UNIT 3 MATHEMATICAL METHODS
SAC 1, 2019 - PART B

Reading time: N/A

Writing time: 35 minutes

Marks available: 17

Name: ______________________________

Teacher: **APW** **JDR** **MRC** **NJM** **REC**

Instructions

During the assessment there should be no items on the desk other than those noted here:

- Pens
- Pencils
- Highlighters
- Eraser
- Sharpener
- Ruler
- This Assessment paper

Unless stated, diagrams shown are not to scale.

Write your **name** in the space provided on the front page of this paper **circle your teacher's initials**.

All questions are to be answered in the spaces provided.

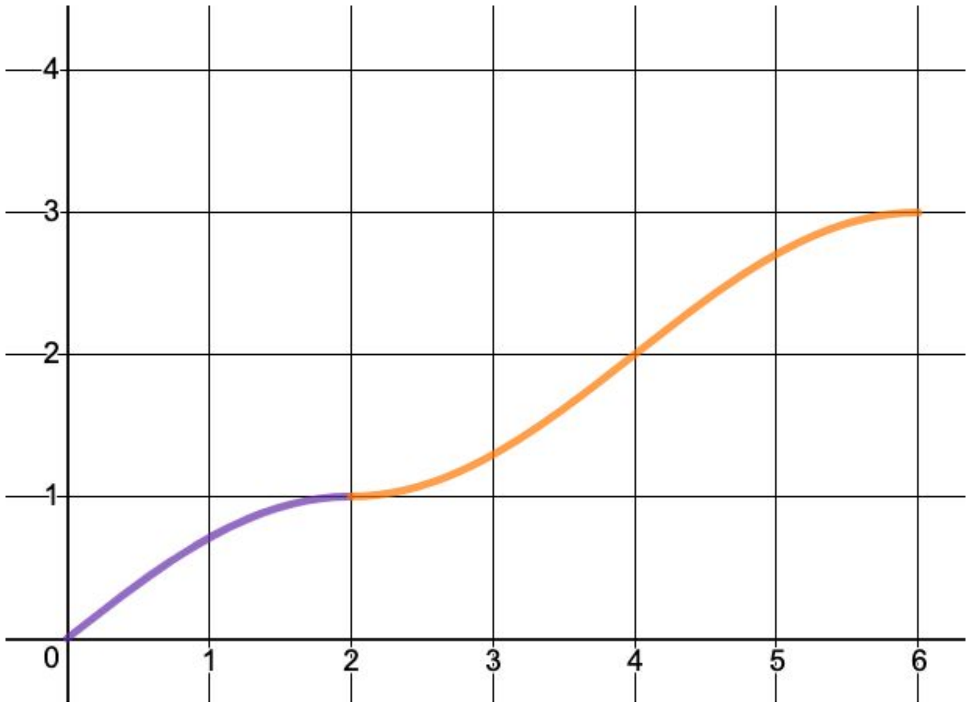
Answers must be in blue or black ink or grey lead pencil.

Students should NOT bring mobile phones and/or any other electronic communication devices into the assessment room.

Part B (17 marks)

Question 1 (6 marks)

Engineers at the Boring Company are continuing to plan for new tunnels to test their digging machine. Another proposed model is sketched below. This model uses two sine graphs with different restricted domains.



a. Consider a rule *p(x)* such that the first (purple) section of this tunnel can be modelled by:

$$p : [0, 2] \rightarrow R,\ p(x) = \sin(kx)$$

Given that there is a stationary point at (2,1), write down the value of *k*.

1 mark

b. Solve the equation $p(x) = \frac{1}{2}$.

