Camberwell Girls Grammar School
An Anglican School - Educating Tomorrow's Woman

STUDENT NUMBER

									Letter
Figures									
Words									

Student Name			
Teacher	Ms. Lobo	Mrs. Bergamin	Mr. Levitt

MATHEMATICAL METHODS
School Assessed Coursework - Calculus Application Task
Thursday 22nd June 2017
Reading time: 10 minutes
Writing time: Part A: 25 mins
Part B: 65 mins

PART B: Technology Enabled

Number of questions	*Number of questions to be answered*	*Number of marks*
4	4	**32**

- Students are permitted to bring into the Assessment room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are **not** permitted to bring into the Assessment room: notes of any kind, blank sheets of paper and/or white out liquid/tape.
- **Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room. Students must not disclose the contents of the task; to do so will be a breach of VCE guidelines and will be dealt with according to VCAA regulations.**

Materials supplied

- Question and answer book of 9 pages.
- Working space is provided throughout the book.

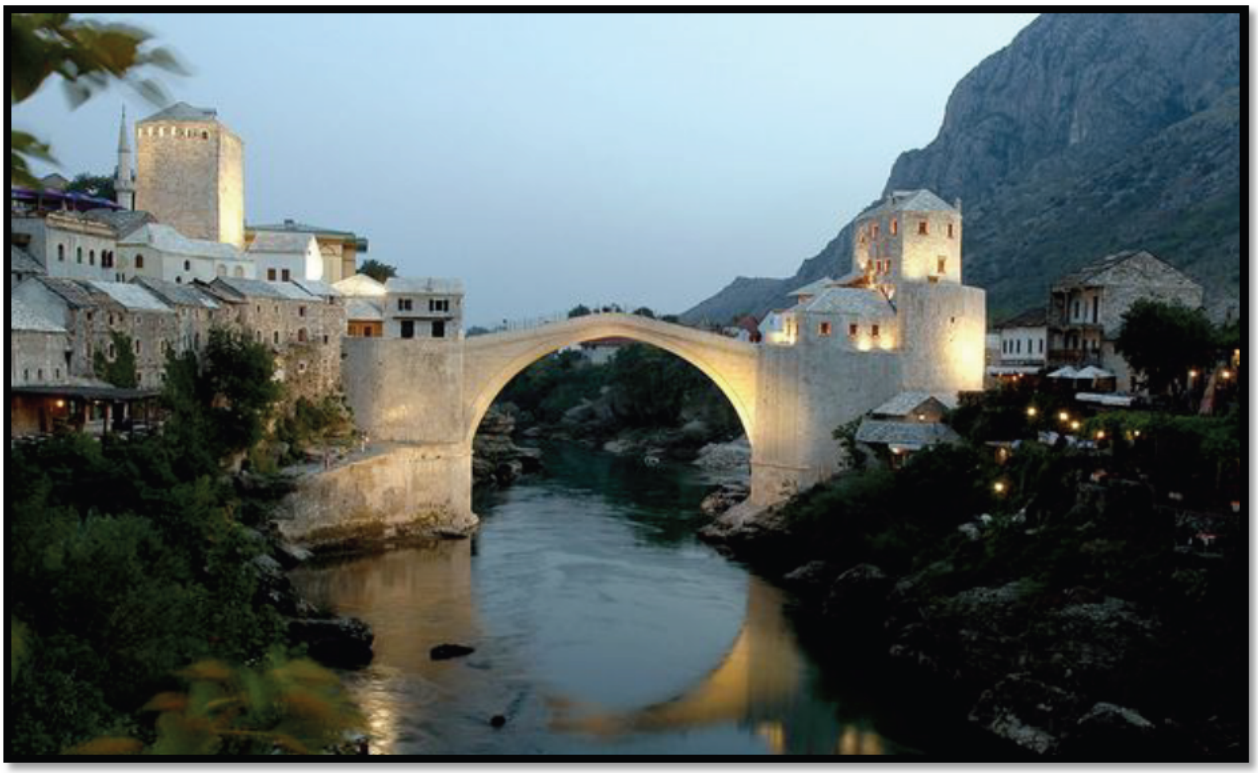
Instructions

- Write your name in the space provided above on this page.
- All responses must be written in English.
- You may use your calculator for Part B & C of the assessment. Part A must be handed in prior to this.
- Answer **all** questions in the spaces provided.
- Unless otherwise specified an **exact** answer is required to a question.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

The Stari Most

The Stari Most (this literally means old bridge) pictured below is in the town of Mostar, Bosnia Herzegovina.

