Camberwell Girls Grammar School
An Anglican School - Educating Tomorrow's Woman

STUDENT NUMBER

										Letter
Figures										
Words										

Teacher	Mr. Woodlock	Mrs. Bergamin	Mr. Trufitt

MATHEMATICAL METHODS
School Assessed Task – 1.3

Wednesday 23rd May 2018
Reading time: 10 minutes
Writing time: 120 minutes

Modelling Task

Number of questions	*Number of questions to be answered*	*Number of marks*
4	4	**70**

- Students are permitted to bring into the Assessment room: pens, pencils, highlighters, erasers, sharpeners, rulers, one CAS calculator, one bound notebook
- Students are <u>**not**</u> permitted to bring into the Assessment room: blank sheets of paper and/or white out liquid/tape.
- **Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room. Students must not disclose the contents of the task; to do so will be a breach of VCE guidelines and will be dealt with according to VCAA regulations.**

Materials supplied
- Question and answer book of **11** pages.
- Working space is provided throughout the book.

Instructions
- Write your name in the space provided above on this page.
- All responses must be written in English.
- Answer **all** questions in the spaces provided.
- Unless otherwise specified an **exact** answer is required to a question.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

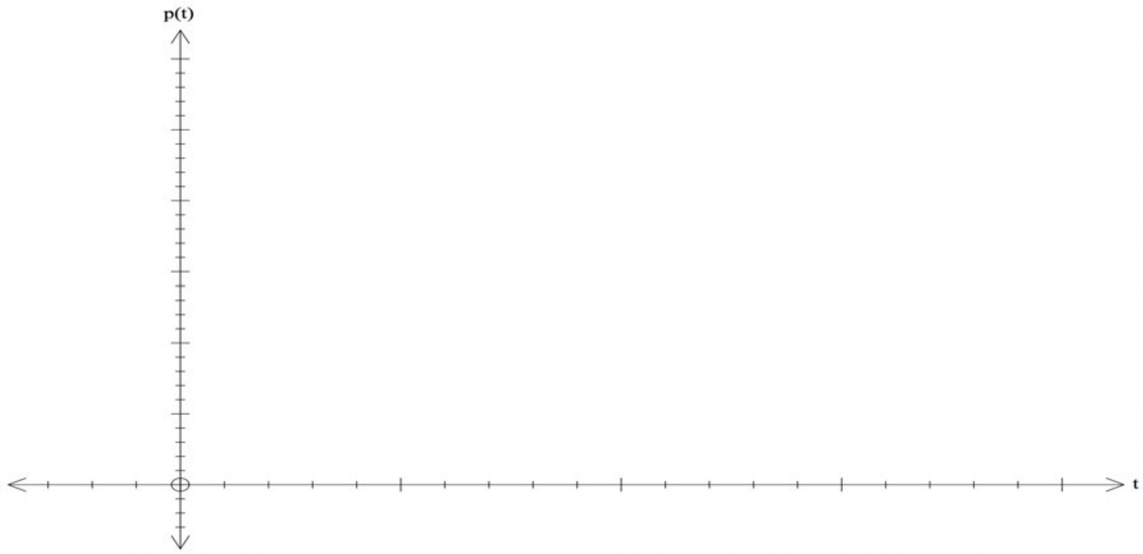
Question 1 (22 marks)

At the GSV swimming carnival, Bronte, in lane 4, is competing in the 50 metres freestyle race. Her position from the starting block, as the function of time, t seconds, is modelled by the function

$$P(t) = \frac{-1}{23}t(t-69), t \geq 0$$

a. Graph a function to model her swim for the entire 50 m race correct to 2 decimal places .

50m = 1 lap of the pool



2 marks

b. Use the graph in part **a** above to determine the race finishing time for Bronte (to 2 decimal places)

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1 mark

c. Determine the instantaneous velocity and acceleration in appropriate units after Bronte has swum for exactly 10 seconds.

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4 marks

Meanwhile Cate's position swimming in lane 5 for the same race (50 m Freestyle) is modelled by the function

$$Q(t) = 60(1 - e^{-0.06t}), \qquad t \geq 0$$

d. Graph a function to model this function for the entire 50 m race stating the domain, correct to 2 on the same axis above

2 marks

e. Based on the information determine the finish time for Cate (to 2 decimal places)

1 mark

f. Who between Bronte and Cate is the winner and by what margin of time? (to 2 decimal places)

2 marks

g. i. Determine the average speed between 3 and 5 seconds for Bronte(to 2 decimal places)

ii. Determine the average speed between 3 and 5 seconds for Cate(to 2 decimal places)

2 marks

h. Based on the above calculations, who has swum more distance between the 3 and 5 second interval and by how much? (to 2 decimal places)

2 marks

i. Find the time interval during which Bronte is swimming faster than Cate. (3 decimal places)

2 marks

j. Hence, or otherwise determine the average acceleration for both Bronte and Cate in this time interval (3 decimal places)

2 marks

k. Discuss the implications of these figures (from **i** and **j**) with respect to what is happening in the race at this time.

