Camberwell Girls Grammar School
An Anglican School

CREATE YOUR TOMORROW

Student Name		
Teacher	Mr Trufitt	Mr Woodlock

MATHEMATICAL METHODS UNIT 3

SAC 1: Application Task

PART 3 – "SPECIFICATION OF CONTEXT"

Wednesday 15 May 2019

Reading time: 10 minutes
Writing time: 75 minutes

Structure of Task

Section	*Number of questions*	*Number of questions to be answered*
A. Neck of the Wine Carafe	4	4
B. The Cylindrical Stopper	3	3
C. The Smooth-Tapered Stopper	3	3

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one CAS calculator and/or one scientific calculator, and one approved bound reference.
- Students are not permitted to use: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 8 pages.
- Working space is provided throughout the book.

Instructions

- Write your name in the space provided above on this page.
- All responses must be written in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students must not disclose the contents of the task; to do so will be a breach of School guidelines.

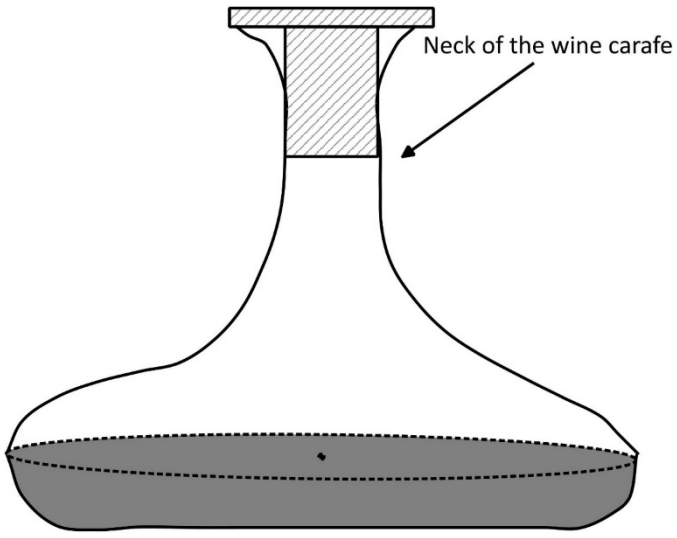
MATHEMATICAL METHODS UNIT 3

2019 SAC 1: Application Task

PART 3: "SPECIFICATION OF CONTEXT"

Michelle is the head designer with Camberwell Glass Gifts Specialists. Michelle specialises in the design of wine carafes.

Michelle is currently modelling the stopper to fit the neck of the new design.



A. The Neck of the Wine Carafe

The glass neck of the carafe can be modelled by the equation

$$h: D \to R, h(x) = A\log_e((x-p)^2 - m) \text{ where } m > 0$$

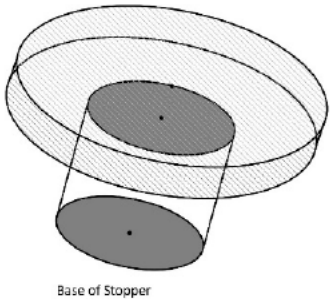
1. Let $p = 1$ and $m = 4$. Vary the values of A and describe the physical effects that changing A has on the shape of the neck of the wine carafe.

2. Let $A = 1$ and $m = 4$. Vary the values of p and describe the physical effects that changing p has on the shape of the neck of the wine carafe.

