Camberwell Girls Grammar School
An Anglican School - Educating Tomorrow's Woman

STUDENT NUMBER

									Letter
Figures									
Words									

Student Name			
Teacher	Ms. Lobo	Mrs. Bergamin	Mr. Levitt

MATHEMATICAL METHODS
School Assessed Task – Part 1.1
Tuesday 31st May 2017
Reading time: 10 minutes
Writing time: 1 hour

Modelling Task

Number of questions	*Number of questions to be answered*	*Number of marks*
3	3	**47**

- Students are permitted to bring into the Assessment room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are **not** permitted to bring into the Assessment room: notes of any kind, blank sheets of paper and/or white out liquid/tape.
- **Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room. Students must not disclose the contents of the task; to do so will be a breach of VCE guidelines and will be dealt with according to VCAA regulations.**

Materials supplied

- Question and answer book of 9 pages.
- Working space is provided throughout the book.

Instructions

- Write your name in the space provided above on this page.
- All responses must be written in English.
- You may use your calculator for Part B & C of the assessment. Part A must be handed in prior to this.
- Answer **all** questions in the spaces provided.
- Unless otherwise specified an **exact** answer is required to a question.
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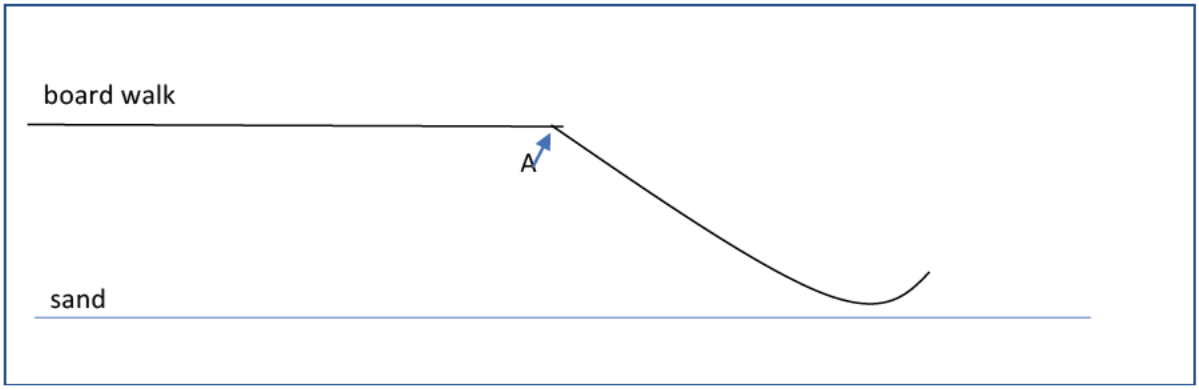
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

The Board Walk and the Beach

Part A (19 marks)

Maggie walks on the beach on a regular basis and is increasingly concerned that the waves are rising higher and higher up the sand towards the sand dunes. Maggie notices that the path through the sand dunes, made up of parallel wooden boards that lead down the beach, has collapsed under the weight of the waves. Being mathematically minded, Maggie notices that the end of the boardwalk appears to now follow the shape of a **positive parabola**, which finishes abruptly above the sand. From a side view the path looks like



Given the points $A\left(0,\frac{5}{2}\right)$, $B\left(2,\frac{1}{2}\right)$, $C(1,\frac{3}{4})$ for the parabolic section, Maggie finds a mathematical model that describes this set of data based on the general equation $f(x) = ax^2 + bx + c$ where f(x) represents the height above the sand and x represents the horizontal distance from the start of the parabolic section.

a) Show that $a = \frac{3}{4}, b = -\frac{5}{2}\ and\ c = \frac{5}{2}$ in Maggie's mathematical model.

