Camberwell Girls Grammar School
An Anglican School

CREATE YOUR TOMORROW

Student Name	
Teacher	Mrs Bergamin Mr Trufitt Mr Woodlock

MATHEMATICAL METHODS UNIT 4

SAC 3: Probability Task

Wednesday 15 August 2018

Reading time: 10 minutes
Writing time: 120 minutes

Structure of Task

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
Extended response	6	6	75

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one CAS calculator and/or one scientific calculator, and one approved bound reference.
- Students are not permitted to use: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 15 pages.
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your name in the space provided above on this page.
- All responses must be written in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students must not disclose the contents of the task; to do so will be a breach of School guidelines.

Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (12 marks)

Soccer fever swept across the globe in June and July 2018 as 32 nations took part in the FIFA World Cup in Russia. Social media was lit up with millions of posts from soccer fans all over the world.

The number of daily tweets that an average World Cup soccer fan makes on 'Twitter' is a discrete random variable, T, with the probability distribution shown below

t	0	1	2	3	4	5
$\Pr(T = t)$	0.01	0.13	0.24	0.22	a	0.15

a. Show that $a = 0.25$. 1 mark

b. What is the probability that a World Cup soccer fan will make one or two tweets in a day? 1 mark

c. What is the expected number of tweets that an average World Cup soccer fan will make in one day, correct to two decimal places? 2 marks

d. Find the standard deviation of the random variable T, correct to 3 decimal places. 2 marks

e. Find the probability that over the first weekend (Saturday and Sunday), a person will make exactly 4 tweets in total. 2 marks

f. Find the probability that a person who made 4 tweets on the first weekend made more tweets on Saturday than Sunday. 2 marks

g. On Wednesday of the first week, ten randomly selected World Cup fans were surveyed regarding the number of tweets they had made on the previous day. Find the probability that eight of them had made at least 2 tweets? Give your answer correct to 3 decimal places. 2 marks

Question 2 (11 marks)

One group of Australian soccer fans invented a dice game to keep them amused in between matches. The game involves rolling two unbiased tetrahedral (4-sided) dice, with faces numbered 1 to 4. The recorded result is the number on the side facing **downwards.**

Die 1 **Die 2**

a. Using either a two-way table, a tree diagram or a list, write down the sample space for the game described. 1 mark

b. From the sample space above, complete the probability distribution table below where the random variable X represents the sum of the numbers facing downwards on the two dice. 1 mark

x	2	3	4	5	6	7	8
$Pr\ (X = x)$							

c. State all conditions that indicate this is a probability distribution. 1 mark

d. Find $\Pr(X > 3 | X \leq 5)$ 2 marks

e. If $A = \{\text{sum of numbers facing downwards on the two dice is greater than } 6\}$
$B = \{\text{the number facing downwards on each die is the same}\}$

Find $\Pr(A \cap B)$ 1 mark

f. Determine if events A and B are

i. Mutually exclusive, giving a reason. 1 mark

ii. Independent, giving a reason. 1 mark

The Australian fans invite the English fans to join in, but charge them 3.00 RUB (Russian Rubles) to play. The prizes are 8.00 RUB if a sum of more than 6 is obtained and 1.00 RUB if a sum of 6 or less is obtained.

g. What is the expected gain/loss for this game, in RUB to two decimal places? 2 marks

h. Is this game fair? Explain. 1 mark

WORKING SPACE

Question 3 (11 marks)

Twenty-four World Cup fans have won a competition for the chance to be a special guest commentator on the SBS television coverage of the Match of the Day. As part of the competition, each of these fans were asked to nominate a soccer league that they would most identify being a fan of. The results are as follows

Soccer league	**Number of fans**
English Premier League	9
Serie A (Italy)	8
La Liga (Spain)	3
Bundesliga (Germany)	4

Each day, one fan is chosen at random from this group to be the special guest commentator. All twenty-four fans are eligible for selection on any given day, even if they have been selected before.

a. Find the probability that

i. On a particular day, an English Premier League fan is **not** chosen. 1 mark

ii. An English Premier League fan is chosen on the first day, a La Liga fan on the second, and Serie A fan on the third. 1 mark

iii. Over three consecutive days, two English Premier League fans and one La Liga fan are chosen. 2 marks

b. Over a period of twenty days, find **to four decimal places**

i. The probability that a Bundesliga fan is selected exactly five times. 2 marks

ii. The probability that the fan chosen is a non-English Premier League fan for between five and 10 days inclusive. 2 marks

c. How many days would need to pass before the probability that any La Liga fan has been chosen exceeds 0.9? 3 marks

Question 4 (16 marks)