2 marks

- **Note: It would be beneficial to clear your calculator before starting this question**

Question 2 (14 marks)

Both Bronte and Cate are racing again in the same lanes in the 100 m freestyle event. Bronte's position, ***B*** metres, from the starting block at any given time, ***t*** seconds, is modelled by the function:

$$B(t) = \begin{cases} \dfrac{t^2}{28} + \dfrac{t}{2} & 0 \le t \le 3\sqrt{161} - 7 \\ 50 & 3\sqrt{161} - 7 < t \le 32 \\ 50 - \dfrac{1}{23}(t-32)^2 & t > 32 \end{cases}$$

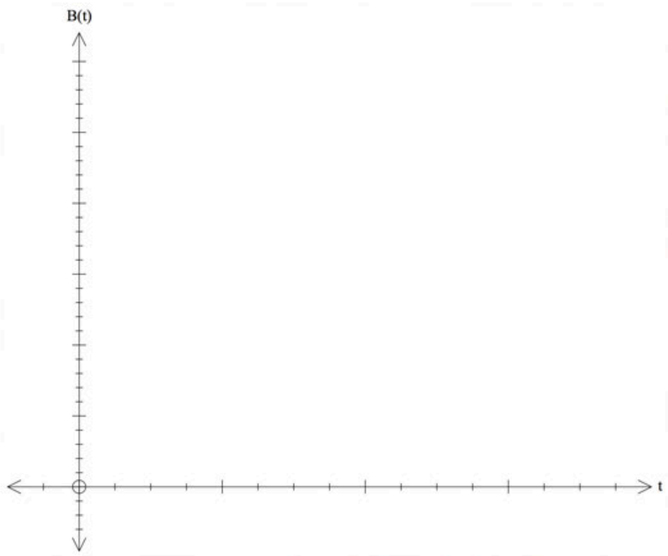
Cate's position, ***C*** metres, from the starting block at any given time, ***t*** seconds, is modelled by the function:

$$C(t) = \begin{cases} \dfrac{25t^2}{512} & 0 \le t \le 32 \\ 50 - \dfrac{20(t-32)}{13} & t > 32 \end{cases}$$

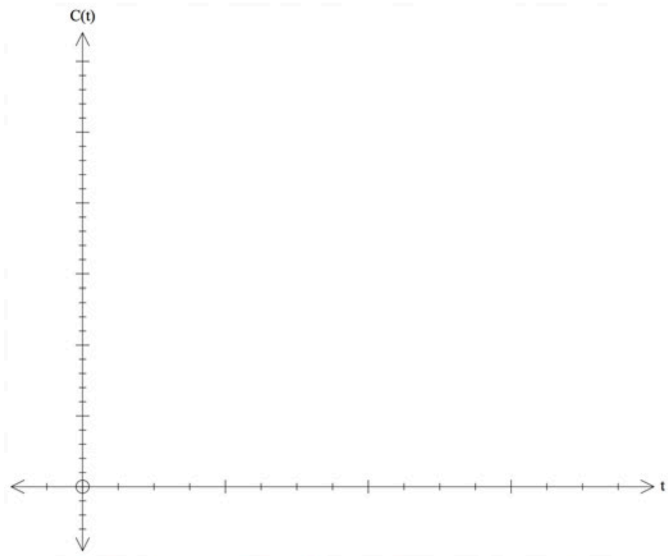
Using the above models:

a. Sketch the function for the entire 100 m race for both Bronte and Cate

Bronte



Cate



6 marks

b. i. Who is the winner of the 100 m freestyle race

ii. What is the winning margin in seconds (to 2 decimal places)

2 marks

c. State the **first** instance, to 2 decimal places, when the 2 swimmers have the same **velocity**.

2 marks

d. Aside from at the start, were the two swimmers level at any time during the race? Justify your answer mathematically.

2 marks

e. Who was leading for the last 50 m of the race?

1 mark

f. What was Cate's average speed for the last 50 m of the race?(to 2 decimal places)

1 mark

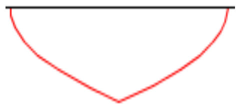
Question 3 (24 marks)

Flags are to be hung above the swimming lanes so that when doing backstroke swimmers can use these flags as a warning they are near the end of the pool. The shape of the lower edge of the 1st flag in lane one is modelled by the function

$$g(x) = \begin{cases} -0.9\sqrt{x} + 1.75, & x \in [0, 0.25] \\ -0.9\sqrt{0.5 - x} + 1.75, & x \in (0.25, 0.5] \end{cases}$$

Where g(x) is the distance, in metres, from the top edge of the pool and *x* is the distance, in metres, from the edge of lane 1.

One flag looks like the diagram on the right.



a. How many flags fit into a pool width of 25 metres, if no gap is left between any two neighbouring flags?

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1 mark

b. How many individual flags will hang above one lane of width 2.5 metres?

