Camberwell Girls Grammar School
An Anglican School

CREATE YOUR TOMORROW

Student Name			
Teacher	Mr Trufitt	Ms Tan	Ms Bergamin

MATHEMATICAL METHODS UNIT 3

Application Task – Curve fitting

PART 1 – Evaluating different models

Date: May 2020

Writing time: 60 minutes (one on-line class)

Structure of Task

Section	*Number of questions*	*Number of questions to be answered*
Part 1	3	3

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one CAS calculator and/or one scientific calculator, and one approved bound reference.
- Students are not permitted to use: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 6 pages.
- Working space is provided throughout the booklet.

Instructions
- Write your name in the space provided above on this page.
- All responses must be written in English.

Stu dents are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

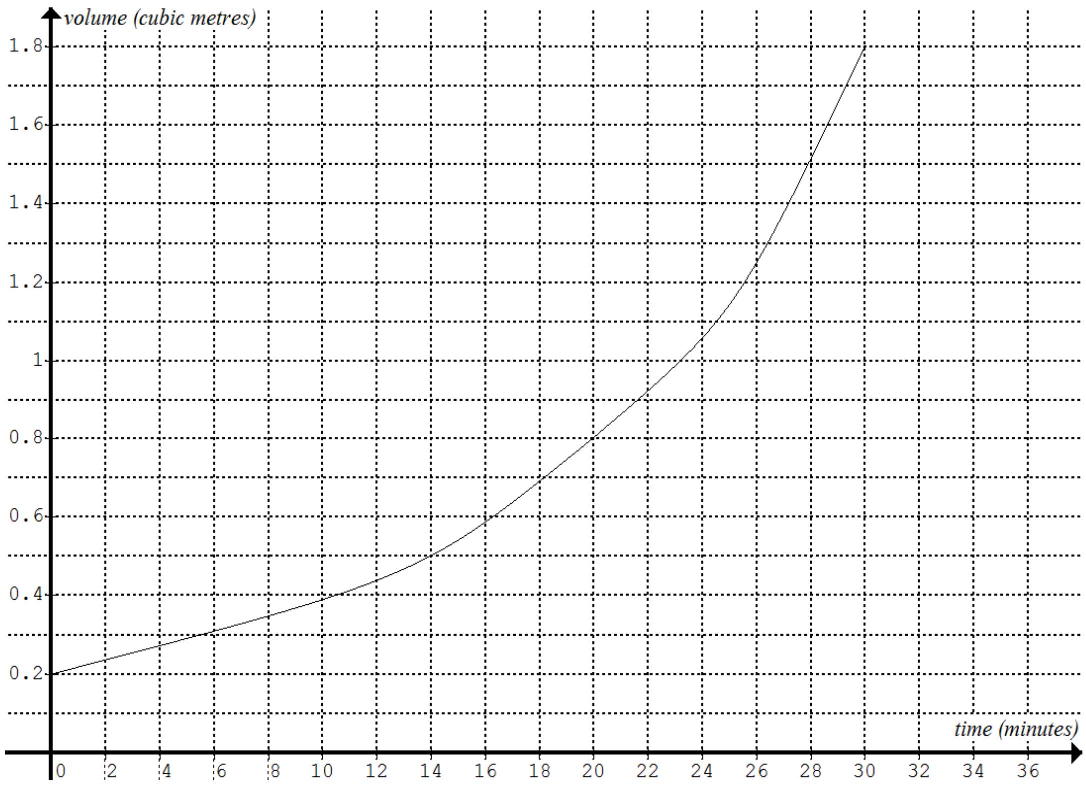
Students must not disclose the contents of the task; to do so will be a breach of School guidelines.

Part 1

This part of the task will require you to further apply your knowledge of different mathematical functions to better model a given curve, and again reflect on your results.

Question 1

(a) To extend and hopefully improve the previous model, use another piecewise-function, but with 4 linear branches to best fit the original curve, reproduced below. Choose appropriate intervals for each branch. Clearly sketch and label your piecewise function.



(b) For this model, find the Residual Sum of Squares (RSS), using the following sampling times, as you did in the introductory part. (Work to 4 decimal places).

t	0	5	10	15	20	25	30
value read from curve (given)	0.2	0.29	0.38	0.55	0.8	1.15	1.8
model $V(t)$							
difference (Δ)							
Δ^2							

RSS =

(c) Briefly compare the RSSs from the 3-branch and the 4-branch piecewise models. Do you believe that the 4-branch model is an improvement?

