Camberwell Girls Grammar School
An Anglican School - Educating Tomorrow's Woman

STUDENT NUMBER									Letter
Figures									
Words									

Teacher	Mr. Woodlock	Mrs. Bergamin	Mr. Trufitt

MATHEMATICAL METHODS
School Assessed Task – Part 1A

Thursday 22nd March 2018
Reading time: 10 minutes
Writing time: 60 minutes

Modelling Task

Number of questions	*Number of questions to be answered*	*Number of marks*
3	3	**40**

- Students are permitted to bring into the Assessment room: pens, pencils, highlighters, erasers, sharpeners, rulers, one CAS calculator and one scientific calculator. One bound notebook

- Students are **not** permitted to bring into the Assessment room: blank sheets of paper and/or white out liquid/tape.

- **Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room. Students must not disclose the contents of the task; to do so will be a breach of VCE guidelines and will be dealt with according to VCAA regulations.**

Materials supplied
- Question and answer book of 10 pages.
- Working space is provided throughout the book.

Instructions
- Write your name in the space provided above on this page.
- All responses must be written in English.
- Answer **all** questions in the spaces provided.
- Unless otherwise specified an **exact** answer is required to a question.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Theme: Determining Curves

General Advice

Often, to solve a mathematical problem, it is necessary to find a particular line or curve which goes through a given set of points. For example, you have probably found the equation of a line which goes through two points. You will remember that ***only one*** straight line can be found that goes exactly through two given points. In this case we say that the two points ***determine*** the line.

One given point and a given slope will also ***determine*** a particular line. However, one point alone is not sufficient.

Sometimes, but not always, a line can be determined from a point on it and the area that is enclosed between the line and the positive parts of the co-ordinate axes. For example, the line $x+ y=2$ (shown in Figure 1) is the only line which passes through (1,1) and encloses an area of two square units with the positive parts of the co-ordinate axes. Therefore, these two pieces of information ***determine*** that line.

However, for the point (2,2) and an enclosed area of nine square units, both $x+2y=6$ and $2x+ y=6$ can be drawn, so the given information does *not* determine a particular line (see Figure 2).

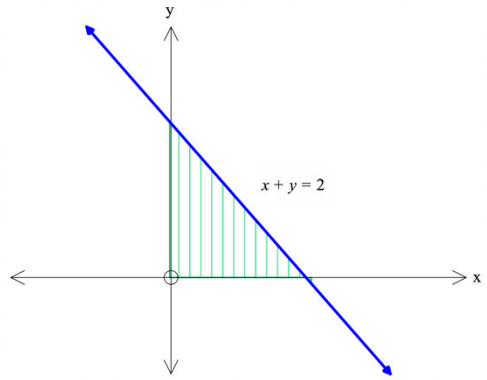


Figure 1. $x+y=2$ is the only line that encloses two square units in the first quadrant.

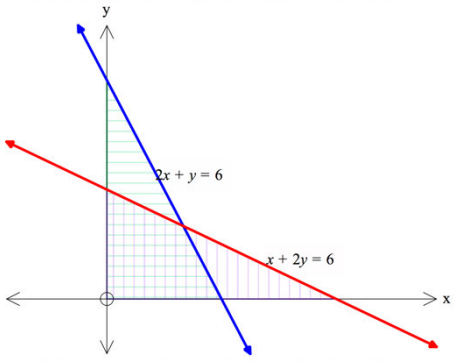


Figure 2. Both $x+2 y=6$ and $2x+ y=6$ enclose an area of nine square units in the first quadrant

In this SAC you will explore what information can be used to determine curves and what curves can be found that fit particular sets of information. For example, graphs like those shown in Figure 3, all of pass through the points (-3,0), (0,0) and (5,0)