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1 mark

c. Determine, to 2 decimal places:

i. $g(0)$

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ii. $g(\frac{1}{4})$

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2 marks

d. If the water level in the pool is 25 cm below the top edges of the pool

i. How high above the water level are the top of the flags?

ii. How high above the water level is the lowest point of the flag?

iii. State the function m(x) for the first flag in lane 4 if this flag has the same shape as the first flag in lane 1.

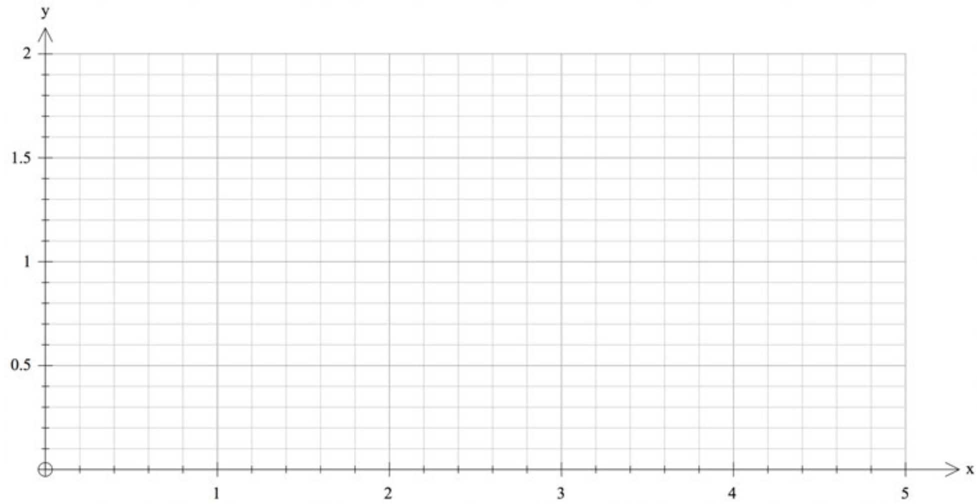
4 marks

It is later decided to have only one flag over each lane such that its lowest point is exactly in the middle of each lane.

e. Propose a transformation that will take the function ***g(x)*** to the rule for the lane 2 flag ***h(x).*** Give the function ***h(x).***

5 marks

f. Sketch the graph for the function $h(x)$ over the new domain and write the coordinates of the point where the function $h(x)$ is pieced



3 marks

g. i. Determine $h'(x)$ and also state the domain of the derivative

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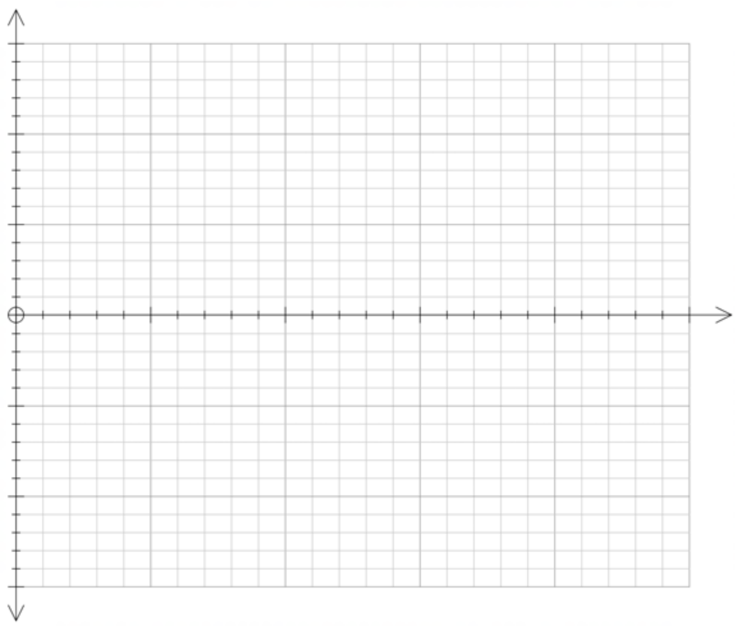
2 marks

h. i. Using your CAS, sketch the graph of the derivative of the transformed function



3 marks

ii. Show mathematically that the derivative function $h'(x)$ fails all three continuity tests at the points where the function h(x) is pieced together.

3 marks

Question 4 (10 marks)

In the 400m Freestyle event, Ariane was swimming in lane 5. Her position from the start at any time, t minutes, was given by the function

$$D(t) = \begin{cases} a\cos(nt) + 25, & 0 \leq t \leq \frac{3}{2} \\ b\cos\left(m(t - \frac{3}{2})\right) + 25, & \frac{3}{2} < t \leq 4 \end{cases}$$

At t = 1.5 mins Ariane had completed 4 laps of her 8 lap event, at t= 4.00 mins she had won the race in record time. (1 lap equals 50 metres)

a. Show that $a = -25, n = \frac{8\pi}{3}, b = 25 \ and \ m = \frac{8\pi}{5}$

5 marks

b. Find Ariane's velocity 2 mins into the race.

2 marks

c. How far had Ariane swum at this point? (Give your answer to 2 decimal places)

3 marks

End of Paper