UNIT 3 MATHEMATICAL METHODS
SAC 1, 2019 - PART A

Name: _____

Teacher: **APW** **JDR** **MRC** **NJM** **REC**

Instructions

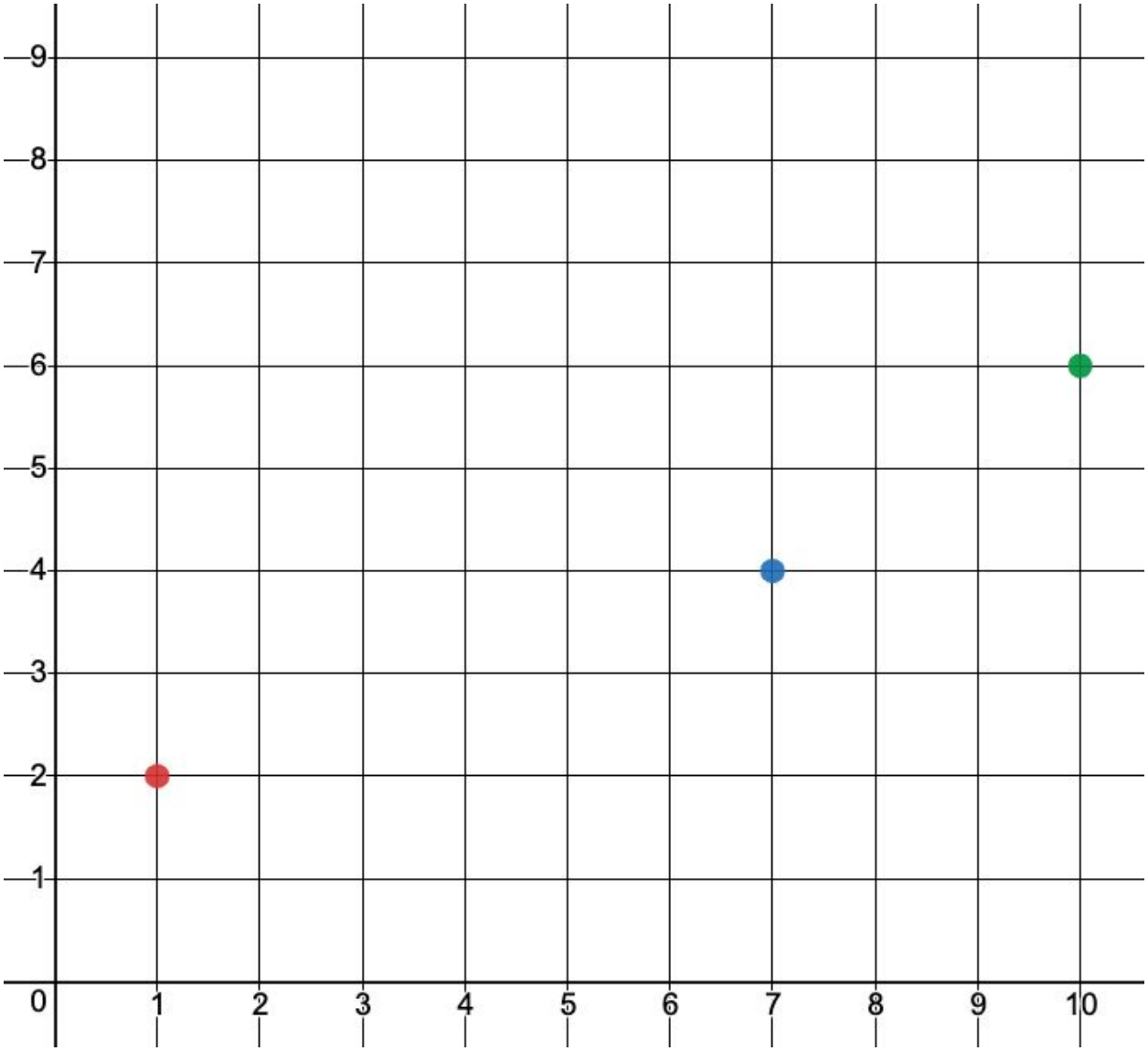
- This is a take-home assignment due at 8:45 am on Wednesday 8th of April. You should hand your submission to your classroom teacher.

- You are permitted to consult your notes and use your CAS calculator while completing this task.

Part A (26 marks)

The Boring Company is an infrastructure and tunnel construction company. For this task you will act in the role of an engineer making a mathematical model for a proposed test tunnel. This tunnel is designed to allow transport sleds to move in one direction between a series of stops or stations.

Modelling the tunnel's route on a Cartesian plane, the first section of the tunnel will join Station A at point (1,2) with Station B at point (7,4). There is another station, Station C, located at (10,6).



Question 1 (4 marks)

If the tunnel is dug in a straight line from A to B,

a. complete the function definition below with a domain and rule that follows this route.

$$f : \_\_\_\_\_ \rightarrow R,\ f(x) = \_\_\_\_\_$$

3 marks

b. calculate the length of this tunnel.

1 mark

Question 2 (7 marks)

The engineering team is also considering modelling the proposed tunnel route from Station A to Station B so that it follows the rule:

$$g(x) = a \log_e(x + b)$$

a. Show that $b = 2$ and hence $a = \frac{2}{\log_e(3)}$.

3 marks

b. Sketch g on the axes on the first page over an appropriate domain.

2 marks

c. Write down the derivative of g with respect to x and evaluate this derivative when $x = 7$.

2 marks

Question 3 (6 marks)

The second section of the tunnel, from Station B to Station C at point (10,6). It is modelled by the rule:

$$h(x) = m\, e^{j\, x} + k$$

a. For the sled to move safely through the tunnel at speed, the two tunnel sections must be facing the same direction when they meet. This is equivalent to the derivatives of each section at that point being equal.

   Using your answer from **Question 2 part c.**, find the values of m, j and k, to 3 significant figures.

   [Note: some CAS calculators experience problems solving the set of equations implied by this question. If this happens to you, try setting a domain constraint so that m>0.]

6 marks

Question 4 (9 marks)

Both sections of the tunnel (i.e from Station A to B to C) can be modelled by a single quadratic function.

a. Write down the rule for this function in the form $y = ax^2 + bx + c$. (Stating the domain is not required.)

3 marks

b. Sketch this quadratic on the axes on the first page over an appropriate domain.

2 marks

c. Imagine that the tunnel modelled by this quadratic has been partly built, from Station A to Station B. If the tunnel modelled by exponential function in **Question 3** is built to complete the section from Station B to C explain whether the sled can move safely down the tunnel at speed, using calculations to support your answer.

4 marks