Question 1

A Ferris wheel is proposed for the park. The Ferris wheel rotates such that the distance, d metres, from the ground is given by the rule:

$$d = 8 - 6\cos\frac{\pi t}{24}$$

where t is the time in seconds.

a. How far is the ride above the ground when $t = 0$?

(1 Mark)

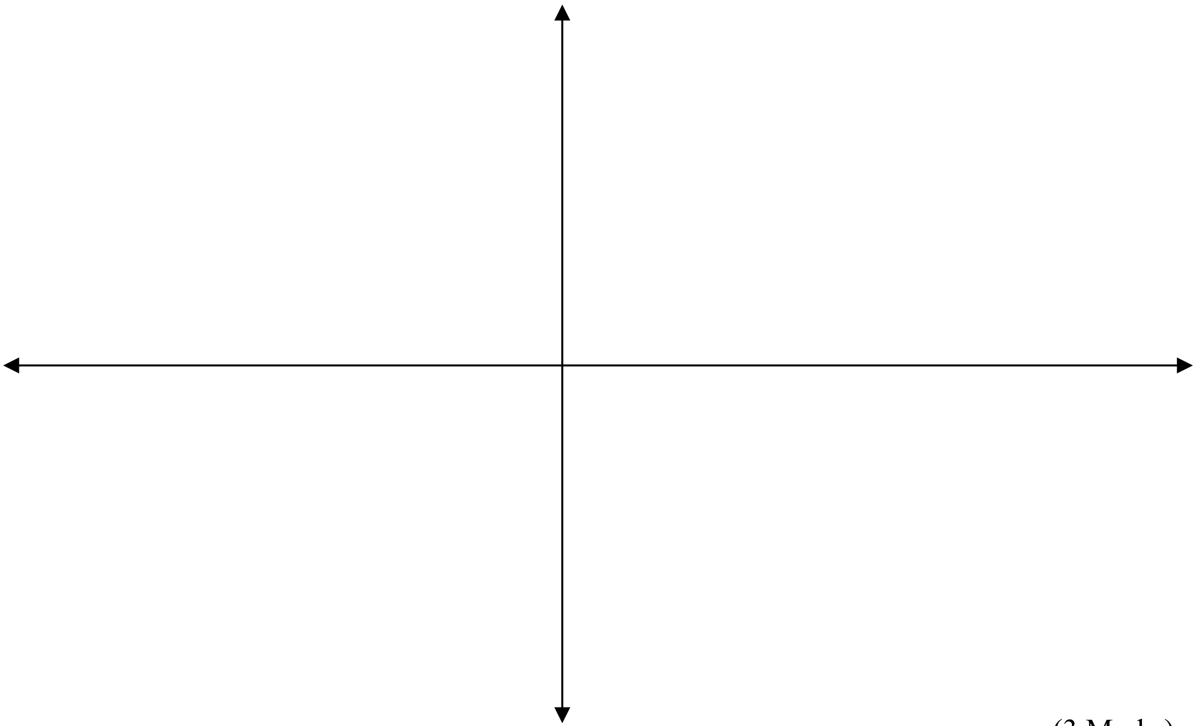
b. If the ride lasts for four minutes. How many rotations does the Ferris Wheel complete during this time?

(2 Marks)

c. Find the maximum distance the Ferris wheel is above the ground

(1 Mark)

d. Sketch the graph of d against t



(3 Marks)

e. In the first rotation, find the intervals of time when the Ferris wheel is at most 10 metres above the ground, to two decimal places.

(2 Marks)

Question 2

The population of butterflies in the park, t weeks after a virus is introduced, is modelled by $P(t) = 1000e^{-0.4t}$ where P is the number of butterflies.

a. How many weeks does it take for the population to halve, to the nearest week?

(2 Marks)

b. What is the rate of decrease of the population after 2 weeks, correct to two decimal places?

(2 Marks)

After 15 weeks the virus has become ineffective and the population of the butterflies start to increase again based on the model

$$P = P_0 + 30(t - 15)\log_e(t - 30)$$

where t is the number of weeks since the virus was first introduced

c. Find the value of

(2 Marks)

d. What is the population after one year?

(1 Mark)

e. What is the rate of change of the population after one year?

(2 Marks)

f. How many weeks does it take for the population to get back to its original number?