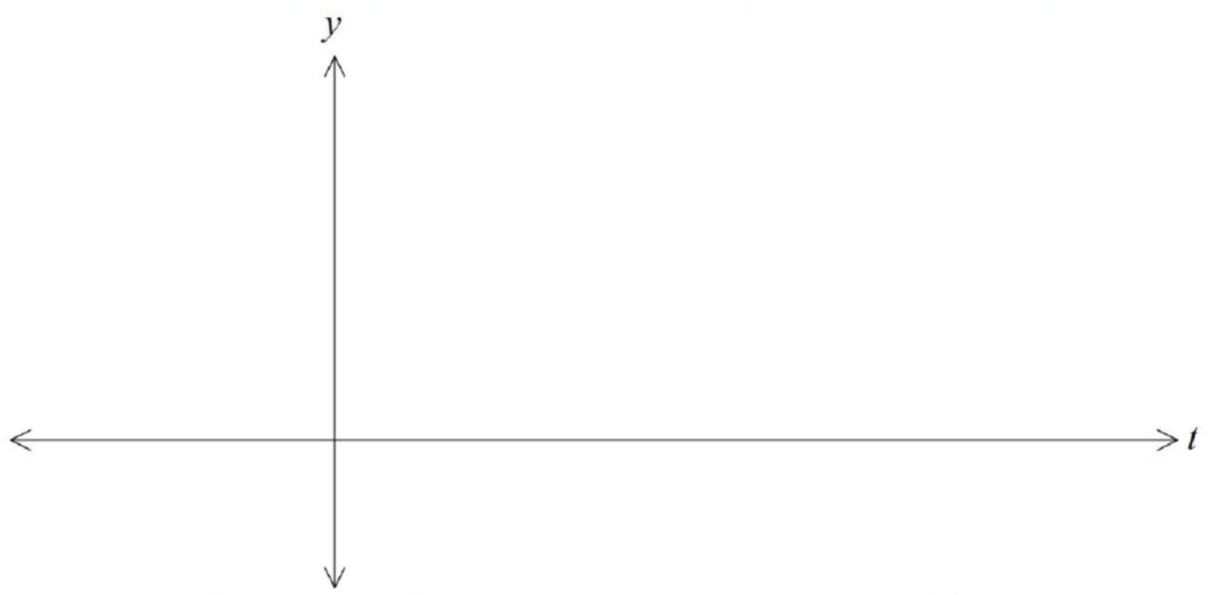
The extra-time, t minutes, played at the conclusion of a half of World Cup soccer is a continuous random variable described by the probability density function f, where

$$f(t) = \begin{cases} k\left(\sin\left(\frac{\pi t}{4}\right) + 2\right), & 0 \le t \le 6 \\ 0, & elsewhere \end{cases}$$

and k is a real number.

a. Show that $k = \frac{\pi}{4(3\pi+1)}$ 3 marks

b. Sketch the graph of $y = f(t)$, clearly labelling all key features **to three decimal places**. 3 marks



c. Find the probability that the extra-time played at the World Cup is between four and five minutes. Give your answer to four decimal places. 2 marks

d. Find the expected length of extra-time played at the World Cup, $E(T)$, correct to the nearest second. 2 marks

e. Find the median length of extra-time played at the World Cup, correct to the nearest second. 2 marks

f. Find the probability that no more than five extra minutes are played, given that at least the median length of extra-time was played. Give your answer to four decimal places. 2 marks

The Video Assistant Referee (VAR) is a football assistant referee who reviews decisions made by the head referee with the use of video footage.

If the VAR is called upon, the amount of extra-time played at the conclusion of the half of football is affected according to the function υ, where

$$\upsilon(t) = \begin{cases} \frac{t^3}{200}(t+6)(7-t), 0 \le t \le 6 \\ 0\,, \quad elsewhere \end{cases}$$

g. Find $E[\upsilon(T)]$, the expected value of extra-time when the VAR is called upon, correct to the nearest second. 2 marks

WORKING SPACE

Question 5 (10 marks)

All answers to Question 5 are to be given ***correct to four decimal places****.*

The time, T minutes, before the first goal of a World Cup soccer match is scored is normally distributed with a mean of 52 minutes and a standard deviation of 17 minutes.

a. Find the probability that the time before a first goal is scored in a World Cup match is:

i. Between 35 and 69 minutes — 1 mark

ii. Over 80 minutes. — 1 mark

A World Cup soccer match consists of two halves of 45 minutes each.

b. Find the probability that the first goal of a World Cup match is scored in the first half. — 1 mark

There are eight matches played in the 'knockout phase' of the World Cup.

c. Find the probability that the first goal is scored in the first half:

i. In exactly 6 matches of the knockout phase — 2 marks

ii. In less than 6 matches of the knockout phase — 1 mark

iii. In exactly 6 matches of the knockout phase, given that the first goal was scored in the first half in at least 4 matches of the knockout phase. 2 marks

The time, T minutes, before the first goal of an English Premier League match is scored is also normally distributed with a mean of 50 minutes and a standard deviation of σ minutes. Each half of football in the English Premier league is also 45 minutes.

d. If the first goal is scored in the first half of 40% of English Premier League matches, calculate the standard deviation of the length of time, in minutes, before a first goal is scored. 2 marks

WORKING SPACE

Question 6 (15 marks)

The light globes in the roof of the Luzhniki Stadium in Moscow that hosted the World Cup final are manufactured by Staybright International Lighting Company Ltd.

Staybright claims that 89% of their light globes last longer than 110 hours, but due to the high fail-rate during testing, stadium engineers claim that this is not true. Independent inspectors take a random sample of 450 globes and find that 366 of the sample last longer than 110 hours.

a. Find a 98% confidence interval for the proportion of light globes that last more than 110 hours. Give your answer correct to four decimal places. 2 marks

b. If the inspectors took another 1000 random samples of 450 light globes, how many of the 98% confidence intervals would be expected to capture the value of the population proportion? 1 mark

c. According to the results from the sample, do stadium engineers have a right to complain? Briefly explain your answer. 2 marks

d. What is the largest sample size that Staybright could have agreed to so their claim that 89% of their globes last longer than 110 hours is within a 98% confidence interval, assuming the sample proportion p is 0.814? 3 marks

It was found that 81.4% of Staybright light globes last longer than 110 hours. Six light globes were selected at random.

e. What is the probability that more than two of them last longer than 110 hours? Give your answer correct to three decimal places. 2 marks

f. Given that only 3 of them last longer than 110 hours, what is the probability it was the first three selected? Give your answer correct to three decimal places. 2 marks

The length of time that Staybright light globes last is normally distributed. The inspectors also found that 4% of the light globes lasted longer than 130 hours.

g. Find the mean and standard deviation of the distribution correct to three decimal places. 3 marks

END OF SAC 3