Name:

Teacher: **AP** **SL** **ZZ**

Ivanhoe Girls' Grammar School

School Assessed Coursework

Mathematical Methods (CAS) Unit 3

Application Task

Part 1 – Question and Answer Booklet

Number of questions: 2

Writing time: 50 minutes

Instructions for students:

- Students are permitted to bring one bound reference, one approved CAS calculator and if desired one scientific calculator.
- This SAC is in three parts. This is part 1 of 3.
- Answer all questions in the spaces provided.
- Unless specified, give all answers as exact values.

At the completion of this part your bound reference and this question and answer booklet will be collected and returned to you during part 2 of this task.

Water World is a new theme park opening in Melbourne. The consultancy firm Willa-Wonda Workers have hired you to help them design and model some of the attractions they are planning.

Question 1

The first attraction you have been asked to model is a section of the kids' pool. The pool is to be created in a rectangular field block of land with a grassed area and a pool (water area). The boundary between the grass and water will be paved and needs to be modelled with a mathematical function. The landscape of the pool is shown below in Figure 1, where x and y represent the distance in meters from the corner of the pool label O.

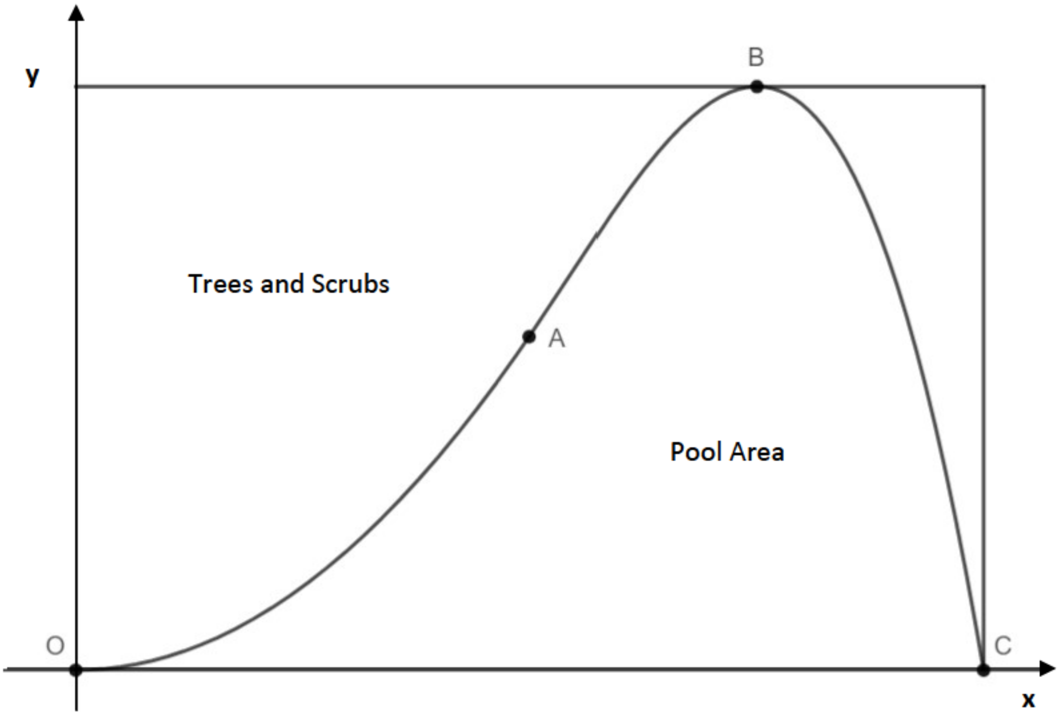

Figure 1

The boundary, separating the pool area (the area between the curve and the *x*-axis) from the trees and shrubs, is made of two curves, by the function.

$$f(x) = \begin{cases} \dfrac{x^2}{10} & for \quad 0 \leq x \leq x_A \\ ax^3 + bx^2 + cx + d & \quad\;\; x_A < x \leq 20 \end{cases}$$

The first part of the function will be modelled with a quadratic function need to meet the cubic function at $y = 10$.

a Show that x_a, the x-coordinate where the two functions meet will be 10.