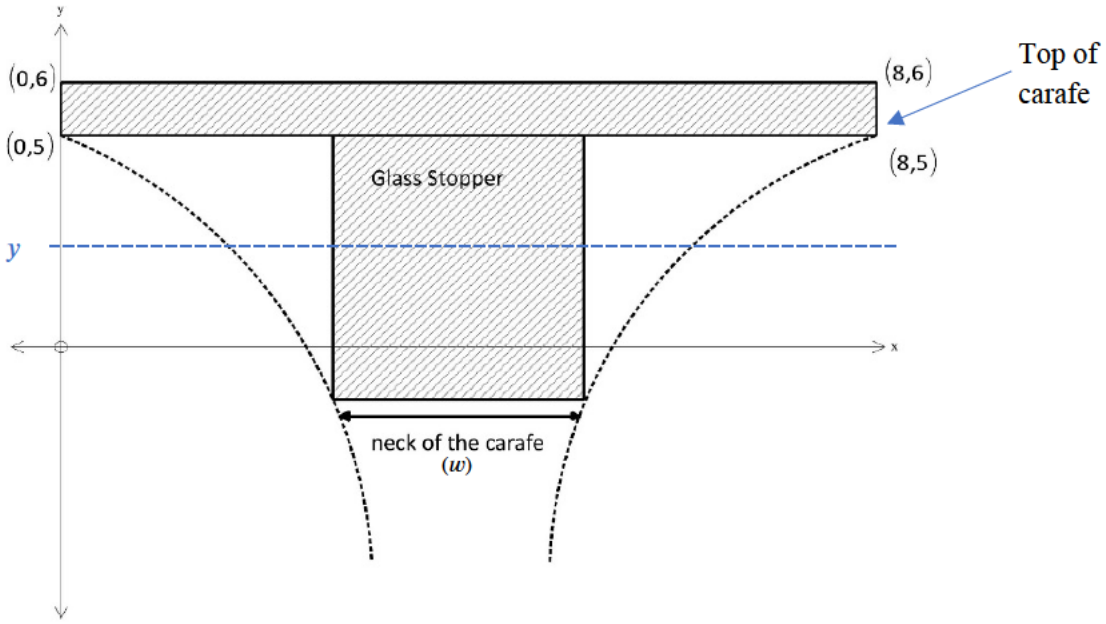
3. Let $p = 1$ and $A = 1$. Vary the values of m and describe the physical effects that changing m has on the shape of the neck of the wine carafe.

4. Express the width of the narrowest section of the neck of the carafe in terms of m.

B. The Cylindrical Stopper



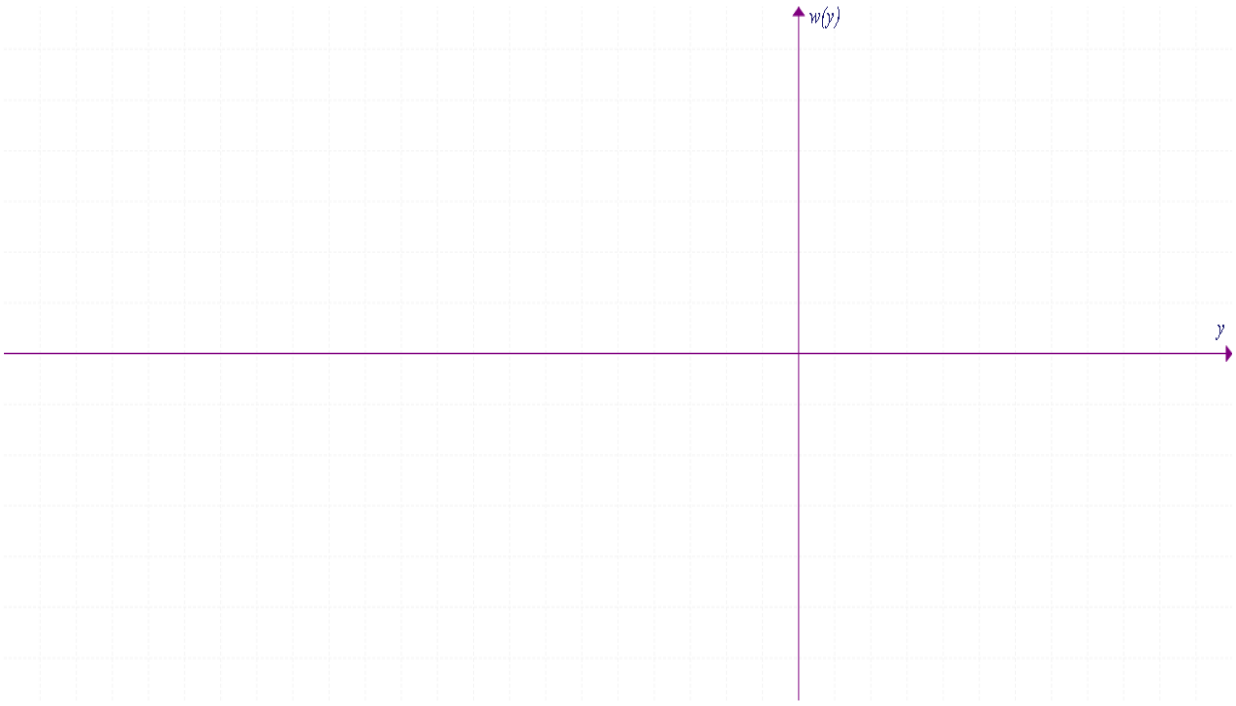
Michelle is sketching an initial design and produces the neck of a carafe with the dimensions as shown in the diagram. Into the neck of the carafe she inserts a cylindrical glass stopper as shown in the diagram below. The top of the carafe is indicated.



