Camberwell Girls Grammar School
An Anglican School - Educating Tomorrow's Woman

STUDENT NUMBER									Letter
Figures									
Words									

Student Name			
Teacher	Ms. Lobo	Mrs. Bergamin	Mr. Levitt

MATHEMATICAL METHODS
School Assessed Task – Part 1.2
Wednesday 3rd May 2017
Reading time: 10 minutes
Writing time: 60 minutes

Modelling Task

Number of questions	*Number of questions to be answered*	*Number of marks*
4	4	**44**

- Students are permitted to bring into the Assessment room: pens, pencils, highlighters, erasers, sharpeners, rulers, one CAS calculator
.
- Students are **not** permitted to bring into the Assessment room: blank sheets of paper and/or white out liquid/tape.

- **Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room. Students must not disclose the contents of the task; to do so will be a breach of VCE guidelines and will be dealt with according to VCAA regulations.**

Materials supplied
- Question and answer book of 9 pages.
- Working space is provided throughout the book.

Instructions
- Write your name in the space provided above on this page.
- All responses must be written in English.
- Answer **all** questions in the spaces provided.
- Unless otherwise specified an **exact** answer is required to a question.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (18 marks)

Linh's family is caught in traffic on their way to the Port Fairy Music Festival and she is worried that they will be late. Linh knows that the crowd at the music festival grows exponentially once the bands start playing. The pattern of crowd growth appears to follow a model:

$C(t) = -750\left(\frac{2}{3}\right)^{\frac{t}{4}} + 1000;$

where C is the size of the crowd (rounded **down** to the nearest whole number) and t describes the time (**in minutes**) from the moment the bands start playing.
(Treat the graph of $C(t)$ as a continuous function)

a) Evaluate $C(0)$

1 mark

b) Evaluate, rounded **down** to the nearest whole number, $C(10)$

1 mark

c) The maximum audience that the music festival venue can fit is 1000 people. Based on the equation given above will the audience for the festival ever fill this area? Why/why not?

2 marks

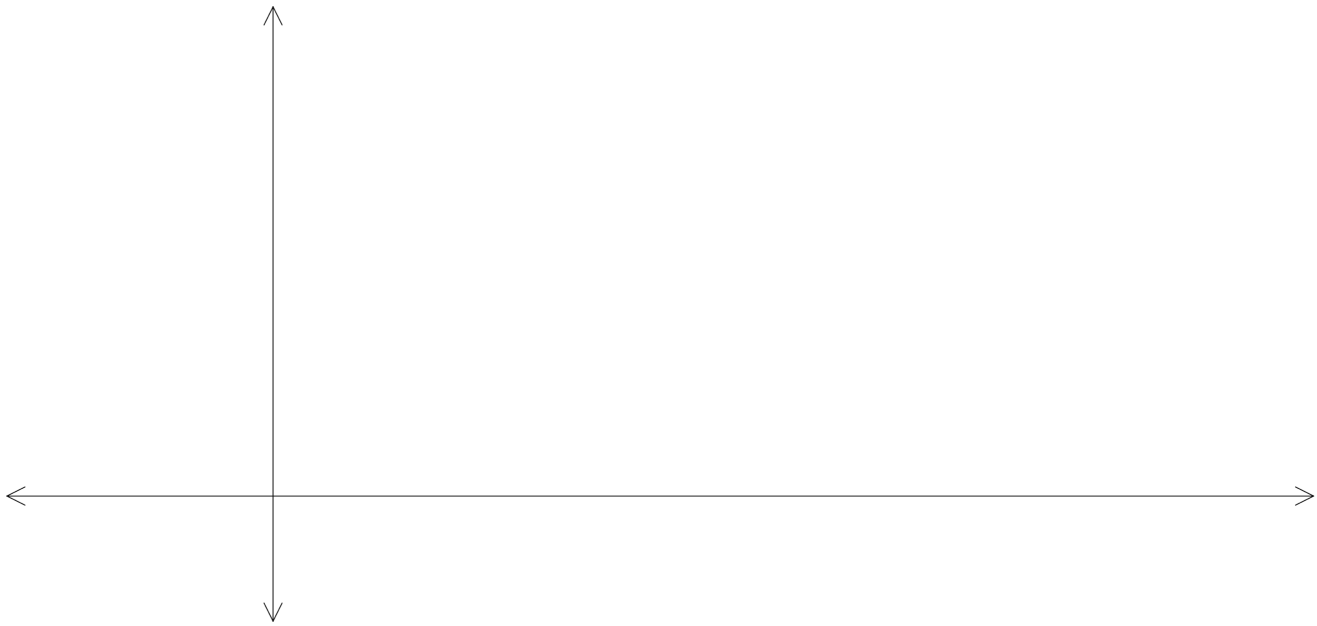
d) State the transformations, in a correct order, that are carried out to change the function

$$C_1(t) = \left(\frac{2}{3}\right)^t \quad \text{to} \quad C(t) = -750\left(\frac{2}{3}\right)^{\frac{t}{4}} + 1000$$

1.	
2.	
3.	
4.	