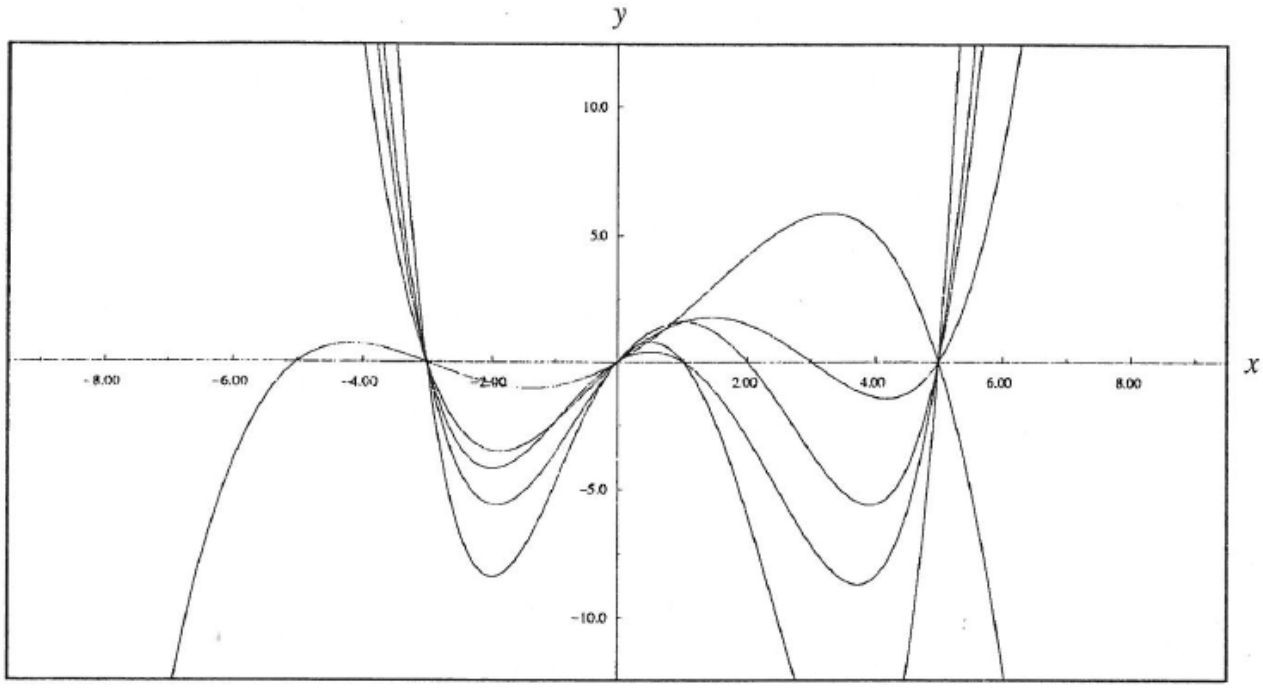


Figure 3. Polynomials of degree 4 pass through the points (-3,0), (0,0) and (5,0)

Types of information you may use to determine curves include: points, gradients, area beneath the curve, symmetry, asymptotes, maxima or turning points**. In this advice the word curve is used to include straight lines.**

Examples of types of curves are straight lines, circles, quadratics, cubic equations, and power functions.

Remember, to test whether a given set of information determines a curve of a given type, you must show two things:

i) that there is a curve of that type which fits the information (this usually means finding its equation)