This stone bridge was built in 1566 on the orders of Suleiman the Magnificent, the Sultan of the Ottoman Empire. Unfortunately in 1993 the bridge was destroyed due to war. The bridge was a UNESCO World Heritage Site and in 2004 it was rebuilt using the original techniques as a way to bring peace to the people.

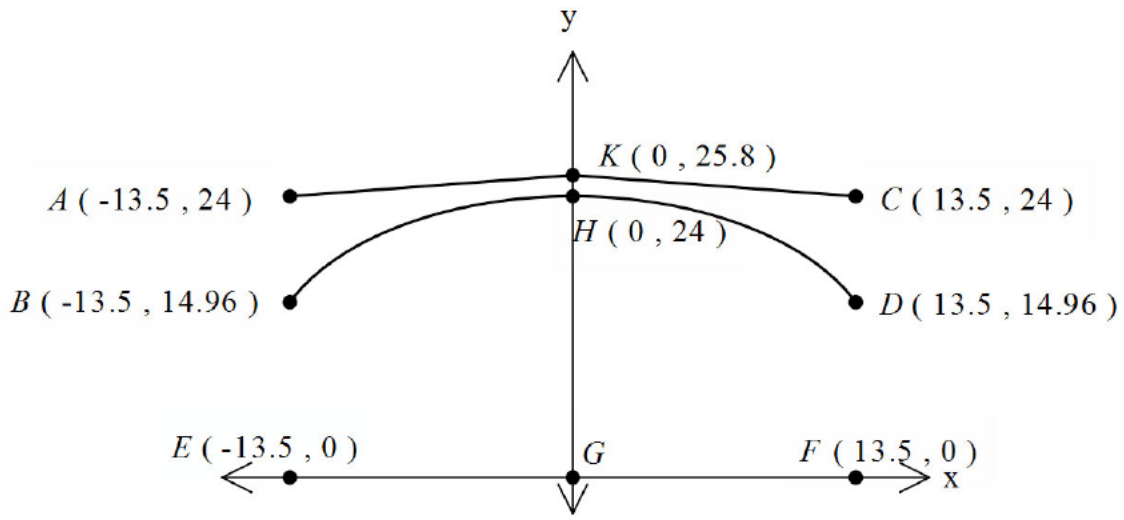


Some more detail about the bridge is given on the next page.

Required Information about the Stari Most

The end-points of the functions given on the graph represent the arch section and the top of the walkway of the bridge.

The y-axis represents an imaginary vertical line through the centre of the bridge. The *x*-axis represents the water line in the month of July. The *x* and *y* axes represent horizontal and vertical distances respectively, measured in metres.



The under surface of the arch (from B to D) can be approximated to the following curve,

$$a: \left[-13.5 \;\; 13.5 \right] \rightarrow \mathbb{R},\; a(x) = \sqrt{213.16 - x^2} + 9.4$$

The top of the walkway is a hybrid (piece –wise) function made up of two linear functions.

Part B

Question 1

a.i. Show that the height of the arch at point D is 14.96m.

1 mark

ii. What is the distance, in metres, of the walkway between A to K. Give your answer to 2 decimal places?

1 mark

b.i. By forming the two **trapeziums** EBHG and GHDF, find the **approximate** area of the cross-section below the arch of the bridge to the nearest square metre.

2 marks

ii. Using calculus finds the area of the cross-section below the arch of the bridge, in square metres, to 2 decimal places.

2 marks

The section of the bridge above the arch and below the walkway is made of local stone. This will be obtained from the same quarry that the original stone came from 429 years ago. Assume that this section of the bridge is uniformly 4 metres wide and that the front and rear stone surfaces are vertical.

The two linear sections that make up the walkway form a hybrid function *w(x)*. The two parts of this function are reflections in the *y*-axis.