4 marks

e) Sketch a graph of the continuous function $C(t) = -750\left(\frac{2}{3}\right)^{\frac{t}{4}} + 1000$, for the **first hour** of the bands playing. Label the coordinates of the endpoints with values correct to the nearest integer.

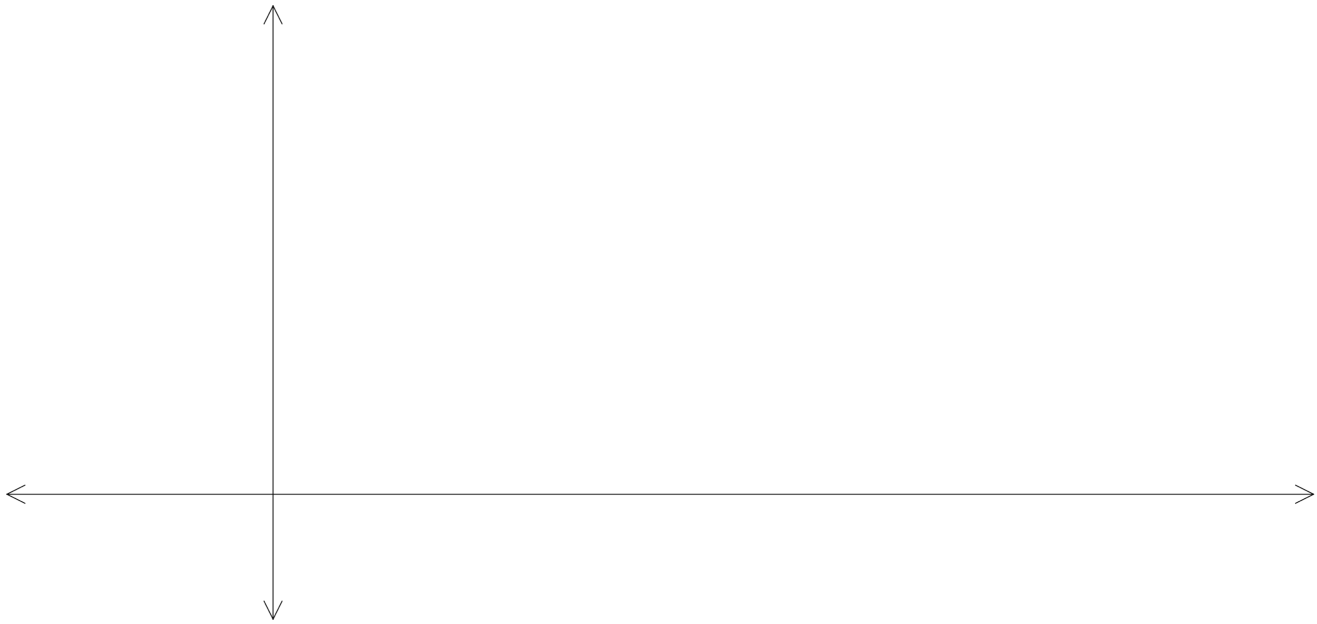


3 marks

f) Find the inverse $C^{-1}(t)$ of $C(t) = -750\left(\frac{2}{3}\right)^{\frac{t}{4}} + 1000$ where $t \in [0,60]$ giving your answer in full function notation, with the domain to **the nearest integer**.

3 marks

g) Sketch the graph of $C^{-1}(t)$, labelling all key features including coordinates of endpoints to the **nearest whole number**.