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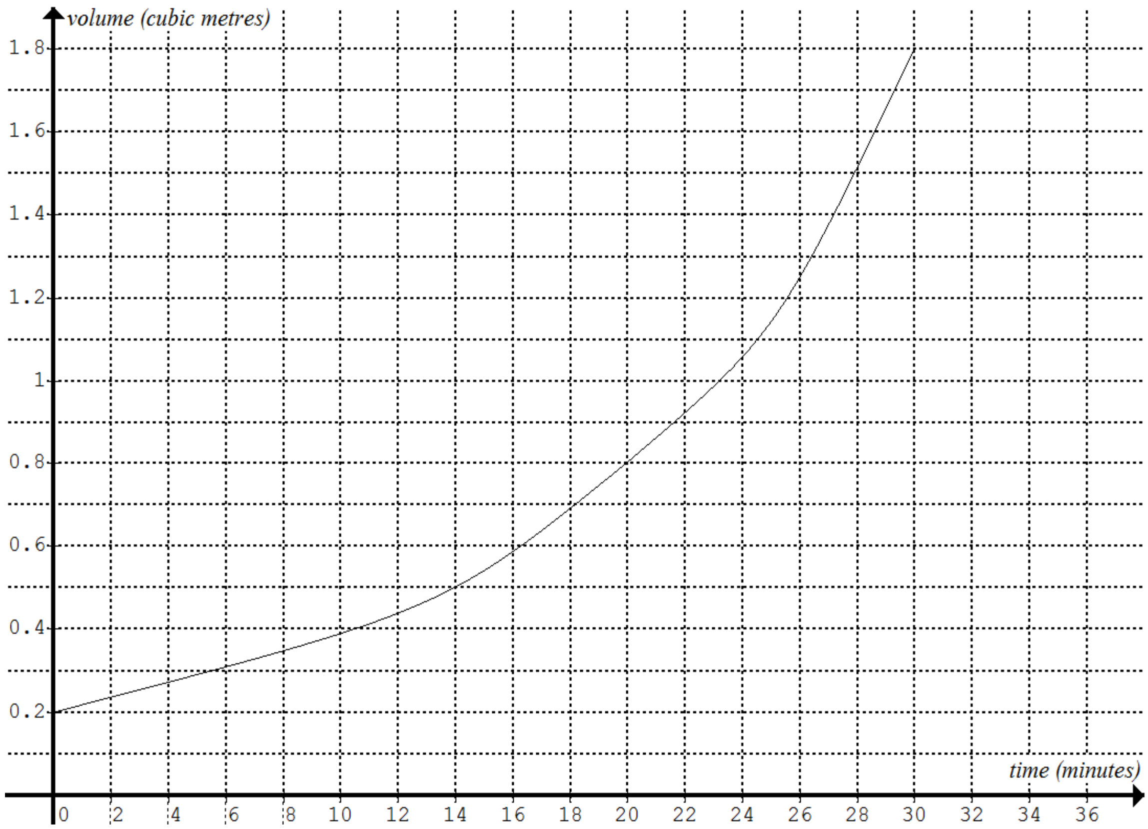
Question 2

The piecewise-linear model of the water leakage curve has some weaknesses, not least that piecewise models with a large number of multiple-branches are clumsy to work with. The technicians now apply a quadratic function, with appropriate transformations, to, hopefully improve the model.

New model: $y = at^2 + c$

(a) Selecting the end-points of the curve, find appropriate values for a and c, giving final values to 4 decimal places. State your new model. Clearly sketch and label on the following diagram



(b) For the quadratic model, find the Residual Sum of Squares (RSS), using following sampling times. (Work to 4 decimal places).

t	0	5	10	15	20	25	30
value read from curve (given)	0.2	0.29	0.38	0.55	0.8	1.15	1.8
model $V(t)$							
difference (Δ)							
Δ^2							

RSS =

(c) A modified quadratic function, with appropriate transformations is now applied.

