Camberwell Girls Grammar School
An Anglican School

CREATE YOUR TOMORROW

Student Name			
Teacher	Mr Trufitt	Ms Tan	Ms Bergamin

MATHEMATICAL METHODS UNIT 3

Application Task – Curve fitting

PART 2 – Exploring alternate models

Date: May 2020

Writing time: 60 minutes (one on-line class)

Structure of Task

Section	*Number of questions*	*Number of questions to be answered*
Part 2	2	2

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one CAS calculator and/or one scientific calculator, and one approved bound reference.
- Students are not permitted to use: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 7 pages.
- Working space is provided throughout the book.

Instructions

- Write your name in the space provided above on this page.
- All responses must be written in English.

Stu dents are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students must not disclose the contents of the task; to do so will be a breach of School guidelines.

Part 2

The shape of the original curve suggests that an exponential function with appropriate transformations could better fit the curve.

Before transformations: $y = 2^t$

After transformations: $y = a(2)^{bt} + c$

Question 1

(a) State, in correct order, the transformations represented by parameters a, b and c.

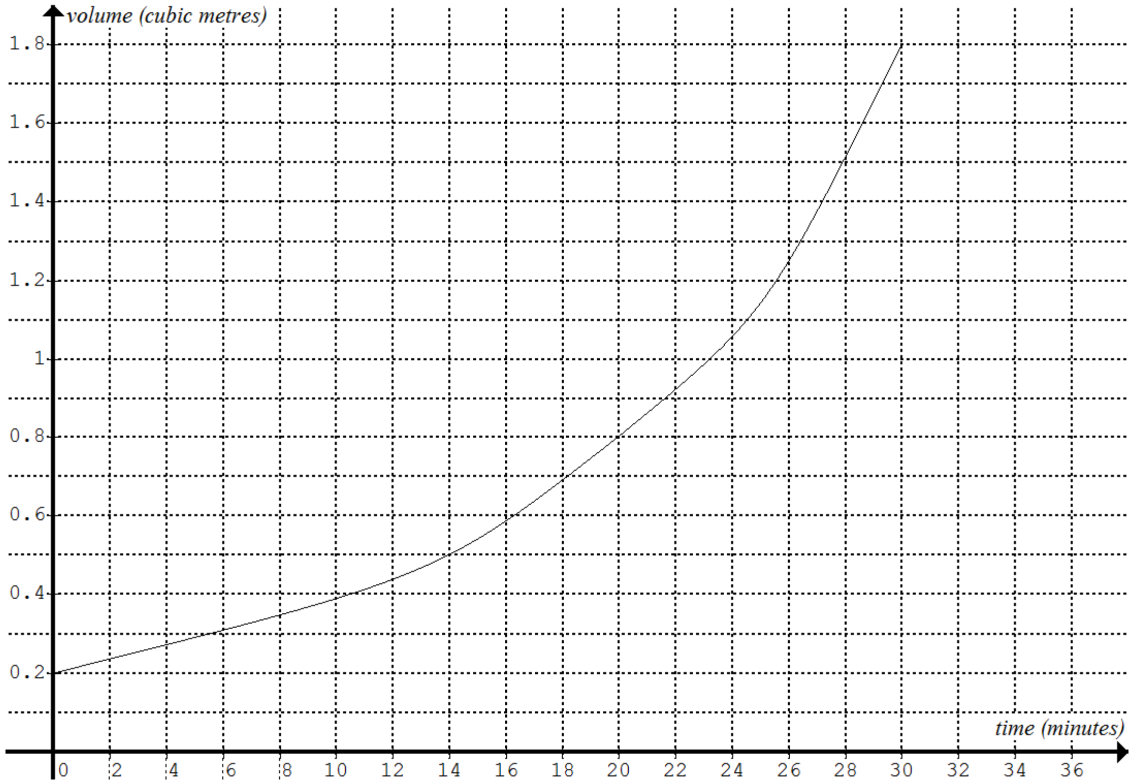
(b) Three pieces of information are required to evaluate the parameters a, b and c. Use the two end points and the point $(20, 0.8)$ on the curve to set up three simultaneous equations involving the parameters. State the equations.

