Camberwell Girls Grammar School
An Anglican School

CREATE YOUR TOMORROW

Student Name		
Teacher	Mr Trufitt	Mr Woodlock

MATHEMATICAL METHODS UNIT 3

SAC 1: Application Task

PART 4 – "FOCUS ON A RELATED ASPECT"

Thursday 23 May 2019

Reading time: 10 minutes
Writing time: 90 minutes

Structure of Task

Section	*Number of questions*	*Number of questions to be answered*
A. Packaging the Wine Carafe	1	1
B. Investigating the General Case for Packaging	1	1
C. Final Packaging Considerations	6	6

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one CAS calculator and/or one scientific calculator, and one approved bound reference.
- Students are not permitted to use: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 10 pages.
- Working space is provided throughout the book.

Instructions

- Write your name in the space provided above on this page.
- All responses must be written in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

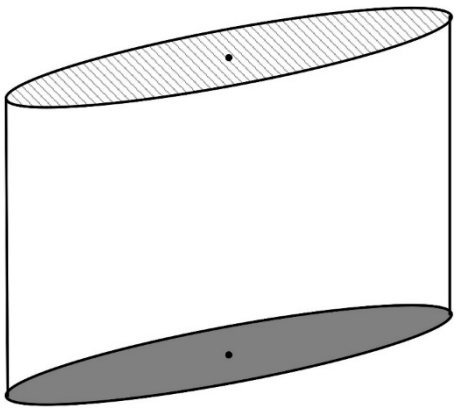
Students must not disclose the contents of the task; to do so will be a breach of School guidelines.

MATHEMATICAL METHODS UNIT 3

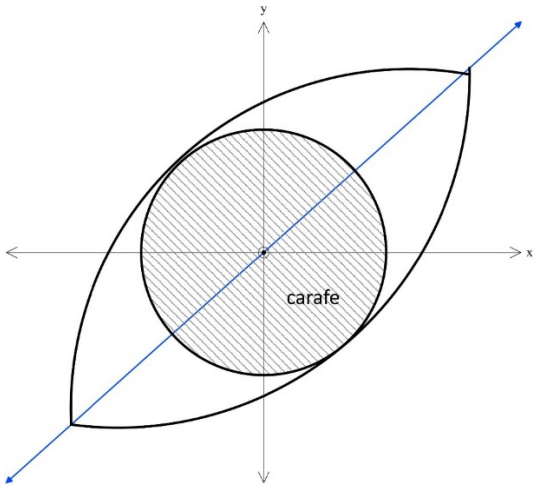
2019 SAC 1: Application Task

PART 4: "FOCUS ON A RELATED ASPECT"

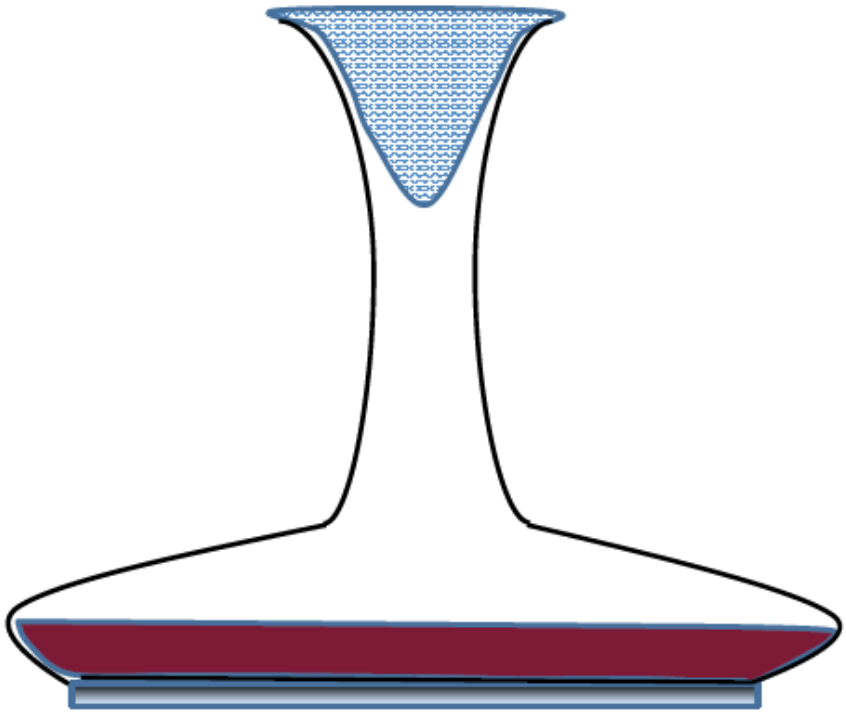
After the wine carafe had been designed, Michelle put her expertise into the design of the packaging.