1. The neck of the carafe approaches a width, w, of 4 cm at its narrowest point (ie $w = 4$). Find the values of A, p and m for this particular design.

2. a. Find an equation, $w(y)$, that can be used to calculate the width of the neck of the carafe, w, at any vertical point, y, along the neck of the carafe (as shown in the diagram on the previous page).

b. Sketch the graph of $w(y)$ for $y \in [-10, 5]$.



3. If the neck of the glass cylindrical stopper is 5 cm wide, how far down into the carafe does it reach?

C. The Smooth-Tapered Stopper

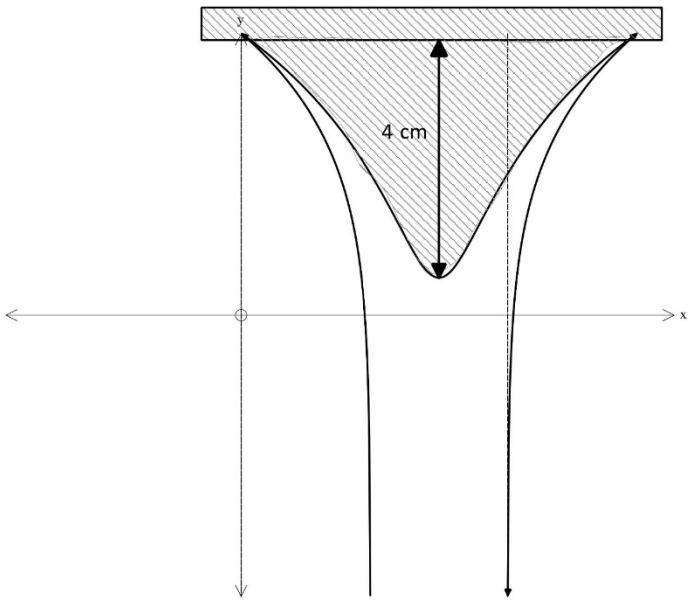
Under certain conditions, the family of functions of the form: $f(x) = \log_e((x-b)^2 - m)$ will contain a local minimum turning point instead of a pair of asymptotes.

1. Find an expression for $f'(x)$ and hence state the values of x and m under which the family of curves given by $y = f(x)$ has a local minimum turning point.

2. (i) For the function $f(x) = \log_e((x-4)^2 - m)$, sketch the graph of $y = f(x)$ for a range of values of m. The values of m for each graph must be stated.

(ii) What effect does m have on the shape of the graph?

3. For the smooth-tapered stopper defined by $f(x) = \log_e((x-4)^2 - m)$, find the value of m such that the stopper reaches 4 cm into the carafe.



WORKING SPACE

END OF PART 3