(2 Marks)

Question 2

A pond is to be added to the park. A bird's eye view of the pond follows the rule

$s(x) = (x - a)^3(x - b)$, where a and b are constants and $x \in [-3,1]$

a. Show that $f'(x) = (x - a)^2[4x - (3b + a)]$

(2 marks)

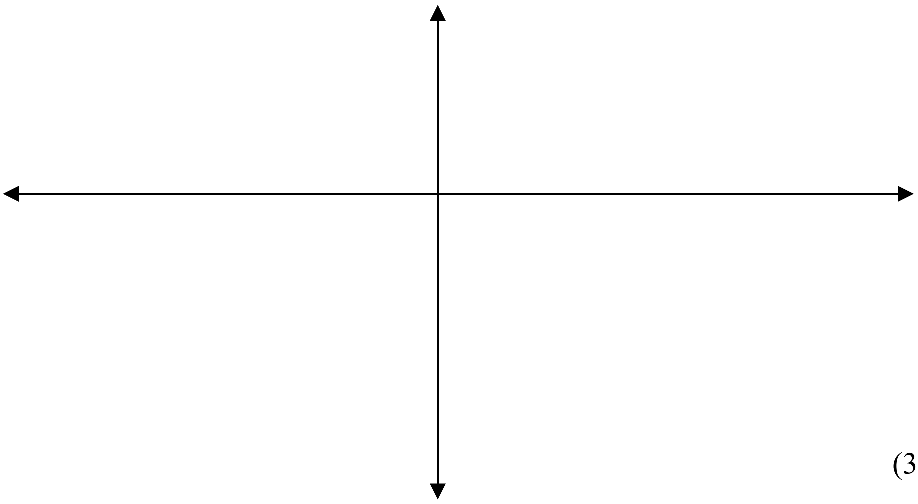
b. i. If $a = -3$ and $b = 1$, show that the coordinates of the stationary points of the graph of $s(x) = (x - a)^3(x - b)$ are (-3,0) and (0, -27).

(2 marks)

ii. Using an appropriate method determine the nature of the stationary points.

(2 marks)

c. Sketch the graph of $s(x)$ when $a = -3$ and $b = 1$. Show all intercepts and turning points in coordinate form.



(3 marks)

Question 3

A bridge is to be constructed in the park, similar to the image below, as shown by the graph below.

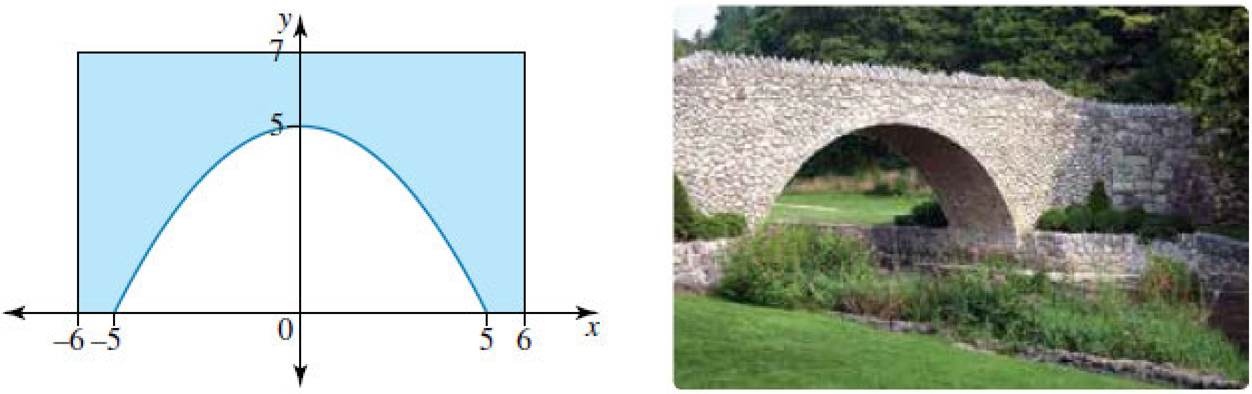


Figure 2: Arch bridge graph and image

The bridge is modelled by a quadratic function for $x \in [-5,5]$, with all measurements in metres.

a. Give the equation of the arch.

(2 marks)

b. Calculate the area of the cross section of the bridge, represented by the shaded area in Figure 2.