4 marks

b) Evaluate

i) $f(0)$

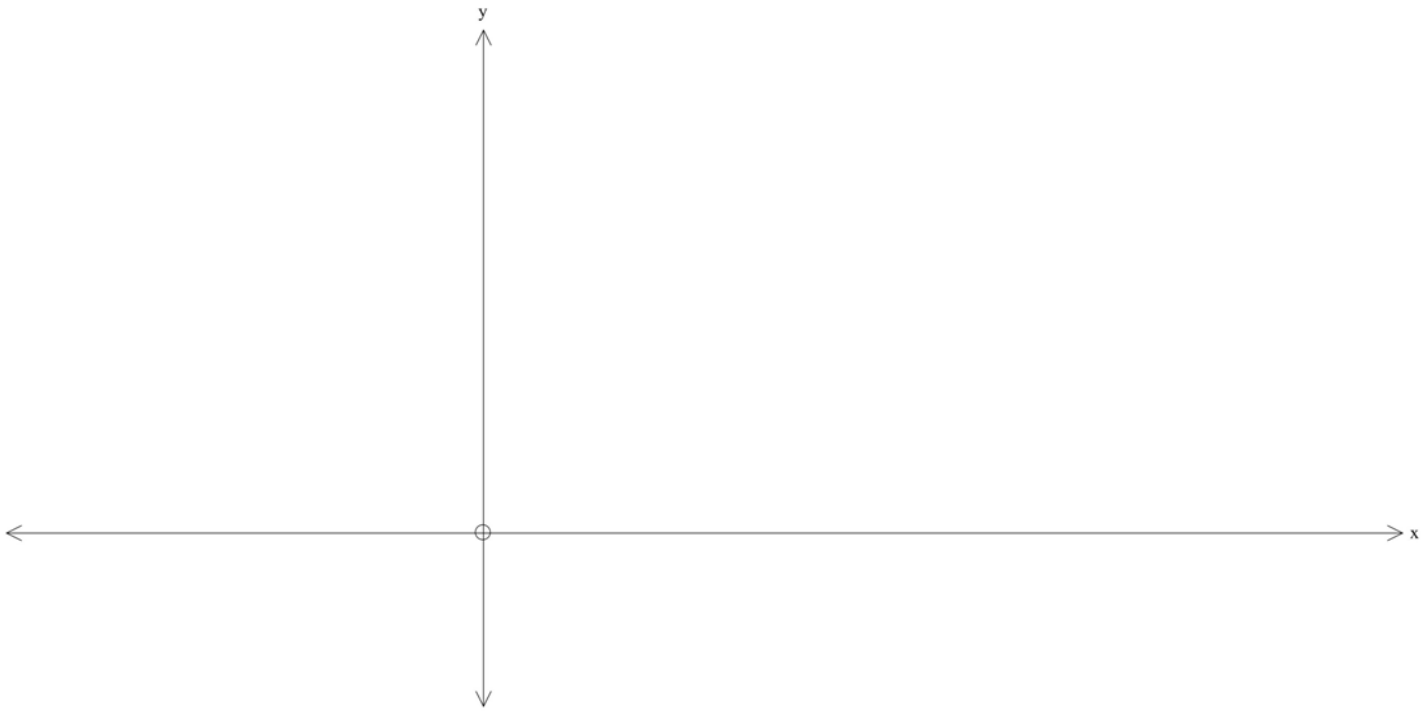
ii) $f(\frac{5}{3})$

iii) $f(3)$

3 Marks

c) Sketch the graph of g(x) for the domain $\boldsymbol{x \in [-2, 3)}$ where

$$\boldsymbol{g(x) = \begin{cases} 2.5 & -2 \leq x \leq 0 \\ \frac{3}{4}x^2 - \frac{5}{2}x + \frac{5}{2} & x > 0 \end{cases}}$$



4 marks

d) Use the process of completing the square to express the quadratic in the form of $f(x) = a(x-h)^2 + k$.

3 marks

e) Within the domain $x \in [-2,3)$, how high in metres is the boardwalk from the sand at its **lowest** point?

1 mark

f) What is the straight-line distance between the point A and the endpoint of the parabolic section?

2 marks

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Part B (12 marks) CAS ACTIVE

Maggie far preferred the shape of the boardwalk before it collapsed. She had on previous visits already formed a mathematical model that described its shape.

$$h(x) = \begin{cases} 2, & x \leq 0 \\ -\frac{1}{30}(x-2)^2 + \frac{7}{4}, & x > 0 \end{cases}$$

where $h(x)$ measures the height in metres from the sand and *x* measure the position in metres from the start of the parabolic section of boardwalk. (not necessarily the same as question 1)