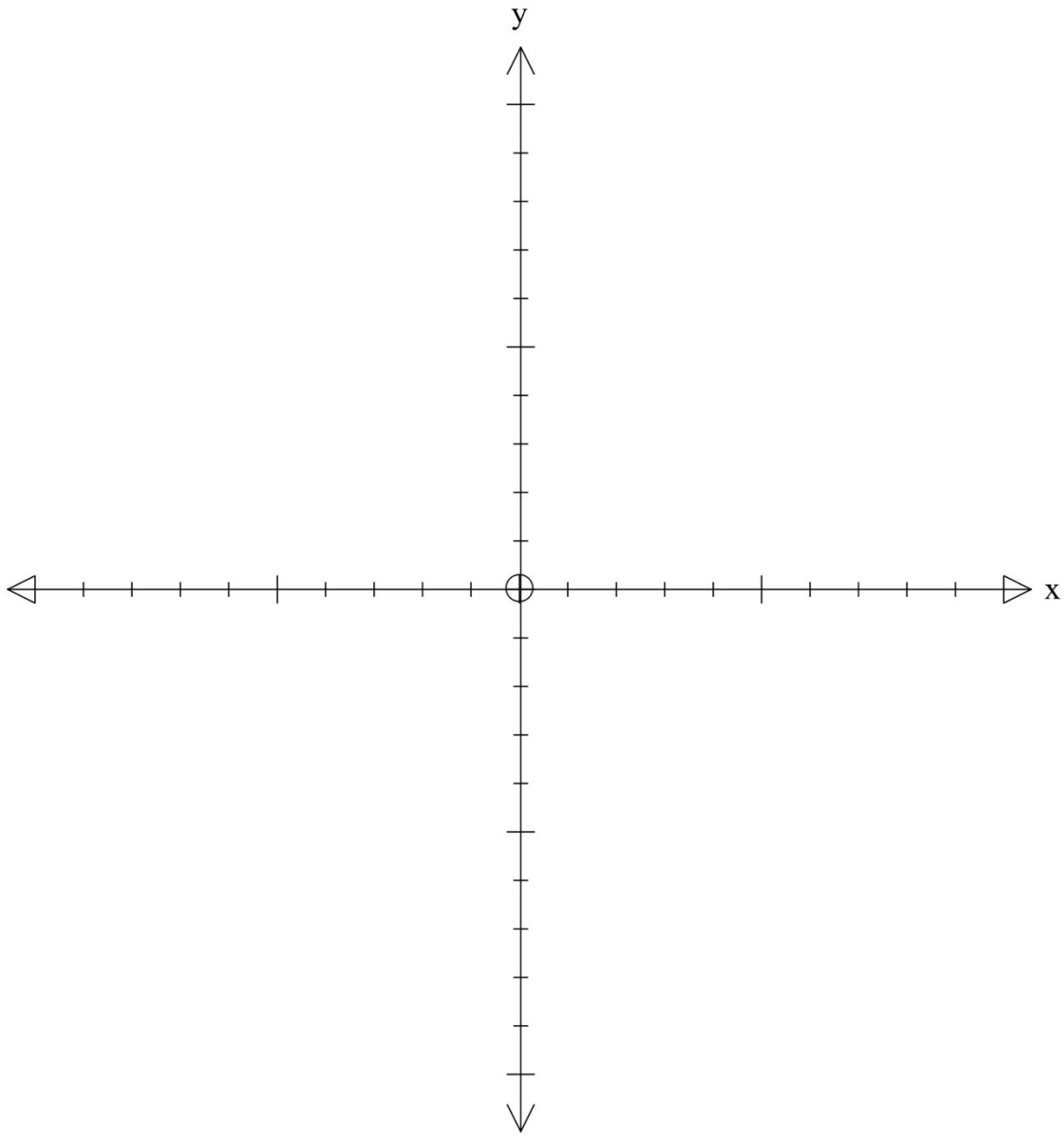
and

ii) that no other curve of that type fits the information.

SAC Part 1.A

Show all working including any assumptions made

Question 1 Fitting curves to points



For the two points (-2,2) and (0,8):

i) Plot the two points on the graph above

1 mark

ii) Find an equation in the form $y = mx + c$ that passes through these points
Sketch this graph on the same graph above clearly labelling any key features

3 marks

iii) Find an equation in the form $y = a(x - h)^2 + k$ that passes through these points
Sketch this graph on the same graph above clearly labelling any key features

3 marks

iv) Find an equation in the form $y = \frac{a}{x-h} + k$ that passes through these points
Sketch this graph on the same graph above clearly labelling any key features

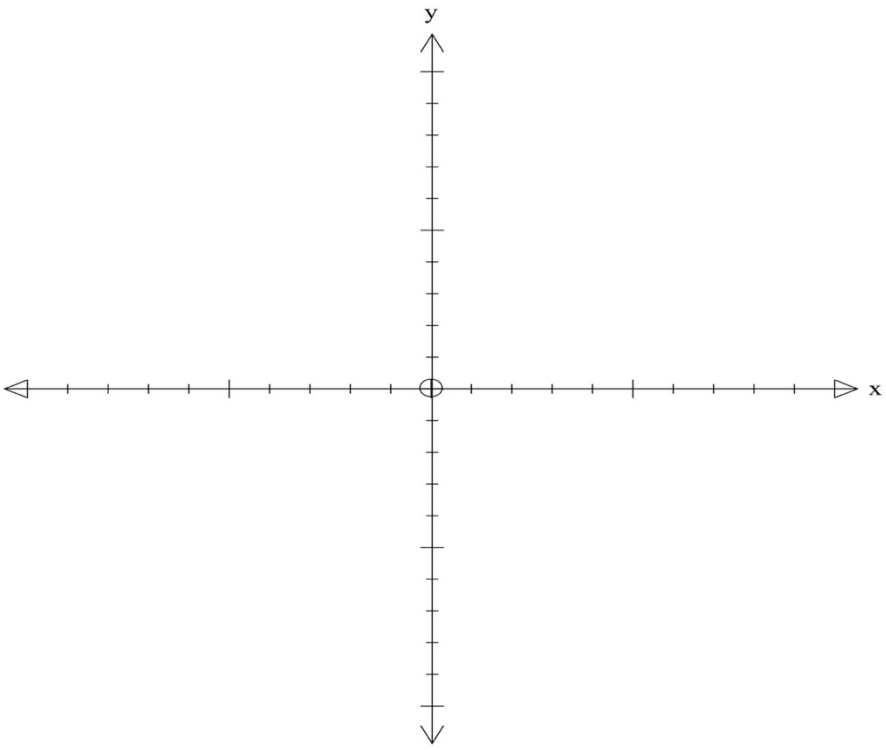
3 marks

v) Using techniques similar to the previous questions, show that the points (0,0), (1,3) and (4,0), (3,3) *determine* the curves

a) $y = \frac{4}{(x-2)^2} - 1$ and b) $y = -x(x-4)$

4 marks

vi) Demonstrate graphically, that it is possible to draw a number of cubics through the points (0,0), (1,3) and (4,0). Label the key features of your curves



4 marks

Question 2. Curves through sets of points

i) Using your CAS find the equation of the curve in the form $\boldsymbol{y = ax^2 + bx + c}$ that passes through the points (1,0), (3, 1) and (5,6).

4 marks

ii) Find two other curves of different types that go through these three points. Which of these curves are *determined* by the three points?

6 marks

iii) Identify one type of curve which *CANNOT* go through all of the above three points. Explain your reasoning using a sketch graph.

2 marks

iv) Find the equation of one curve that passes through all four points (-1,16) , (0,-9), (2,-4); (1,0).

State the general equation of the curve you are fitting to the points at the start of your working.

4 marks

Question 3 – Curves that intersect

a) **Show that** the values of m for which the line $y = mx + 2$ intersects the parabola $y = ax^2 + bx + c$ twice is given by: {m: $m < -2\sqrt{ac - 2a} + b \ \cup m > 2\sqrt{ac - 2a} + b$}

3 marks

b) Give any restrictions on the values of a, b and c

3 marks

END OF PAPER