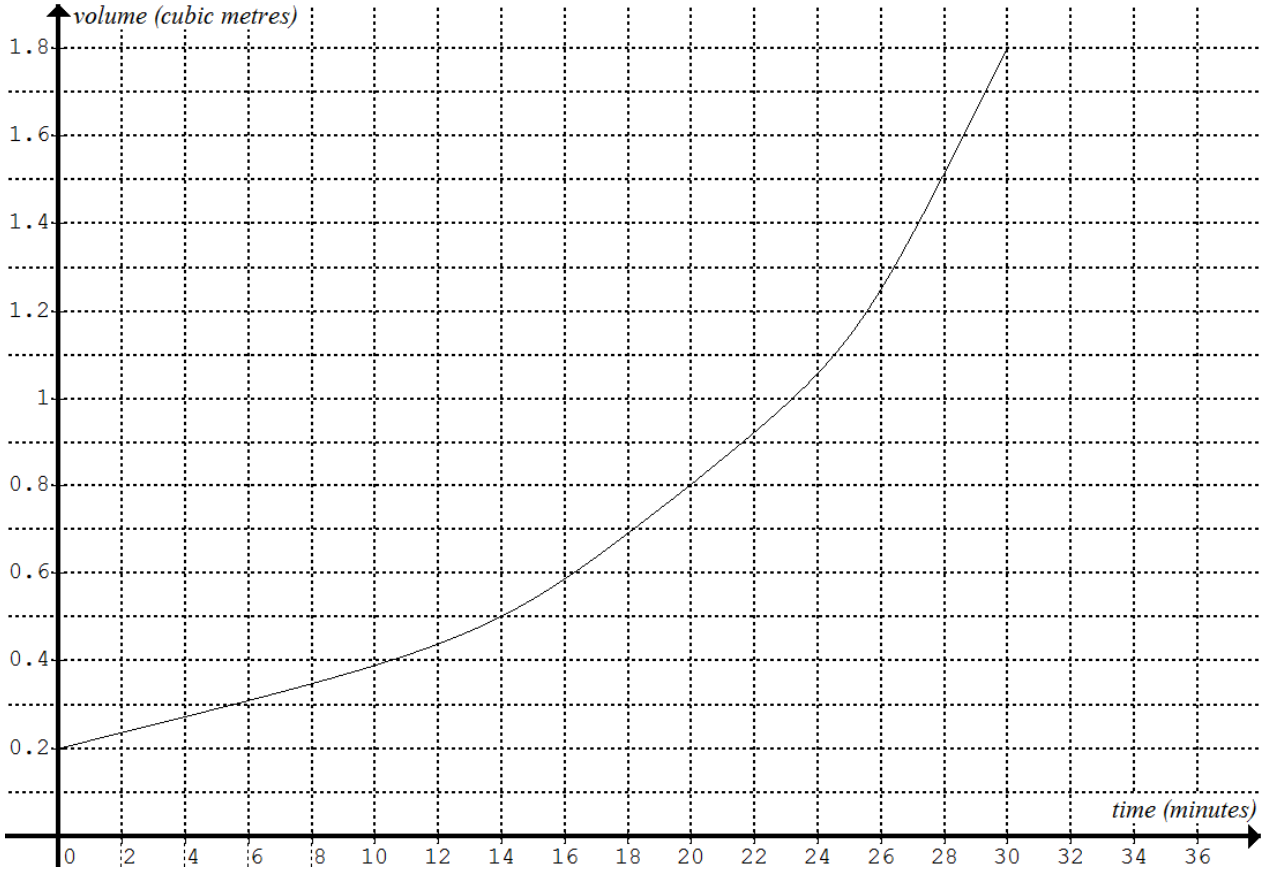
Introduction - to be done in class (at home) – not for assessment

This part of the task (SAC) *will require you to apply your knowledge of different mathematical functions to model a given curve, and reflect on your results. This is an introductory part only.*

The process of papermaking uses a large amount of water, so minimising water loss and recycling is important. For testing purposes, the water leaking from a particular papermaking machine was collected and stored in a tank.

Every minute, over a 30 minute period, $t \in [0, 30]$, the volume of liquid in the tank $(V\ m^3)$ was measured by lab technicians. The data points were plotted and a smooth curve was drawn, as shown. (The data points are not shown).



As a first attempt at modelling the curve, the following piecewise (hybrid) function was tried:

$$V = \begin{cases} 0.02t + 0.1 & 0 \le t < 10 \\ 0.02t + 0.3 & 10 \le t < 20 \\ 0.11t - 1.5 & 20 \le t \le 30 \end{cases}$$

(a) Sketch and label the piecewise function on the graph.

This first model is obviously not a good fit. Let's explore a commonly-used statistical measure to indicate how well the model fits the underlying data – the Residual Sum of Squares.

The Residual Sum of Squares (RSS) measures the overall difference between actual data and the values predicted by an estimation model. The differences are then squared and then added.

(b) For the above model, find the residuals (difference in volumes between the actual data and the model), square and then sum them. Use the following sampling times: $t = 0, 5, 10, 15, 20, 25, 30$. To assist, one column has been pre-filled. (Work to 4 decimal places).

t	0	5	10	15	20	25	30
value read from curve		0.29					
model $V(t)$		0.20					
difference (Δ)		0.09					
Δ^2		0.0081					

$RSS = \sum \Delta^2$ - sum the Δ^2 values RSS =

(c) Use the following piecewise function to better model the leakage curve. Determine the values of the coefficients: a, b, c, d, e and f. State your final function.

$$V = \begin{cases} at + b & 0 \le t < 14 \\ ct + d & 14 \le t < 24 \\ et + f & 24 \le t \le 30 \end{cases}$$

(d) Clearly sketch and label your piecewise function on the same graph.

(e) For your model, find the RSS, using the same sampling times. (Work to 4 decimal places).

t	0	5	10	15	20	25	30
value read from curve							
model $V(t)$							
difference (Δ)							
Δ^2							

$RSS = \sum \Delta^2$ - sum the Δ^2 values RSS =

(f) Which one of the two piecewise straight-line functions gives the best fit to the original curve? How did you decide which one is the best fit?

(g) Give a brief explanation as to why you think the residuals are squared first, before being summed.

(h) What does an RSS of 0 indicate?

BRING THIS BOOKLET TO THE NEXT PART OF THE TASK