Question 2

a. Find the equation that describes the intervals AK and KC respectively and hence complete the hybrid function below. Give the domain for each section.

$$w(x) = \begin{cases} \_\_\_\_\_, & \text{domain} \\ \_\_\_\_\_, & \text{domain} \end{cases}$$

2 marks

b. Find the area between *a(x)* and *w(x),* in square metres, that describes the surface of stone between the top of the walkway and the bottom surface of the arch to 2 decimal places.

2 marks

c. The arch section of the bridge is made of solid stone. Calculate the volume of stone (in m^3) required to fill this space to 2 decimal places.

1 mark

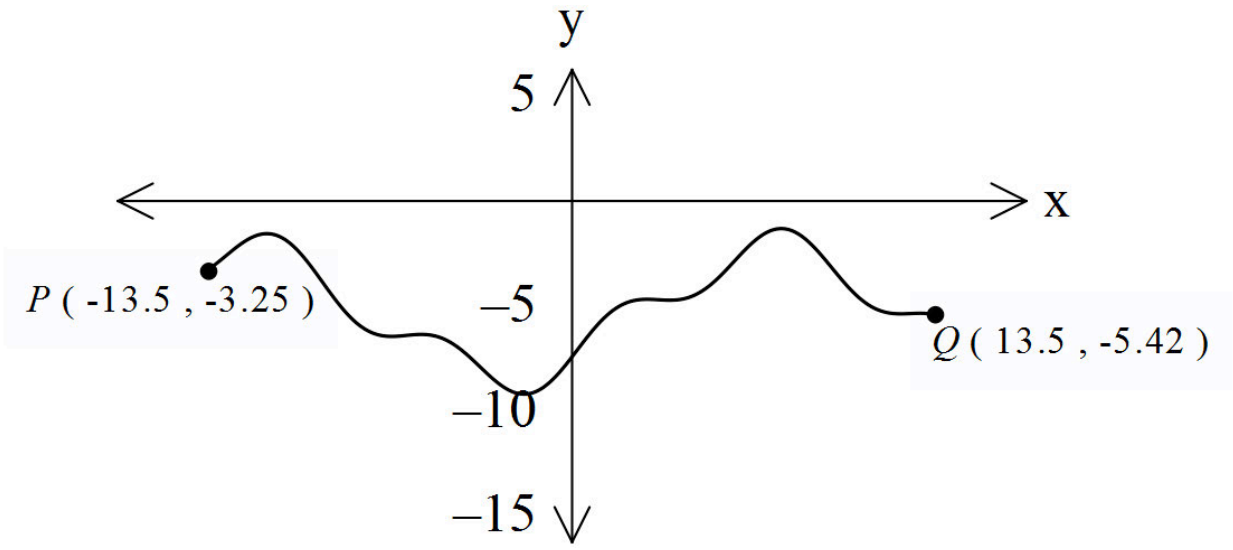
d. The bridge is made from cut stone. The stones sit perfectly next to each other. Cutting stone this precisely is time consuming and so is carefully laying them. On average the stone is cut and laid at the rate of 0.37 m^3/day.

Calculate how long it took to cut and lay the necessary stone. Give your answer to the nearest number of **years and months**. (Assume non-leap years during this period)

2 marks

Below the surface of the water, under the arch, the river bed (ground under the river) closely resembles the function shown below:

$$r(x) = \sin(x) + 3\cos\left(\frac{\pi x}{10} - \frac{3\pi}{4}\right) - 5.3 \quad x \in \left[-13.5 \;\; 13.5\right]$$



Question 3

a. By forming **six** rectangles of equal width of 4.5 units, find an approximation for the area in square metres between *r(x)* and the *x* axis from *x = -13.5* to *x = 13.5,* using **right-end point method**, correct to 2 decimal places.

3 marks

b. How could the approximation found above be more accurate while still only applying the right-end point method?

1 mark

c. Determine the area in square metres between $r(x)$ and the x axis from $x = -13.5$ to $x = 13.5$, using integration, correct to 2 decimal places.

2 marks

d. Find the average depth in metres below the surface of the riverbed (ground under the river) correct to 2 decimal places.