3 marks

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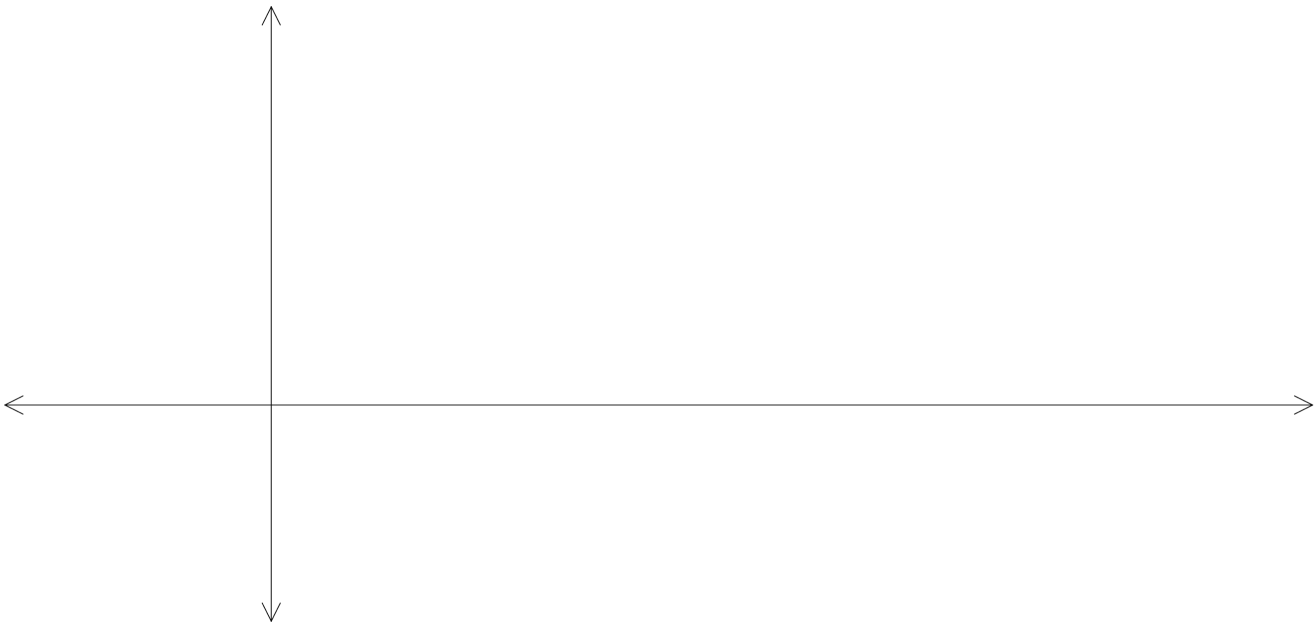
Question 2 (14 marks)

In a break in the music program, Linh's family decide to walk on the beach. They notice that the tide has encroached on the beach much further than they had remembered on previous occasions. Linh visits the beach on several occasions over the weekend and measures the distance from the sand dunes to the farthest reach of the waves on the beach. She finds that this is modelled by the function

$$D(t) = -5\cos\left(\frac{\pi(t-1)}{6}\right) + 5$$

where $D(t)$ is the distance, in metres from the water to the edge of the sand dunes and t is the time of day (in hours) after **7am.**

a) Sketch the graph of $D(t) = -5\cos\left(\frac{\pi(t-1)}{6}\right) + 5$ for $0 \leq t \leq 24$ in the space provided.
Clearly label the t-intercepts and the exact endpoints.



5 marks

b) State how close the water is to the sand dunes at 8am

1 mark

c) When is/are the time(s) of day when the water is at the furthest point from the sand dunes?

2 marks

Linh leaves her jacket on the beach while she returns to town for an ice-cream at 2pm. She leaves her jacket at a distance of 1 metre from the sand dunes.

d) Will the water reach Linh's jacket in a 24 hour cycle? If so, when **and** for how long? Give your answer correct to 2 decimal places.

3 marks

e) Apply the following transformations, **in the order as stated below**, to the function $D(t) = -5\cos\left(\frac{\pi}{6}(t-1)\right) + 5$ and hence state the function $D_T(t)$ that is formed.

1.	Translate the function $D(t)$ in a vertical direction, 5 units downwards.
THEN	
2.	Reflect over the *t*-axis
THEN	
3.	Translate the function parallel to the *t*-axis, 3 units to the right.

The transformed function, $D_T(t)$, is:

3 marks

Question 3 (10 marks)

Linh's family have had a good day, having enjoyed the music and the general atmosphere of the festival. Out of interest, Linh measured the sound level (measured in decibels) at various points around Port Fairy throughout the day. The music appeared particularly loud at her favourite coffee shop: *The Dulcet Tone*.

Linh finds that the mathematical model of the decibel level at different points of the town can be expressed as $y(x) = a\log_e(x+2)+c$, where y is the sound level in decibels, (dB), and x is the distance (in metres) from *The Dulcet Tone*; $a, c \in R$.

Linh takes a reading of 108 decibels (dB) at *The Dulcet Tone* ($x = 0$).
Linh takes another reading of 44 decibels (dB) at 30 metres from *The Dulcet Tone*.