2 marks

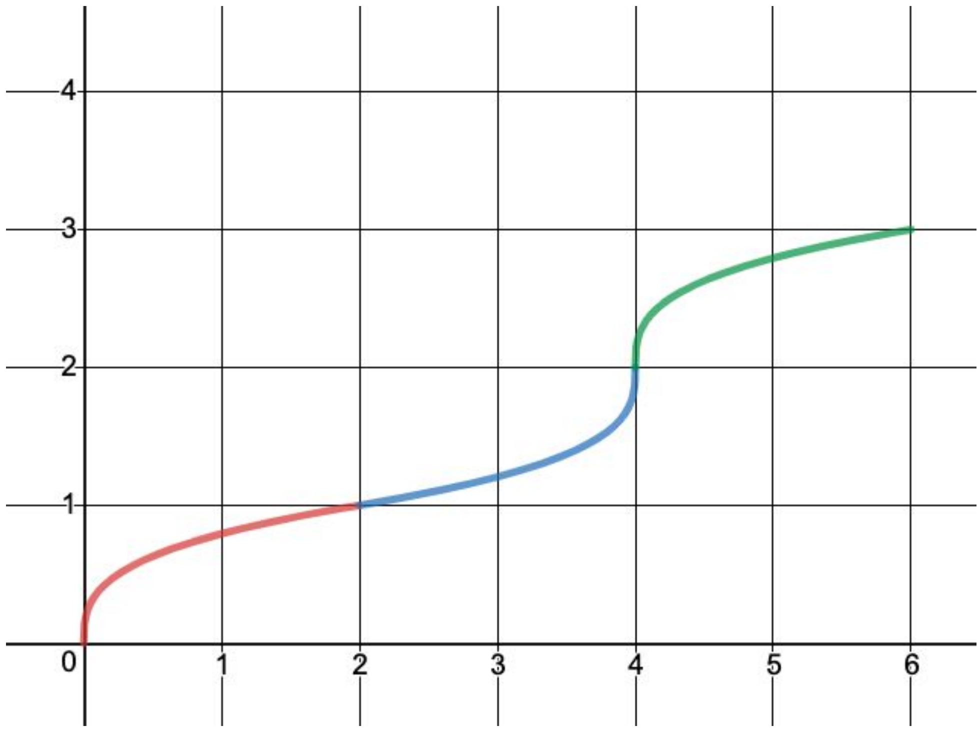
c. Consider the graph of $y = \sin(x)$ (not shown) . Describe how the graph of $y = \sin(x)$ could be transformed to the graph of the third (yellow) section of the tunnel.

Write your answer as a transformation matrix equation.

3 marks

Question 2 (11 marks)

The latest proposal is to build tunnels using the same section repeated. For instance, the tunnel model sketched below shows three separate functions, where the shape of the first (red) section is repeated twice for the second and third (blue and green) sections.



The model for the first (red) section is given by the function

$$r:\ [0,2] \rightarrow R,\ r(x) = \sqrt{\frac{x}{2}}$$

The model for the second (blue) section is given by the function

$$b:\ [2,4] \rightarrow R,\ b(x) = 2 - \sqrt{\frac{4-x}{2}}$$

a. Find a sequence of transformations that takes the graph of $r(x)$ to the graph of $b(x)$.

4 marks

b. Write down the rule and domain for the model of the third (green) section $g(x)$.

3 marks

c. Remember that for the sled to move safely through the tunnel at speed, two sections must be going in the same direction when they meet. This is equivalent to their derivatives at that point being equal.

By calculating derivatives, determine whether the sled can move safely at speed from the first (red) section to the second (blue) section.

4 marks

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) \quad nx^{n \;\; 1}$	$\int x^n dx \quad \frac{1}{n+1}x^{n+1}+c, \;\; n \neq \;\; 1$
$\frac{d}{dx}\left((ax+b)^n\right) \quad an(ax+b)^{n \;\; 1}$	$\int (ax+b)^n dx \quad \frac{1}{a(n+1)}(ax+b)^{n+1}+c, n \neq \;\; 1$
$\frac{d}{dx}\left(e^{ax}\right) \quad ae^{ax}$	$\int e^{ax} dx \quad \frac{1}{a}e^{ax}+c$
$\frac{d}{dx}\left(\log_e(x)\right) \quad \frac{1}{x}$	$\int \frac{1}{x}dx \quad \log_e(x)+c, \;\; x>0$
$\frac{d}{dx}\left(\sin(ax)\right) \quad a\;\cos(ax)$	$\int \sin(ax)dx \quad \frac{1}{a}\cos(ax)+c$
$\frac{d}{dx}\left(\cos(ax)\right)= \quad a\;\sin(ax)$	$\int \cos(ax)dx \quad \frac{1}{a}\sin(ax)+c$
$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)} \quad a\;\sec^2(ax)$	

product rule	$\frac{d}{dx}(uv) \quad u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) \quad \frac{v\frac{du}{dx} \quad u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		