Camberwell Girls Grammar School
An Anglican School - Educating Tomorrow's Woman

STUDENT NUMBER									Letter
Figures									
Words									

Student Name			
Teacher	Ms. Lobo	Mrs. Bergamin	Mr. Levitt

MATHEMATICAL METHODS
School Assessed Task – Part 1.3A

Thursday 25 th May 2017
Reading time: 10 minutes (part A and B)
Writing time: 30 minutes part A

Modelling Task

Number of questions	*Number of questions to be answered*	*Number of marks*
5	5	**21**

- Students are permitted to bring into the Assessment room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are **not** permitted to bring into the Assessment room: notes of any kind, blank sheets of paper and/or white out liquid/tape.
- **Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room. Students must not disclose the contents of the task; to do so will be a breach of VCE guidelines and will be dealt with according to VCAA regulations.**

Materials supplied

- Question and answer book of 9 pages.
- Working space is provided throughout the book.
- VCAA formula sheet

Instructions

- Write your name in the space provided above on this page.
- All responses must be written in English.
- You may use your calculator for Part B of the assessment. Part A must be handed in prior to this.
- Answer **all** questions in the spaces provided.
- Unless otherwise specified an **exact** answer is required to a question.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Part A - Short Answer

Question 1

The graphs of $y = \cos(2x)$ and $y = a\sin(2x)$, where a is a real constant, have a point of intersection at $x = \frac{\pi}{3}$

a) Find the value of **a**

2 marks

b) If $x \in [0, \pi]$, find the x coordinate of the other point of intersection of the 2 graphs

2 marks

Question 2

a) Apply the following transformations to the curve $y = \frac{2}{x^2} + 2$, in the order shown and hence give the rule of the image.

i) Translate 2 units in the negative direction of the y-axis

ii) Dilate by a factor of ½ from the x-axis

iii) Reflect in the line y=x

iv) Translate 1 unit in the negative direction of the x axis

4 marks

b) State the largest possible domain of $f(x) = \frac{2}{x^2} + 2$ such that the inverse function exists.

1 mark

Question 3

A person steps off a building with a speed of $v = ae^{-\frac{3}{4}t} + b\ m/s$, where t is the time in seconds after stepping off from the top. After 10 seconds, her speed is 72 km/hr. In order to survive the fall she must land with a speed of less than 40 m/s.

a) Find the exact values of a and b.

3 marks

b) Calculate the time by which the glider must land in order to survive.

2 marks

Question 4

Use the quotient rule to show that if $f(x) = \tan(2x)$ then $f'(x) = 2\sec^2(2x)$

4 marks

Question 5

a) Differentiate $y = \sqrt{16 - 4x}$ with respect to x.

1 mark

b) If $f(x) = \frac{x}{\sin(x)}$, find $f'\left(\frac{\pi}{2}\right)$.

2 marks

End of Part A

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Student Name			
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MATHEMATICAL METHODS
School Assessed Task – Part 1.3B
Thursday 25 th May 2017
Reading time: with part B
Writing time: 60 minutes

Modelling Task

Number of questions	*Number of questions to be answered*	*Number of marks*
3	3	**41**

- Students are permitted to bring into the Assessment room: pens, pencils, highlighters, erasers, sharpeners, rulers.
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- **Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room. Students must not disclose the contents of the task; to do so will be a breach of VCE guidelines and will be dealt with according to VCAA regulations.**

Materials supplied

- Question and answer book of 9 pages.
- Working space is provided throughout the book.
- Formula sheet