Three-dimensional view



Top view



Side profile

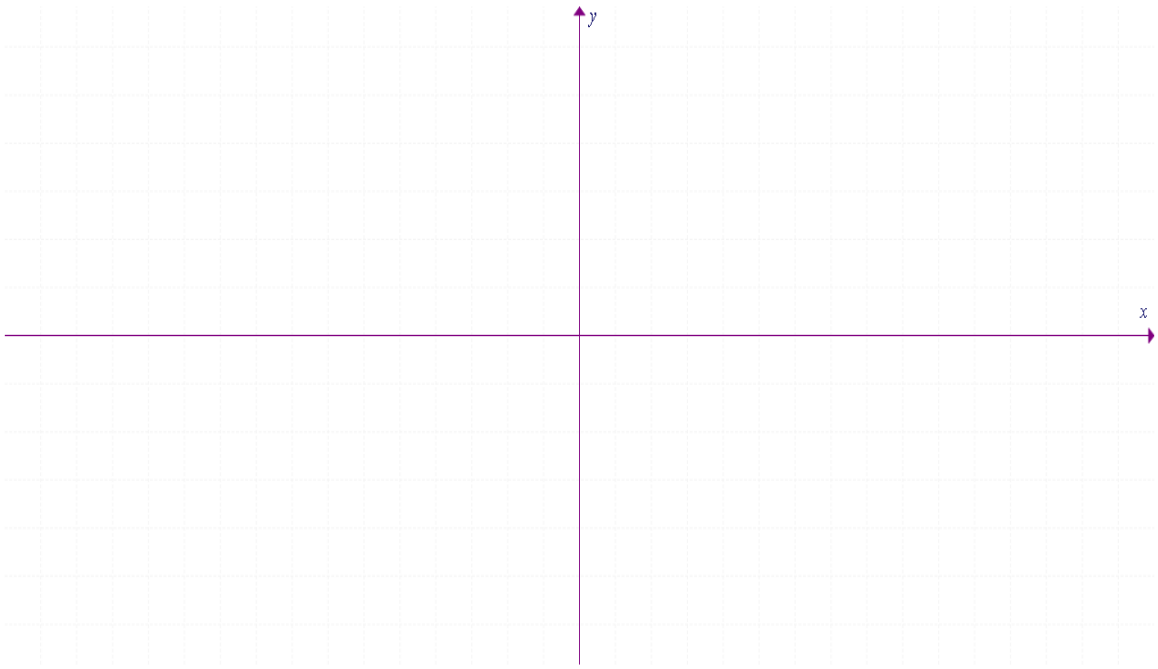
A. Packaging the Wine Carafe

1. The trial design model of the horizontal cross-section of the box to carry the wine carafe is symmetrical in shape and is framed by the functions:

$$g(x) = \frac{2x+1}{x+2} \text{ and } g^{-1}(x).$$

a. Find the rule and state the domain for $g^{-1}(x)$.

b. Sketch the graphs of $g(x)$ and $g^{-1}(x)$ for $x \in [-1, 1]$ on the axes below, labelling key features.



c. Find the coordinates of the point on the curve $g(x)$ such that the distance to the origin is a minimum. Show all working.

d. If the wine carafe has a circular cross-section, write an equation representing the largest carafe that can fit into the box.

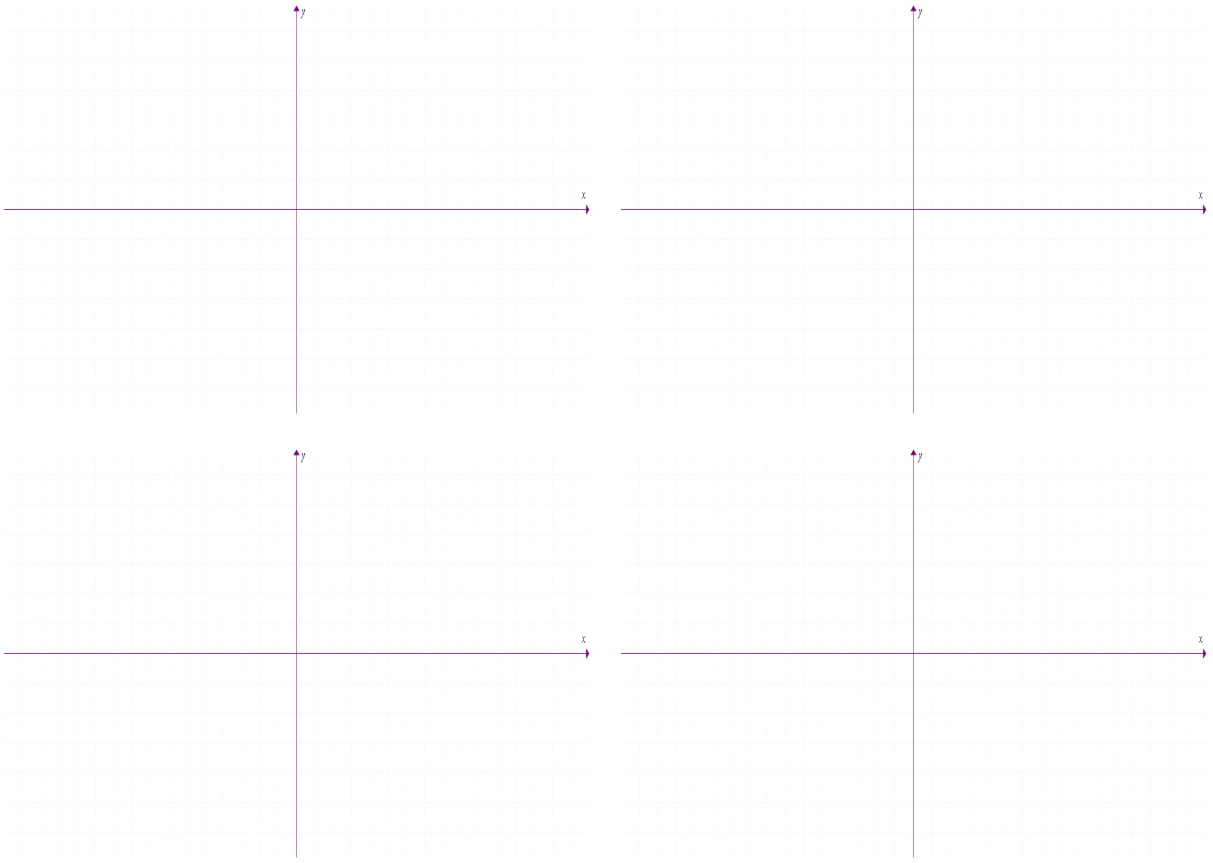
B. Investigating the General Case for Packaging

