VICTORIAN CERTIFICATE OF EDUCATION
2016

NAME:

MATHEMATICAL METHODS UNIT 4

School Assessed Coursework 3 (SAC 3):

Problem-Solving Task 2 (Probability and Statistics) Section 1

Die Hard With a Random Variable

Reading Time: 10 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
5	5	40

- SAC 3 Problem-Solving Task 2 Section 1 consists of one extended-response question and four short answer questions.
- Students are permitted to bring into the assessment room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aides for curve sketching, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the assessment room: notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 11 pages, sheet of miscellaneous formulas.
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the assessment room.

Instructions

Answer all questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.

Bruce Willis is in training for his new movie Die Hard With a Random Variable. *The idea for the movie came to him during* Die Hard 2 *when his character John McClane mused*

"Oh man, I can't fracking believe this. Another basement, another elevator. How can the same Scheisse happen to the same guy twice?"

In order to be (at least a little bit) believable in his role, Bruce has to learn some probability and statistics. The training is brutal and uncompromising. After a particularly ferocious training session that leaves Bruce in a pool of sweat and barely able to breath, his personal trainer, Kermmando, decides to take him to the local Droolworths defmarket to get a packet of feijoas as a special treat.

Question 1 (26 marks)

At the *Droolworths* defmarket the number X of feijoas in a **large** packet is a discrete random variable with probability distribution as shown in the following table:

x	14	15	16	17
$\Pr(X = x)$	$\frac{1}{12}$	$\frac{1}{2-k}$	$\frac{5}{21}$	$\frac{1}{4}+k$

a. Find all possible values of k. 3 marks

Question 1 - continued

The *Droolworths* defmarket also sells **small** packets of feijoas. The number Y of feijoas in a **small** packet is a discrete random variable with probability distribution as shown in the following table:

y	6	7	8	9
$\Pr(Y = y)$	0.21	0.38	0.23	0.18

b. Bruce buys two small packets of feijoas.
Find the probability, correct to four decimal places, that

i. each packet has 8 feijoas in it. 1 mark

ii. the total number of feijoas in both packets is equal to 14. 2 marks

iii. if the total number of feijoas in both packets is equal to 14 then one of the packets has 6 feijoas in it. 3 marks

c. The following day, after another arduous training session, Bruce buys six small packets of feijoas from the *Droolworths* defmarket.
Find the probability, correct to four decimal places, that

i. at least four of the packets have 8 feijoas. 2 marks

ii. if at least four of the packets have 8 feijoas then at least five of the packets have 8 feijoas. 3 marks

Before training on the following day, Bruce buys a number of small packets of feijoas from the *Droolworths* defmarket in anticipation of another strenuous training session.

d. Find the **smallest** number of packets that Bruce could have bought if the probability that at least half of his packets have less than 8 feijoas is greater than 0.86. 3 marks

The length L of feijoas sold at the *Droolworths* defmarket is a normally distributed random variable with a mean of 2.2 cm and a standard deviation of 0.4 cm. A **regular** feijoa at *Droolsworth* has a length of at least 2 cm. This gives Kermmando some ideas for testing Bruce.

e. Find, correct to four decimal places, the probability that:

i. a randomly chosen feijoa is regular. 1 mark

ii. a regular feijoa is longer than 2.2 cm. 2 marks

iii. there are more than 5 regular feijoas in a small packet containing 8 feijoas. 2 marks

A feijoa is also considered **large** if it has a length greater than b where $\Pr(L > b) = 0.25$.

iv. Find, correct to 2 decimal places, the value of b. 1 mark

Koles defmarket is the arch-rival of *Droolworths*. The length of feijoas sold at the *Koles* defmarket is a normally distributed random variable with a standard deviation of 0.5 cm. 83% of all feijoas sold at the *Koles* defmarket have a length greater than 1.9 cm.

f. Find, correct to three decimal places, the mean length of feijoas at the *Koles* defmarket. 3 marks

Many weeks pass. Until finally:

"Bruce!"

Kermmando shouts.

"You're almost ready! You're almost ready to show the world that not only are you a tough guy, but you also know your probability and statistics!! You have one final test!

Kermmando pointed to the dirt on the ground. The dirt was cold and wet because it was raining. There was strange writing in the dirt. Bruce didn't like the look of it. It looked unintelligible. It reminded him of how Sylvester Stallone sounded when he talked.

See those four questions in the dirt! You have to answer them!! While doing push-ups!!!"

Bruce Willis gaped at Kermmando.

Kermmando roared.

Don't stand there gaping, you bonehead! Get down in the dirt and start calculating!! The clock's ticking!!!

Question 2 (3 marks)

Find all possible values of b so that the function

$$f(x) = \begin{cases} 4b^3x^3 + 3x^2 - b & \text{if } \; 0 < x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

is a probability density function. 3 marks

Question 3 (3 marks)

Let the random variable W be normally distributed with mean 1.5 and standard deviation 0.4.

Let Z be the standard normal random variable, such that $Z \sim N(\mu = 0,\ \sigma = 1)$.

Find a such that $\Pr(W < a) = \Pr\left(Z > \frac{a}{3}\right)$.

Question 4 (5 marks)

For events A and B from a sample space, $\Pr(A) = \frac{1}{5}$ and $\Pr(B) = \frac{2}{3}$.

If A' denotes the complement of A, calculate $\Pr(A' \mid B)$ when

a. A and B are mutually exclusive. 1 mark

b. A and B are independent. 1 mark

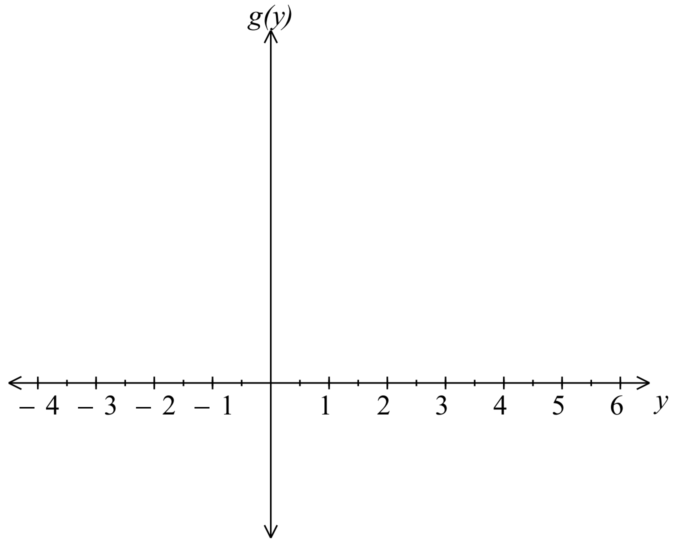
c. $\Pr(A \cup B) = \frac{3}{4}$. 3 marks

Question 5 (3 marks)

A continuous random variable Y has a probability density function given by

$$g(y) = \begin{cases} \dfrac{\log_e(y-1)+1}{e} & \text{if } \; 2 < y \le e+1 \\ 0 & \text{elsewhere} \end{cases}$$

Draw the probability density function of Y on the axes below.



END OF QUESTION AND ANSWER BOOK