a) Evaluate

i) $h(0)$

ii) $h(-6)$

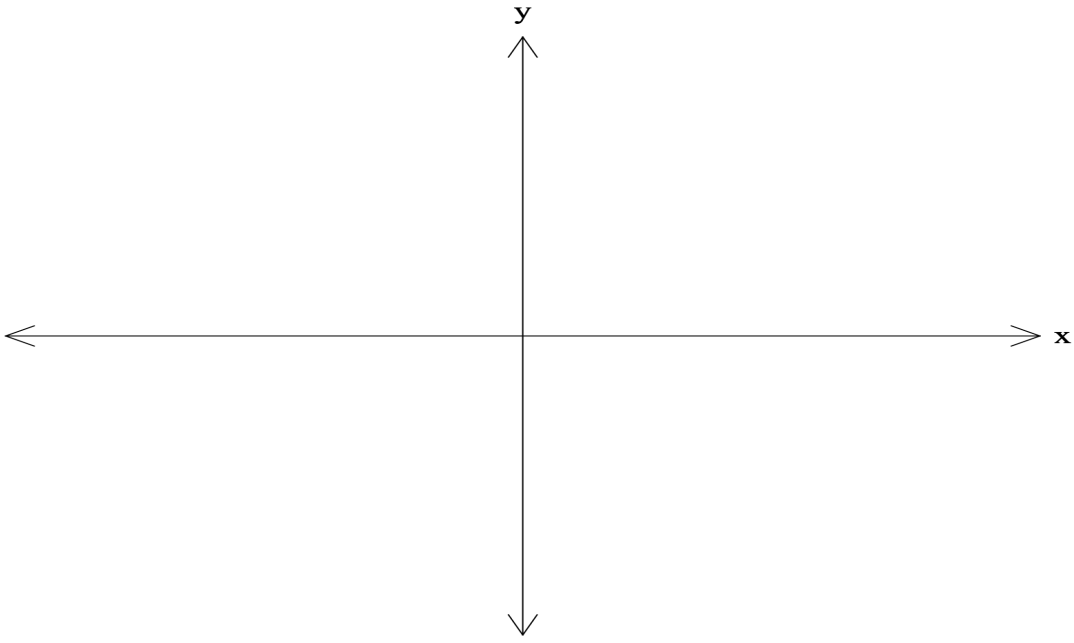
iii) $h(2)$

iv) $h(8)$

4 marks

b) Sketch the graph of the hybrid function $h(x)$ for the domain $\boldsymbol{x} \in [-\mathbf{9}, \mathbf{8}]$**.**

Clearly indicate **all** relevant features including endpoints in coordinate form to 2 decimal places.



4 marks

c) How high in metres is the parabolic section of the boardwalk from the sand at its **highest** point? Give your answer correct to 2 decimal places.

1 mark

d) Maggie jumps off the end of the boardwalk onto the sand. How high in metres is the boardwalk from the sand at its endpoint if the boardwalk finishes at the point $x = 8$? Give your answer correct to 2 decimal places.

1 mark

e) Find the new hybrid equation p(x) after h(x) undergoes the following transformations:

- a dilation by a factor of 4 from the x-axis
- a translation of 2 units right and 1 unit down

3 marks

Part C (19 marks) CAS ALLOWED

Maggie is so upset by the damage to the boardwalk that she pushes down the end to try and smooth out the kink at the end of the boards (breaking the boards). She then notices that she has created two curves, a square root function and a hyperbola. **The hyperbola joins the square root function at the same point at $x = 0$.** (Meaning the graph is continuous at $x = 0$**).**

Maggie also notices that the hyperbola section also goes through the points $\left(\frac{9}{2}, 0\right)$ and $(3, \frac{1}{4})$.

a) For the hybrid function for the boardwalk $g(x) = \begin{cases} \sqrt{x+9}, & x \leq 0 \\ \frac{a}{x+b} + c, & x > 0 \end{cases}$

Show that $a = \frac{11}{3}, b = 1, c = -\frac{2}{3}$.

4 marks

b) Explain your model by calculating where the hyperbola section of the boardwalk reaches the sand.

2 marks

c) Evaluate

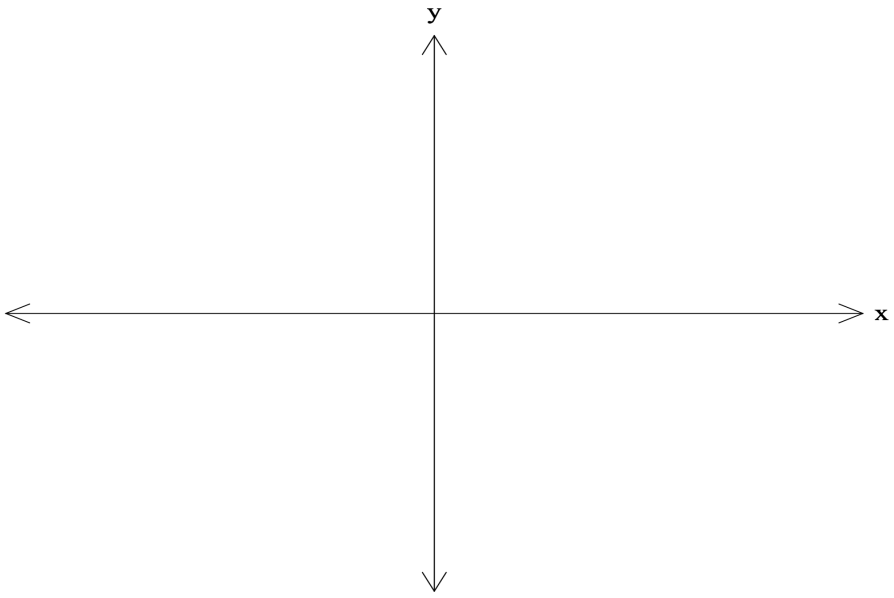
i) $g(0)$

ii) $g(-9)$

iii) $g(4.5)$

3 marks

iv) Sketch the graph of the hybrid function $g(x)$ for the domain $\boldsymbol{x} \in [-\mathbf{9}, \mathbf{4.5}]$



4 marks

v) How high in metres is the boardwalk from the sand at its **highes**t point?

_____

1 mark

vi) Maggie jumps off the end of the boardwalk onto the sand. How high in metres is the boardwalk from the sand at its endpoint if the boardwalk finishes at the point $x = 3.5$? Give your answer to two decimal places.

_____

1 mark

i) Find the inverse of g(x) **in full function notation** over the domain x>0 **only**.

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_____

3 marks

END OF PAPER