Newer model: $y = a(t+10)^2 + c$

Find appropriate values for a and c, giving final values to 4 decimal places. State your new model. Clearly sketch and label on the previous diagram

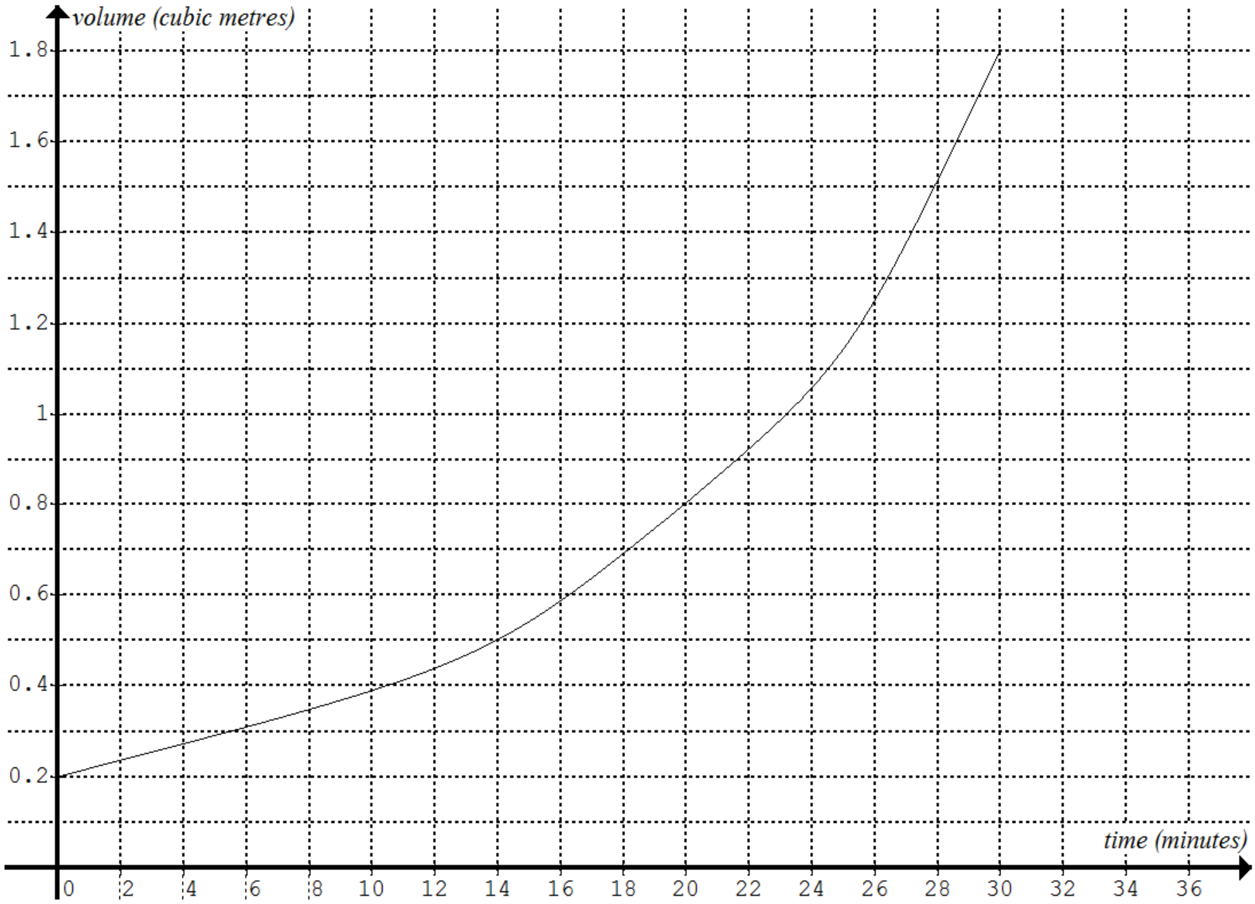
(d) Were you able to improve on previous linear models? Comment briefly

Question 3

A staff member now proposes a model using a power function of the form: $y = a(t+10)^n + c,\ n \in Q$.

Repeat the modelling process, trying greater n values that **may** give a better fit to the original curve. Use your n value to determine the values of a and c. State your models. Comment on any unexpected occurrences or the **limitations of your models**. Give a to 6 decimal places.

(*A copy of the leakage curve, to assist with your working is reproduced over the page*).


volume (cubic metres)
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
time (minutes)
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36

End of part 1