(1 mark)

To ensure that the boundary built between the pool and grassed area is safe and aesthetically pleasing the two sections of the curve must **meet smoothly**. To meet smoothly, the functions must meet at the same point and the gradient of the two functions must also be equal at the join.

The x-coordinate of B, the maximum of the cubic, is halfway between the x coordinates of A(10,10) and C(20,0) and is the furthest point from the x-axis reached by the boundary of the pool and the grass as measured in the positive *y*-direction.

b Use the above information to write down four equations which could be used to find the values of *a*, *b* *c* and *d*. Show that $a = -\frac{1}{25}, b = \frac{13}{10}, c = -12$ and $d = 40$.

(3 marks)

c What is the distance reached from the positive x-axis by the furthest point of boundary of the pool and the grass as measured in the positive *y*-direction.

(1 mark)

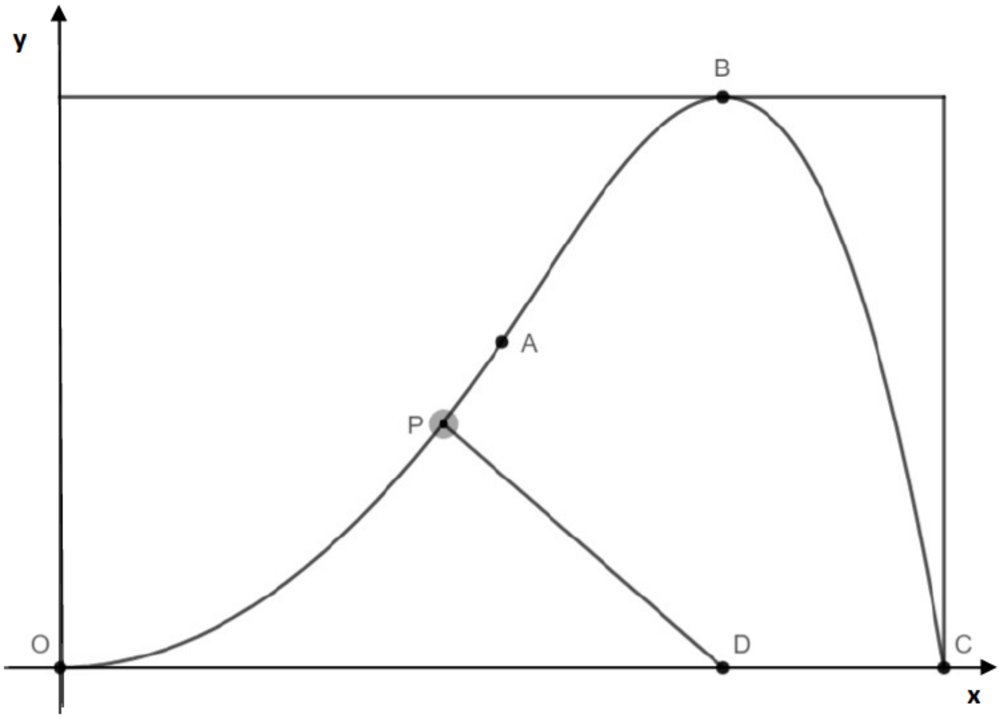

Figure 2

d Willa-Wonda Workers decides that the with a pool this big they need to create a bridge to allow pedestrian to walk across the pool without having to go all the way around. The path will begin at $D(15,0)$ and ends at P, along the boundary of the pool in a straight-line path.

i. If the *x*-coordinate of P is *p* and $0 < p < 10$, find an expression in terms of *p*, for the length of the path from Point D to the boundary.

(2 marks)

ii. If Willa-Wonda wants this path to be as short as possible, find the value of *p* and the length of the path (You are not required to show that this is a minimum). Give your answers correct to two decimal places.