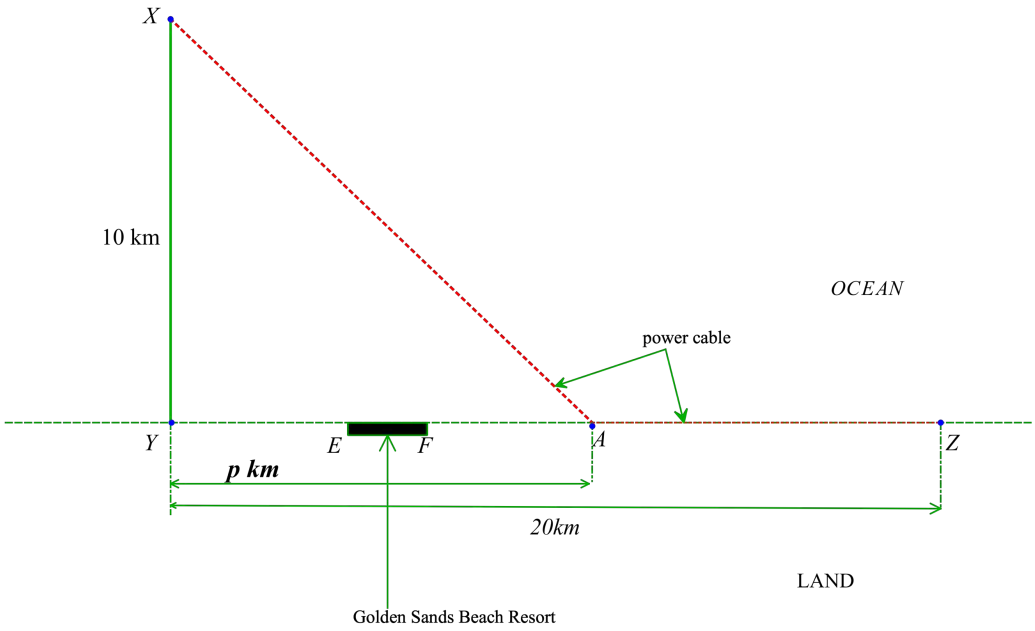
Instructions

- Write your name in the space provided above on this page.
- All responses must be written in English.
- You may use your calculator for Part B of the assessment. Part A must be handed in prior to this.
- Answer **all** questions in the spaces provided.
- Unless otherwise specified an **exact** answer is required to a question.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Part B (41 marks) CAS ACTIVE

Question 1 (12 marks)

An island is located at X, 10 km from the nearest point Y on a straight beach. Electric power is to be provided by laying cable between X and a power generation plant located at Z, 20 km along the beach from Y.

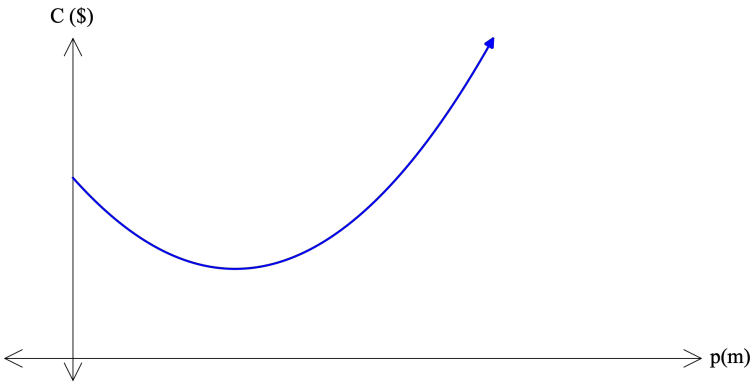


The cable contractor decides that the cable will go along the sea bottom from X to A, a point on the beach p kilometres from Y $(p \geq 0)$. It will then run along the beach to Z. The cost of laying the cable is \$100,000 per kilometre along the beach and $\boldsymbol{w}\times$ \$100,000 per kilometre along the sea bottom, where w>1.

a) Let the total cost of laying the cable be $C\times\$100{,}000$. Show that $C = 20 - p + w\sqrt{100 + p^2}$

2 marks

The general shape of the graph of the function representing the cost of laying the cable is shown here. It shows that the function has a single local minimum.



b) Use Calculus to show that the minimum cost of laying the cable occurs when $p^2 = \frac{100}{w^2-1}$.
(You don't need to demonstrate that th s s spec f ca y m n mum)

4 marks

The Golden Sands Beach Resort is an expensive holiday destination located on the beach front between E and F as shown in the diagram. E and F are 9 and 10 kilometres from Y respectively.

c) If $w > \sqrt{2}$, show that if the cost of laying is to be a minimum the cable will pass along the beach in front of a part or all of the resort.

3 marks

d) If $w = \sqrt{5}$, find the position of A for which the total cost of laying the cable is a minimum **and** the value of this total cost.

3 marks

Question 2 (12 marks)

