



Student Name	
Teacher	Mr Trufitt Mr Woodlock

MATHEMATICAL METHODS UNIT 4

SAC 3: Probability Problem-Solving Task

Wednesday 28 August 2019

Reading time: 10 minutes

Writing time: 120 minutes

Structure of Task

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
Extended response	6	6	65

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one CAS calculator and/or one scientific calculator, and one approved bound reference.
- Students are not permitted to use: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 13 pages.
- Formula sheet
- Working space is provided throughout the book.

Instructions

- Write your name in the space provided above on this page.
- All responses must be written in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students must not disclose the contents of the task; to do so will be a breach of School guidelines.

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (11 marks)

Good friends Zara and Lauren meet on the first Friday night of the month at the movies. Before the movie, it has become tradition that the friends compete against each other in the car racing game at the video arcade inside the cinema complex.

The car racing game, Gaytona, can have up to four people racing against each other in four different cars, given individual identification (ID) numbers C1, C2, C3 and C4.

On Friday night, the probability that each of the cars are occupied by other players when the two girls arrive is shown in the table below.

Car ID number	C1	C2	C3	C4
<i>Pr(occupied)</i>	0.8	0.6	0.5	0.3

a. Calculate the probability, correct to three decimal places, that

i. All four cars are occupied

1 mark

ii. No cars are occupied

1 mark

iii. Three cars are occupied.

2 marks

Let X represent the discrete random variable that represents how many of the 'Gaytona' cars are being used at any one time during any particular day. The probability distribution for X is shown in the table below.

x	0	1	2	3	4
$\Pr(X = x)$	0.05	0.15	a	b	0.1

- b.** If the expected value of X is 2.25, show that the values of a and b are 0.4 and 0.3, respectively 3 marks

- c.** Calculate the standard deviation of X , correct to two decimal places. 2 marks

- d.** On any particular day, determine the probability that exactly three cars are being used given that at least one car is being used. 2 marks

Question 2 (13 marks)

For the previous six months, Zara and Lauren have competed against each other in the car race game 'Gaytona'.

The probability of Zara winning in a given month is 0.6 and the outcome of each month's competition is independent of the previous month.

- a. Calculate the probability that Zara won five of the monthly competitions, correct to three decimal places. 2 marks

- b. Calculate the probability that Lauren **only** won the first and third of the monthly competitions, correct to three decimal places. 2 marks

- c. Determine the probability that Zara lost at most three of the monthly competitions, correct to three decimal places 1 mark

- d. What is the minimum number of monthly competitions that they need to compete in for Lauren to have at least a 95% chance of winning at least two months. 3 marks

The two girls decide to invite their mutual friend Ella to join in their monthly Friday movie nights from now on. Zara challenges Ella to a race in the 'Gaytona' car racing game.

If Zara beats Ella one month, the probability that she will beat her the next month is 0.75, and if Zara loses one month then the probability that she will win the next month is only 0.55.

e. If Ella beats Zara the first month, calculate the probability that

i. Zara wins the next five monthly competitions, correct to three decimal places.

2 marks

ii. Zara wins only one of the next three monthly competitions, correct to three decimal places.

3 marks

WORKING SPACE

Question 3 (10 marks)

The three girls have been playing at the games arcade for nearly one hour, which feels a little longer than normal and Zara is getting a little anxious that they may miss the start of the movie.

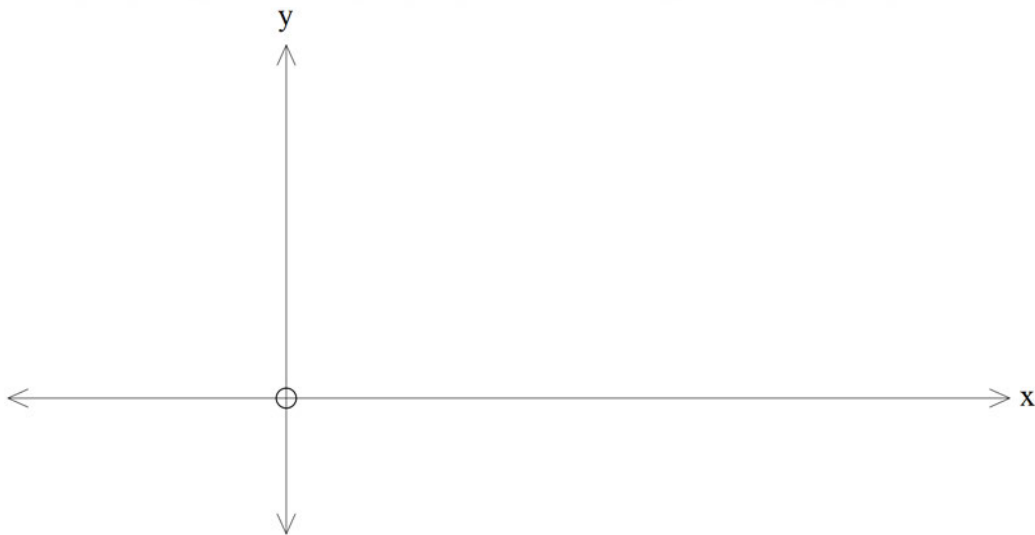
Fortunately, Lauren is an excellent Mathematical Methods student who can determine when they need to leave the arcade to make the start of the movie.