(2 marks)

Question 2

Willa-Wonda Workers wants to create the tallest water slide that has ever been built. To start the design a cubic function will be used to model the tallest section of the waterslide. The graph with the function $w(x) = \frac{1}{5}(x+1)(x-4)^2 + 1$ for $0 < x \leq 4$ represents the height of the water slide in meters as a function of x, the horizontal distance in meters from the starting point.

a Use a by hand method to show that derivative of $w(x)$, $w'(x)$ is $\frac{1}{5}(x-4)(3x-2)$.

(2 marks)

b Find the co-ordinates of the turning points of the graph of $y = w(x)$.

(2 marks)

c Hence or otherwise find the values of n for which the equation $w(x) = n$ has three distinct solutions.

(2 marks)

To ensure that the water slide is safe for customers the slide cannot be too steep.

d Determine the gradient of the waterslide at $x = 2$.

(1 mark)

e What angle will a tangent to the function at $x = 2$ make with the positive direction of the x-axis? Give your answer in to the nearest degree.

(2 marks)

f For what value of x will the gradient of the waterslide be largest?

(1 mark)

The waterslide will be deemed safe if the slide if it does not create an angle greater 60 degrees from the ground.

g Will this waterslide be deemed safe?

(3 marks)

To scale the model to the desired height and ensure that the slide will be the tallest ever built the height must reach 52m.

h The graph of $y = w(x)$ is dilated by a factor of k from the x-axis to form another function $W(x)$ so that the vertical distance from the ground to its maximum is 52 units. Find this value of k.

(1 mark)

End of Part 1

Name:

Teacher: **AP** **SL** **ZZ**

Ivanhoe Girls' Grammar School

School Assessed Coursework

Mathematical Methods (CAS) Unit 3

Application Task

Part 2 – Question and Answer Booklet

Number of questions: 2

Writing time: 50 minutes

Instructions for students:

- Students are permitted to bring one approved CAS calculator and if desired one scientific calculator.
- Students are NOT permitted to bring extra notes of any kind.
- This SAC is in three parts. This is part 2 of 3.
- During this task you will receive part 1 and part 2 of this question and answer booklet as well as your bound reference.

At the completion of this part your bound reference and part 1 and 2 of the question and answer booklet will be collected and returned to you during part 3 of this task.

Question 3

Willa-Wonda Workers decides that the design they wish to go ahead with for the section of the water slide is $w(x) = \frac{1}{5}(x+1)(x-4)^2 + 1$ for $0 \le x \le 4$, where x is the horizontal distance along the rollercoaster in meters and w is the height of the slide in meters.

The chief engineer wants to investigate the possibility of supporting the tracks using vertical supports spaced 1 meter apart, starting at $x = 0$ as illustrated in the diagram below.