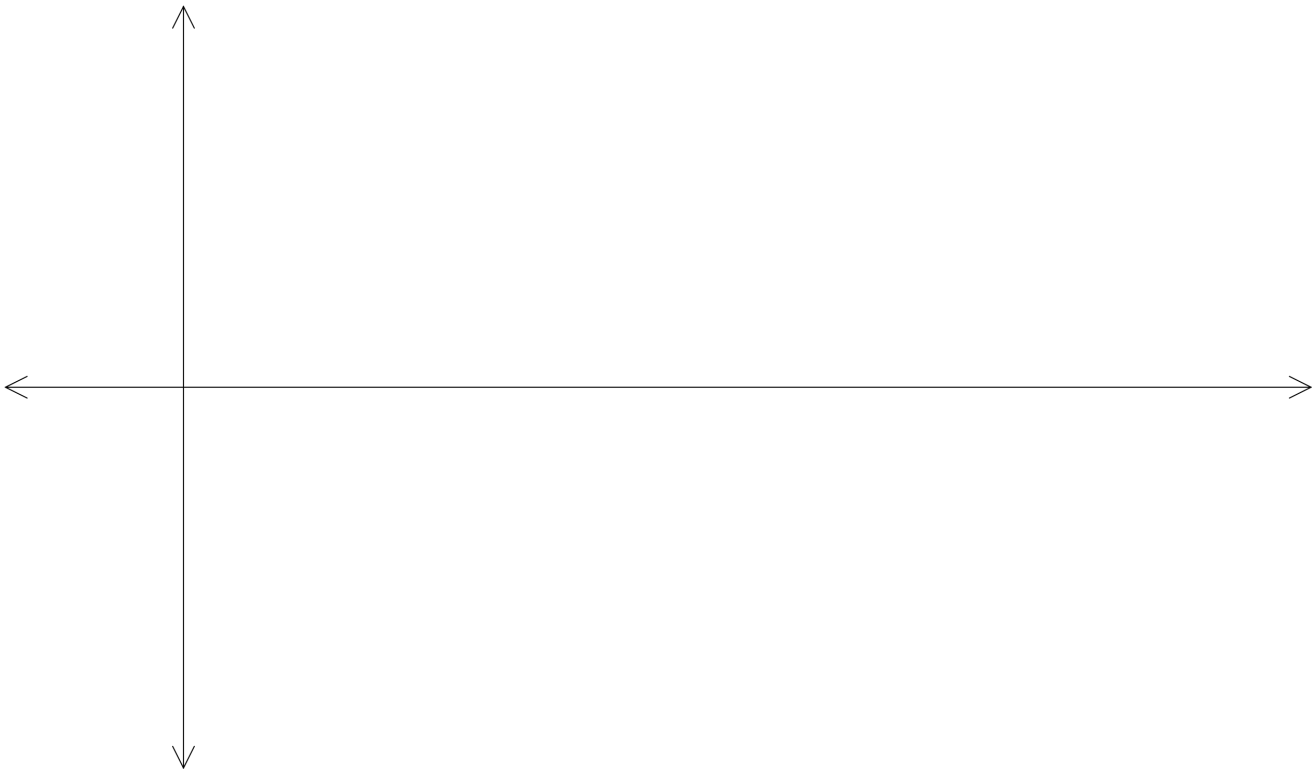
a) Show that the value of a and c correct are $\frac{-16}{\ln(2)}$ and 124 respectively.

4 marks

b) If the sound level reaches 95 dB this is considered to be dangerous.
At what distance, as measured from *The Dulcet Tone,* is this point? (correct to **2 decimal places**),
(Use your **values for** a, c found in part b)).

2 marks

c) Sketch a graph of $y(x) = a\log_e(x+2)+c$ where $x \in [0,\infty)$, using your values of a, c. Label the coordinates of the endpoint and the intercepts correct to **2 decimal places**.



3 marks

d) Mark on this graph in d) the coordinates of the point (correct to **2 decimal places**) where the sound level becomes dangerous.

1 mark

Question 4 (3 marks)

Sound is produced by variations in air pressure. The Noise level of sound can be determined from the air pressure using the following relationship,

$$N = 10 \log_{10} \left(\frac{p}{2 \times 10^{-6}} \right)^2$$

Where N is the Noise level in decibels (dB)
And p is the air pressure in Pascals (Pa)

When 2 sounds are combined (added together), the total noise can be calculated with the aid of the following rule,

$$(p_{total})^2 = (p_1)^2 + (p_2)^2$$

Where p_1 and p_2 are the 2 individual air pressures and p_{total} is the resulting total air pressure

Linh's family and their neighbours parked their cars next to each other and both had their car stereos turned up to the maximum noise level (100 dB each).

Determine the noise level of the combination of both car stereos. Give your answer to **2 decimal places**.

(3 marks)

End of Paper