2 marks

Question 4.

a. A diver dived from the centre of the walkway and a person standing at point A recorded a video so the divers trajectory could be analysed. The shape of the fall was **half** of a perfect parabola. When the diver hit the water, he was 5m forward of his initial vertical position. Determine the quadratic equation, *d(z)* that describes the diver's trajectory (distance out from the bridge as seen from the side). State the domain showing all decimal values to 2 decimal places.

2 marks

b. During the dive, the diver is accelerating vertically downwards at $A(t) = 10\ m/s^2$. Use calculus to determine the diver's velocity *v(t),* in m/s, with respect to time. Assume that the positive direction is upwards and that the initial velocity was zero.

2 marks

c. Using the result from b. above, again using calculus, determine a rule *S(t)* for the diver's height above water, in metres, with respect to time.

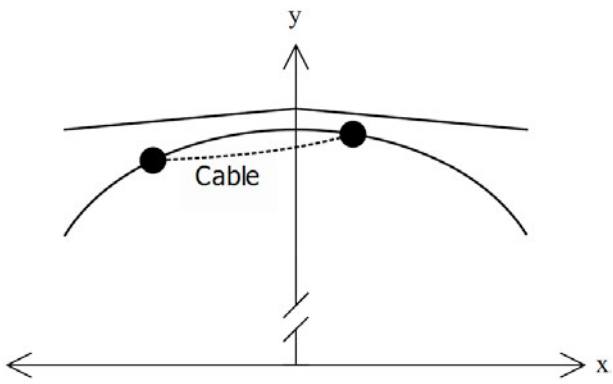
2 marks

d. How long, in seconds, did it take for the diver to hit the water correct to 2 decimal places?

1 mark

Diving spectators suspended a Slovenian flag on a cable between two points on the bridge, near the top, as shown in the diagram provided. The cable follows a shape described by the equation

$$c(x) = \frac{3}{2} e^{\frac{x}{6}} + 21$$



e. i Express the area between the cable and the arch of the bridge as a definite integral, giving the values of the end points to 3 decimal places.

3 marks

ii. Hence, calculate the area between the cable and arch of the bridge, in square metres, to 1 decimal place.

‘

1 mark

Camberwell Girls Grammar School
An Anglican School - Educating Tomorrow's Woman

STUDENT NUMBER

									Letter
Figures									
Words									

Student Name			
Teacher	Ms. Lobo	Mrs. Bergamin	Mr. Levitt

MATHEMATICAL METHODS
School Assessed Coursework - Calculus Application Task

Thursday 22nd June 2017
Reading time: 10 minutes
Writing time: Part A: 25 minutes
Part B: 65 mins

PART A: Technology Free

Number of questions	*Number of questions to be answered*	*Number of marks*
1	1	**15**

- Students are permitted to bring into the Assessment room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are **not** permitted to bring into the Assessment room: notes of any kind, blank sheets of paper and/or white out liquid/tape.
- **Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room. Students must not disclose the contents of the task; to do so will be a breach of VCE guidelines and will be dealt with according to VCAA regulations.**

Materials supplied

- Question and answer book of 2 pages.
- Working space is provided throughout the book.

Instructions

- Write your name in the space provided above on this page.
- All responses must be written in English.
- You may use your calculator for Part B & C of the assessment. Part A must be handed in prior to this.
- Answer **all** questions in the spaces provided.
- Unless otherwise specified an **exact** answer is required to a question.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Part A

1. **a.** Find an antiderivative of $\sqrt{2-3x}$

2 marks

b. Find $\int \frac{5}{1-4x}\,dx$

2 marks

c. i. Show that $\frac{1-2x}{x+1} = -2 + \frac{3}{x+1}$

2 marks

ii. Hence find $\int \frac{1-2x}{x+1}\,dx$

2 marks

d. Differentiate $\log_e(5x^2+3)$ with respect to x and hence evaluate $\int \frac{x}{(5x^2+3)}\,dx$

3 marks

e. If $f(x) = 11x\log_e(3x^2)$ find $f'(x)$ and hence find $\int \log_e(3x^2)\,dx$

4 marks