1. For the function $h(x) = \frac{ax+1}{x+a}$,

   a. Express $h(x)$ in the form: $A + \frac{B}{x+a}$.

   b. Find $h'(x)$.

   c. **Quickly** sketch the graph of $h(x)$ and $h^{-1}(x)$ for $x \in [-1, 1]$ for a range of values of a.



d. What effect does a have on the shape of the box?

e. Investigate the effect on the size of the carafe that can fit into the box as a varies.

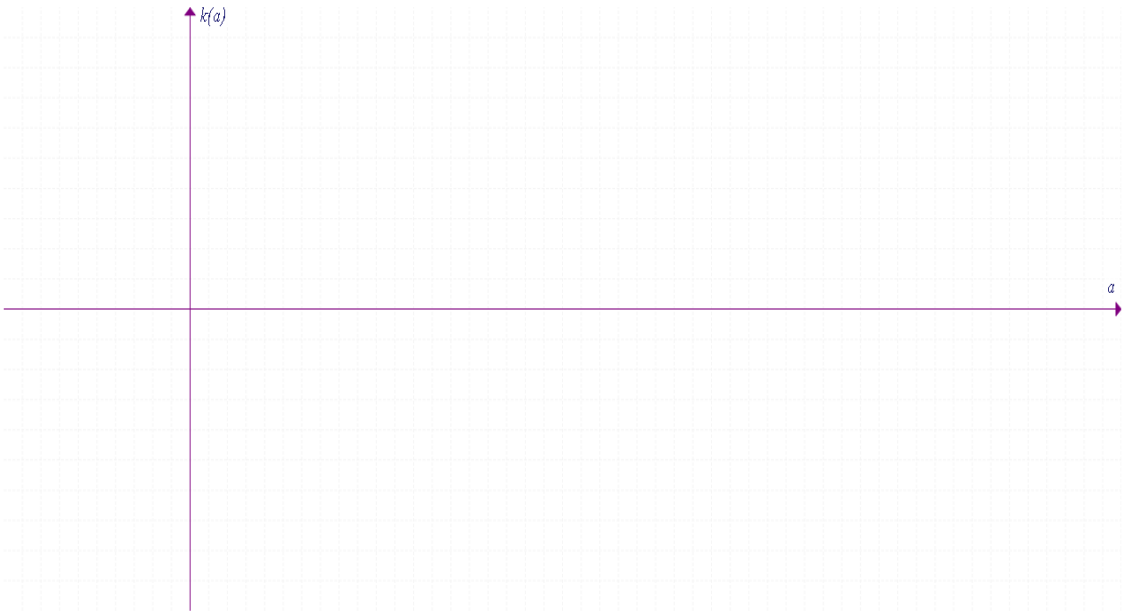
C. Final Packaging considerations

The material that the carafe box is made from is a high-quality cardboard, essential for the protection of the carafe. The cardboard, however, has a limited ability to bend to the required shape for this box. For this particular product, the bending factor, k, is defined as:

$$k(a) = \frac{h'(-1)}{h'(1)} \text{ for } h(x) = \frac{ax+1}{x+a} \text{ where } a \in (1, \infty).$$

1. Write down the rule for $k(a)$.

2. Sketch the graph of k vs a for $a \in (1, \infty)$.



WORKING SPACE

3. Find the values of a for which the bending factor $k \leq 8$.

4. Solve $k(a) = d$, where d is the value of the bending factor. Express a in terms of d.

5. Using the formula, verify the value of a required to achieve a bending factor of 8 for question 4.

WORKING SPACE

6. Design a container that can fit a carafe of base diameter 20 cm and height 25 cm. Note that the bending factor is maintained at $k \leq 8$. Include calculations and diagrams.

END OF SAC