(3 marks)

c. The width of the bridge is 3 metres. Determine the volume of building material (ie concrete and stone) used in the construction of the bridge.

(2 marks)

Question 4

A garden bed in the park is bound by the rules $h(x) = 0.5\sin\left(\frac{x}{2}\right) + 2$ and

$l(x) = 0.5\cos\left(\frac{x}{2}\right) + 2$, where $0 \leq x \leq 4\pi$. The garden bed has edges defined by $x = 0$ and

$x = 4\pi$ All measurements are in metres.

a. Sketch the graphs of $h(x) = 0.5\sin\left(\frac{x}{2}\right) + 2$ and $l(x) = 0.5\cos\left(\frac{x}{2}\right) + 2$ on the axes below. Show all intercepts, intersection points and end points in coordinate form.

(4 marks)

b. Calculate the area of the garden bed, correct to the nearest square metre.

(4 marks)

Question 5

A fountain is installed in the park. The volume of water, V litres in the base of the fountain after t seconds, is given by $V = \frac{2}{3}t^2(15 - t)$, $0 \le t \le 10$.

a. Determine the volume of water in the base after 10 seconds.

(1 mark)

b. At what rate is the water flowing into the base at t seconds?

(2 marks)

c. Give the rate of flow after 3 seconds.

(2 marks)

d. Determine when the rate of flow is greatest and give the rate of flow at this time.

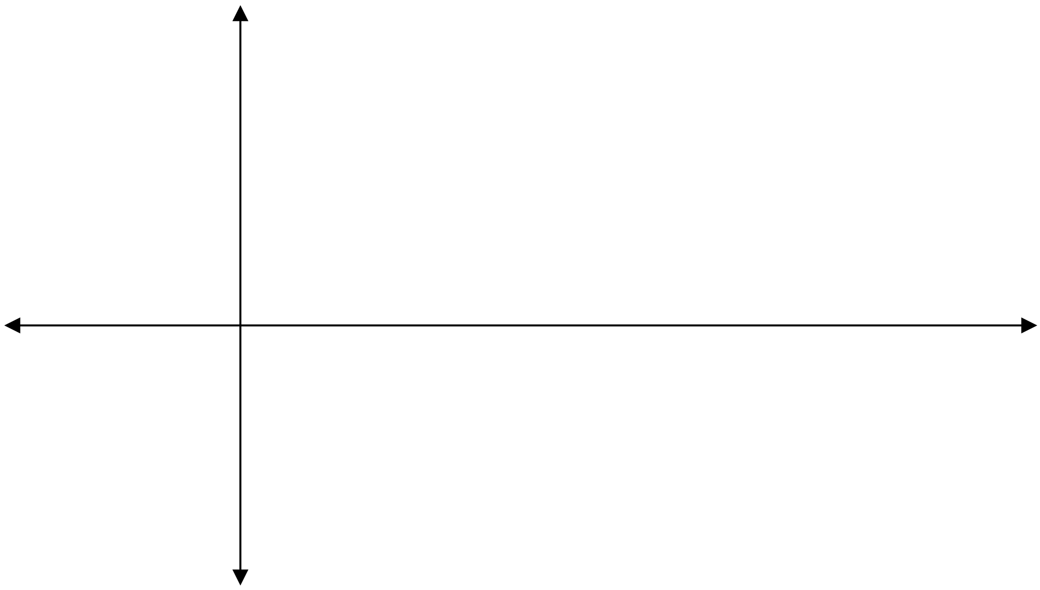
(3 marks)

Question 6

A child is playing with a ball and the velocity of the ball v metres per second, is defined by the rule

$$v(t) = e^{-0.5t} - 0.5,\ t \geq 0, \qquad \text{where } t \text{ is time in seconds.}$$

a. Sketch the graph of $v(t)$ the motion of the ball, labelling all intercepts in exact coordinate form.



(3 marks)

b. Give the acceleration of the ball, a m/s^2, in terms of t.

(2 marks)

c. Find the displacement of the ball, x metres, if $x = 0$ when $t = 0$.

(2 marks)

d. Find the displacement of the ball after 4 seconds.

(2 marks)

e. Find the distance covered by the ball in the first four seconds. Give your answer correct to four decimal places.

(2 marks)

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