The time taken, in minutes, for the girls to leave the arcade and be seated in the movie theatre is modelled by the function

$$f(x) = \begin{cases} 0.2 & 0 < x \leq 1 \\ \frac{2x(10-x)}{405} & 1 < x \leq 10 \\ 0 & \textit{elsewhere} \end{cases}$$

- a. Sketch the graph of $y = f(x)$ on the set of axes below, labelling all key features.

4 marks



- b. Show that $f(x)$ is a probability density function.

3 marks

c. Find the median time it takes the girls to be seated in the movie theatre, correct to the nearest second.

2 marks

d. Find the probability that the girls will take between two and six minutes to be seated, correct to two decimal places.

1 mark

WORKING SPACE

Question 4 (6 marks)

The time, T minutes, that the three girls usually spend at the games arcade before the movie is modelled by the probability density function

$$f(t) = \begin{cases} k \left(\sin \left(\frac{\pi t}{15} \right) + 1 \right) & 30 \leq t \leq 45 \\ 0 & \textit{elsewhere} \end{cases}$$

a. Show that $k = \frac{\pi}{30+15\pi}$

4 marks

b. Find the mean time that the girls spend at the games arcade.

2 marks

Question 5 (10 marks)

For all parts of Question 5, answers are to be given to **four decimal places**.

The three girls leave the arcade with plenty of time to spare before the movie, so they decide to buy some snacks to take in with them. Ella loves popcorn so she decides to buy a large serve.

The mass of popcorn in a large box is normally distributed with a mean of 150 grams and a standard deviation of 12 grams.

- a. Calculate the probability that Ella's large box of popcorn will contain between 144 and 165 grams of popcorn. 2 marks

- b. Large boxes of popcorn that are filled with less than 129 grams of popcorn are rejected by customers as they appear too under-filled. Find the probability that a large box of popcorn is rejected by customers. 1 mark

- c. There are eight pre-filled large boxes of popcorn sitting in the warmer for sale. Find the probability that the customers reject less than three of the boxes. 2 marks

Zara decides to buy a 'Choc Top' ice-cream that also has a normally distributed weight.

Coyts Cinemas have received complaints that many of their 'Choc Top' ice-creams are too small, so they have given their work experience student the task of analysing each batch of 'Choc Tops' received from their supplier and rejecting those that are too small. The student has determined that 15% of Choc Tops are under 90 grams and must be sent back to the supplier.

- d. Find the standardised weight (Z-score) equivalent to 90 grams in the distribution of 'Choc Top' weights. 1 mark

Cinema Ritz, Melbourne's premiere boutique cinemas, have indicated that they only want 'Choc Tops' less than 125 grams as their customers prefer a smaller ice-cream.

78% of the 'Choc Tops' that Coyts Cinemas **does not send back** to the supplier are light enough to sell to Cinema Ritz.

- e. Find the mean and standard deviation of all the 'Choc Top' ice-creams analysed by the work experience student (including the ones sent back to the supplier).

4 marks

WORKING SPACE

Question 6 (15 marks)

Zara is health-conscious and knows that the 300 calories that she consumes from eating the ‘Choc Top’ will be easily burned off in her Saturday morning gym workout.

Zara has used a smartwatch to track the number of calories burnt in her gym workouts over the last 12 months and has found that 5% of her workouts have burned more than 455 calories and 90% of her workouts have burned between 170 and 455 calories. Zara’s workouts are normally distributed with a mean of μ and standard deviation of σ .

- a. Show that $\mu = 313$ and $\sigma = 87$, correct to the nearest whole number.

3 marks

Using $\mu = 313$ and $\sigma = 87$, find the probability that

- i. Zara burns between 250 and 350 calories in her workout (to four decimal places)

1 mark

- ii. Zara burns exactly 300 calories in her workout.

1 mark

Zara has deemed that 80% of her gym workouts over the last 12 months were effective because they burned at least 240 calories.

She selects a sample of 25 workouts from the previous year to analyse.

- b. Calculate the probability that (correct to four decimal places)

- i. the sample proportion is equal to the population proportion.

2 marks

ii. the sample proportion lies within one standard deviation of the population proportion. 3 marks

c. Find an approximate 95% confidence interval for the proportion p of workouts that Zara deems to be effective (correct to four decimal places). 1 mark

d. Interpret what the 95% confidence interval found in part **c.** tells Zara about the proportion of effective workouts. 1 mark

e. Determine the sample size required to achieve a margin of error of 10% in a 95% confidence interval for the percentage of workouts that Zara deems to be effective. 2 marks

If Zara did not know what percentage of workouts were effective, it would still be possible to calculate the required sample size to achieve a certain margin of error using p^* .

f. Very briefly, explain the significance of p^* .

1 mark

END OF SAC