Consider the function $f(x) = \frac{16-x^4}{x^2-4}$

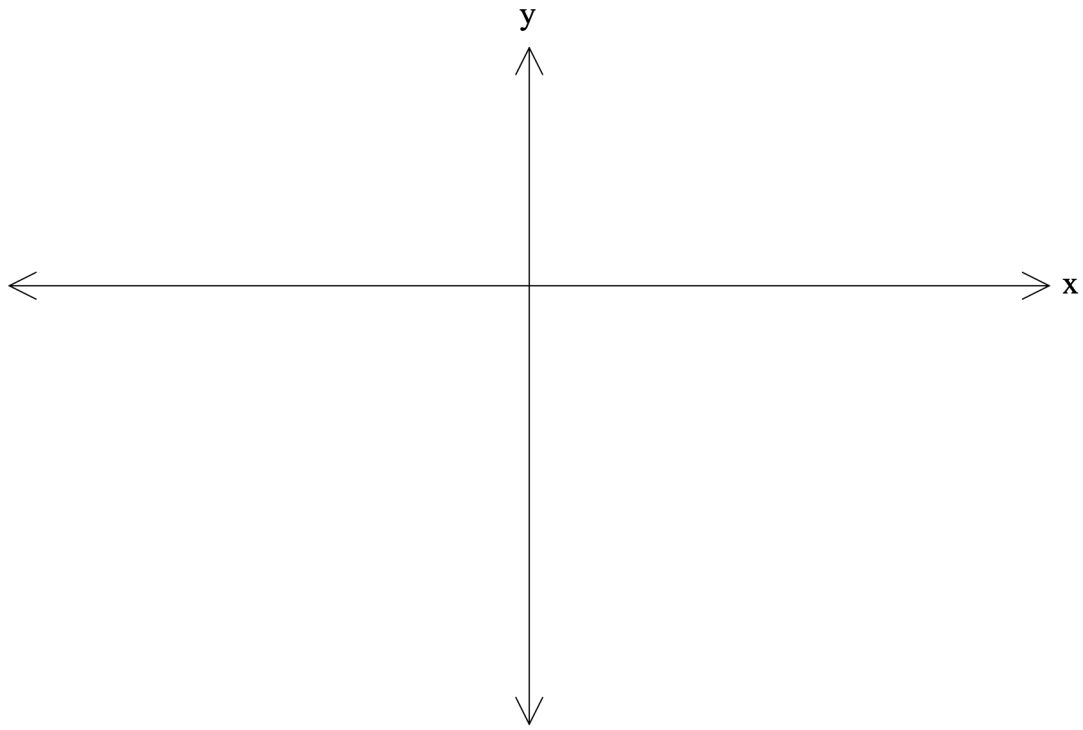
a) What is the domain of f?

1 mark

b) Find $\lim_{x \to 2} f(x)$

3 marks

c) Sketch the graph of f(x) clearly labelling any key features in coordinate form.



4 marks

d) On the same axis sketch the gradient function of f(x) labelling any key features.

2 marks

Question 3(17 marks)

A bungee jumper dives from a tower towards a river. Her height from the river (m) at any time t (s) is given by the function $s(t) = 100\cos\left(\frac{3}{4}t\right).e^{-\frac{t}{5}} + 100, t \in [0,10]$

a) Write an expression for the average velocity of the bungee jumper on the interval [t,t+h]

2 marks

b) How far has the bungee jumper travelled between t=0 and t=10? Give your answer to 2 decimal places.

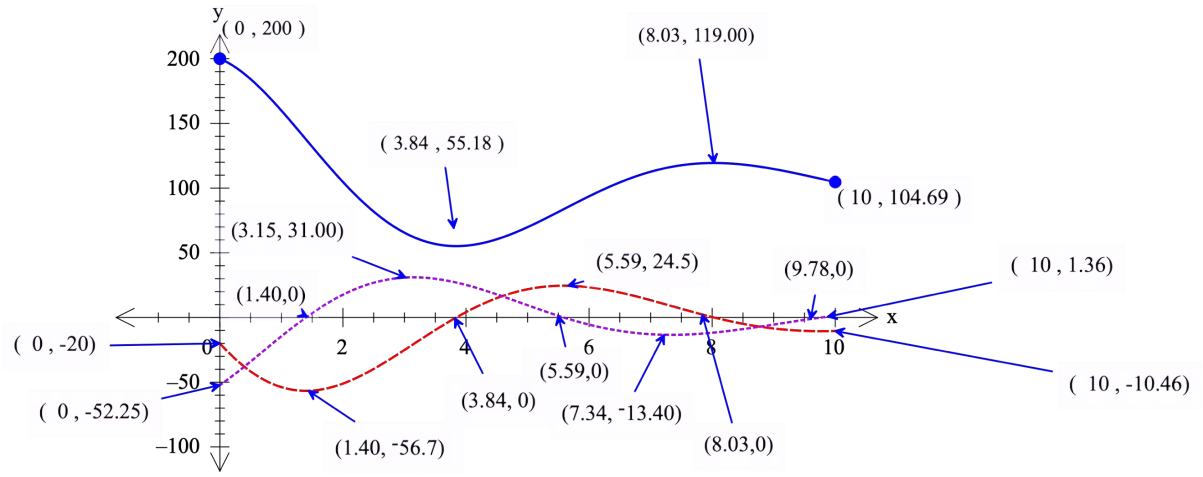
2 marks

c) Find an equation for the instantaneous velocity $v(t)$ of the bungee jumper at time t

1 mark

d) The graph of $s(t)$ and $v(t)$ and $a(t)$ (instantaneous acceleration) are shown below.

i) **Label** each graph with their title.



① Blue, solid line
② Red, small dash line
③ Purple, large dash line

1 mark

Find, correct to 2 decimal places:

ii) the time that the jumper is closest to the river and how far away from the river this is.

_____

_____

2 marks

iii) The maximum rate of change of displacement with respect to time that the jumper experiences

_____

_____

2 Marks

iv) When is the velocity negative? What does negative velocity indicate?

_____

_____

2 marks

v) At what time is the velocity first equal to zero and what is the displacement at this time? Explain the significance of this.

3 marks

vi) When is the acceleration first equal to zero and what is the velocity at this time?

2 marks

END OF PAPER