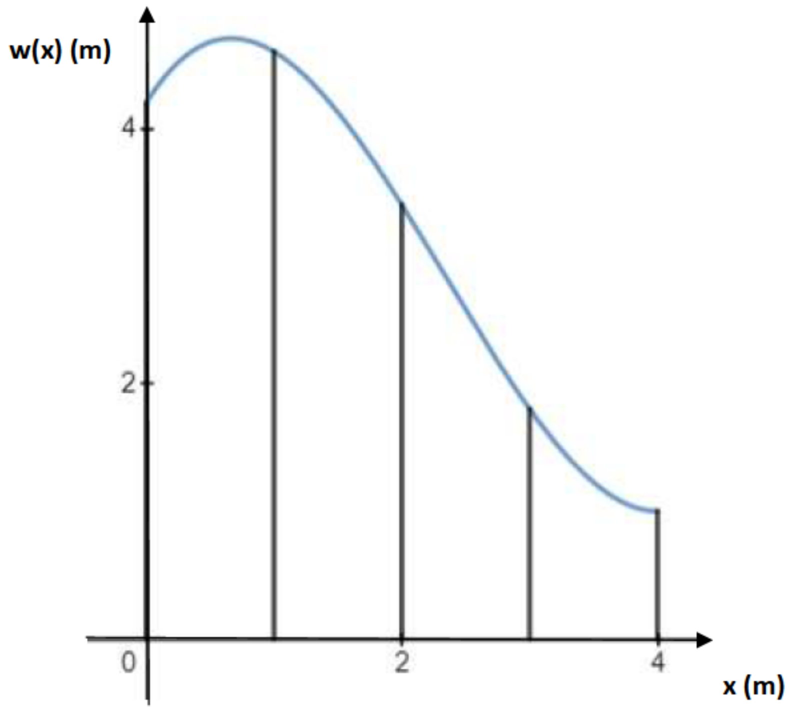

Figure 3

a What is the length of the support at $x = 0$?

b What is the total length of the supports?

The cost of the supports depends on the length of the support required and follows the hybrid function, $C(l)$ in thousands of dollars, where l is the length of the individual supports in meters.

$$C(l) = \begin{cases} 2l \\ \sqrt{l+6}+3 \end{cases} for \begin{matrix} 0 \leq l < 3 \\ 3 \leq l \end{matrix}$$

c What will be the cost of the support at $x = 0$?

d What will the total cost of the supports be? Give your answer to the nearest dollar.

Question 4

Willa-Wonda Workers' next job for you is to model and optimise the boat that people will use to go down the slide. The design being considered is an open half cylinder.

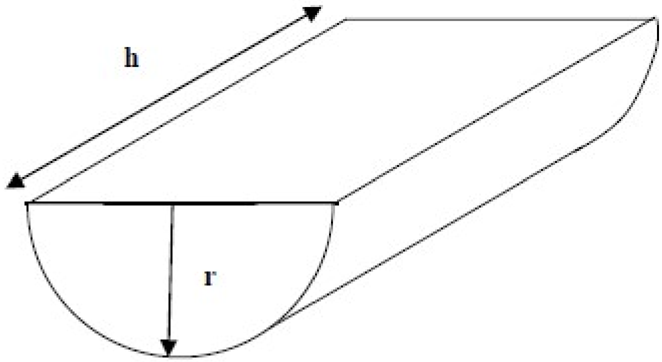

Figure 4

The volume and surface area of the cylinder design are:

$$V = \frac{\pi r^2 h}{2}$$

$$A = \pi r(r + h)$$

Where r and h, the radius and height, are the dimensions of the boat measured in meters. The volume of each boat must be at least $1\ m^3$ to ensure that it will adequately fit an adult.

a Setting the volume of the boat will be $1\ m^3$, find an expression for h in terms of r.

b Hence show that $A = \pi r^2 + \frac{2}{r}$, if the volume of the boat will be $1\ m^3$.

c Using calculus, find the value of r for which this area is a minimum.

d Write down the minimum surface area and the dimensions of the boat. Give your answers correct to 2 decimal places.

e Will these dimensions for the boat be reasonable for people to use? Provide justification for your response.

The cost of creating this boat is a function of the flat surface area and the curved surface area as it is more difficult to create and produce. The costs of the flat and curved surface area are $\$\boldsymbol{p}$ per m^2 and $\$\boldsymbol{q}$ per m^2 respectively. As a result, the cost of the boat is defined by the function.

$$C(r) = \boldsymbol{p}\,\pi r^2 + \frac{2\,\boldsymbol{q}}{r}$$

f Find $C'(r)$ in terms of $\boldsymbol{p}$ and $\boldsymbol{q}$.

g For what value of r will the cost be a minimum? Give your answer in terms of $\boldsymbol{p}$ and $\boldsymbol{q}$.

h If $\boldsymbol{p = 10}$ and $\boldsymbol{q = 30}$ what will the minimum cost of each boat be? Give your answer to the nearest cent.

i If $\boldsymbol{p = 10}$ and $\boldsymbol{q = 30}$ what will the radius and height of the boat be for the cost to be a minimum?

Please turn over

j Will these dimensions for the boat be reasonable for people to use? Provide justification for your response. What suggestions might you make to Willa-Wonda Workers to ensure the best design for the boat?

End of Part 2