(*Another copy of the leakage curve is reproduced on page 4*)

(c) Show that $\dfrac{(0.8-c)^3}{(0.2-c)(1.8-c)^2}=1$.

(d) Hence, **use any technique** to show that $a \approx 0.13,\ b \approx 0.1245$ and $c \approx 0.07$.

(e) Sketch the graph of $v = a(2)^{bt} + c$ on the diagram below, using the above values of a, b and c.



(f) Is this an improvement on the previous models? Comment briefly

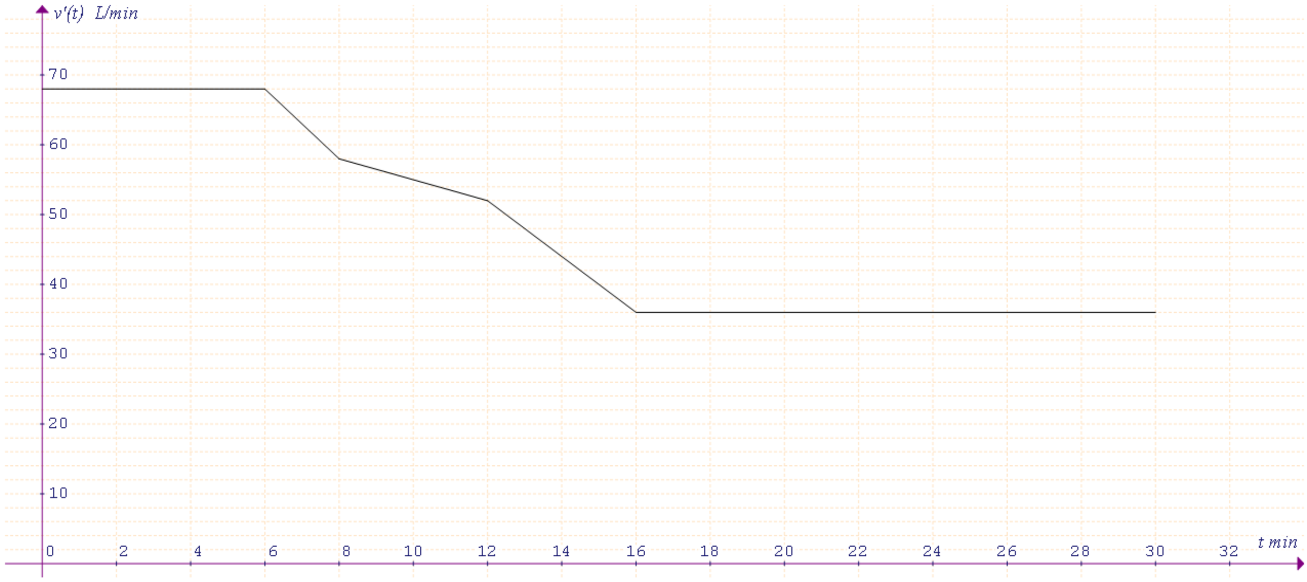
As we have already seen, the process of papermaking uses a large amount of water, so minimising water loss is important. Some modifications were made to the machine and further testing of water leakage was undertaken. This time, the **rate of change** of volume with respect to time was measured and plotted.

READ THIS

As you may have already seen in Methods 1/2, the area under a velocity-time graph gives the displacement. Similarly, the area under a rate of change of volume graph will give you volume. More specifically:

the area bounded by the $v'(t)$ graph, the time axis and times $t=a$ and $t=b$, gives the volume of water used between those two times.

$$(1m^3 = 1000L)$$



Question 2

(a) Over the 30-minute sampling period, have the modifications made a difference to the water loss? Show working and **explain your reasoning**.

Unhappy with the changes so far, the technicians made modifications to the model to further reduce water loss.

They installed a device that activates **after** 4 minutes, then runs for a certain amount of time to slow the rate of water loss. The device is not able to reduce the rate of water loss by any greater than 10 L/min/min. It also cannot reduce the rate of water loss below 25 L/min. Once 25 L/min is reached, the device switches off.

Initially the rate of loss was 65 L/min. The test engineer requires that less than 1000 litres of water are lost over the 30 minute period.

The results of a test run are shown:

(b) Investigate the model with the added device so that the specifications above are met. Show working and your reasoning. State how long the device was switched on for.

End of part 2