VICTORIAN CERTIFICATE OF EDUCATION

2016

MATHEMATICAL METHODS (CAS)

MATH WARS

SOLUTIONS

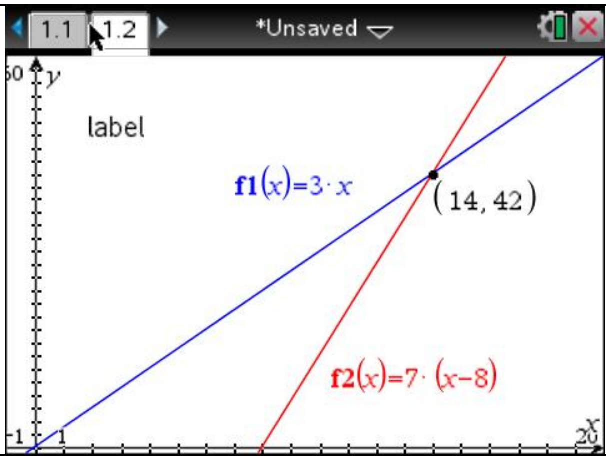
Episode IV – A new graph

Question 1 (9 marks)

a. Assuming that Range2Domain2 travels in a straight line, how far will it travel before Colin Shnierwalker wakes up and notices Range2Domain2 is missing 8 hours later? 1 mark

24 km *students must state km in order to achieve mark	**A1**

b. Colin immediately gets into his speeder and pursues Range2Domain2, travelling at a constant speed 7km/h. Given Range2Domain2 left 8 hours before Colin does, at what time, to the nearest hour, will Colin catch up to Range2Domain2? 2 marks

$D_{R2D2} = 3t$ $D_{Colin} = 7(t-8)$ $3t = 7(t-8)$	**M1**	
$t = 14 hours$	**A1**	

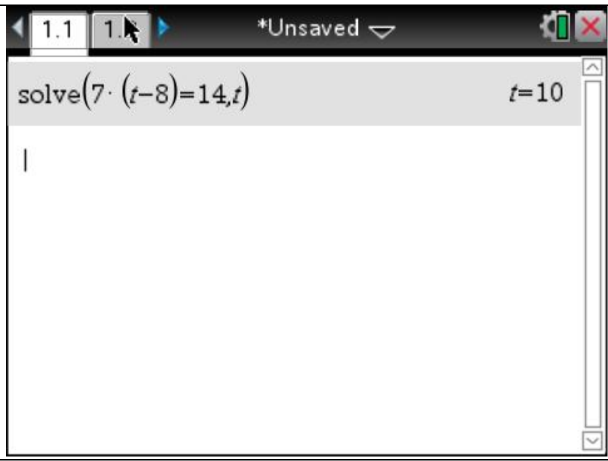
c. 14km away from the moisture farm, in the direction in which Range2Domain2 was heading, the terrain changes from sand to rocky hills. This affects the speed at which Range2Domain2 and Colin Shnierwalker's Speeder can travel. Over the rocky hills, Range2Domain2 can travel at 2km/h and Colin Shnierwalker's Speeder can travel at 4km/h.

Assume that the time at which Range2Domain2 leaves the moisture farm is $t = 0$ hours.

i. Show that the time at which Range2Domain2 reaches the rocky hills is $t = \frac{14}{3}$ hours. 1 mark

$D_{R2D2} = 14 = 3t$ $t = \frac{14}{3}$	**M1**

ii. Show that the time at which Colin Shnierwalker reaches the rocky hills is $t = 10$ hours. 1 mark

$D_{Colin} = 14 = 7(t-8)$ $2 = t - 8$ $t = 10$	**M1**	

The distance of Range2Domain2 from the moisture farm, *t* hours after leaving, can be described by the hybrid function *R(t)* shown below:

$$R(t)=\begin{cases} 3t \quad , \quad 0<t\le\frac{14}{3} \\ \frac{2}{3}(3t+7) \quad , \quad t\ge\frac{14}{3} \\ 0 \quad , \quad elsewhere \end{cases}$$

iii. Find the rule for the function $f(t)$ such that the hybrid function, $C(t)$, shown below, accurately describes Colin Shnierwalker's distance, in km, from the moisture farm at time *t*.

$$C(t)=\begin{cases} 7t-56 \quad , \quad 0<t\le 10 \\ f(t) \quad , \quad t\ge 10 \\ 0 \quad , \quad elsewhere \end{cases}$$

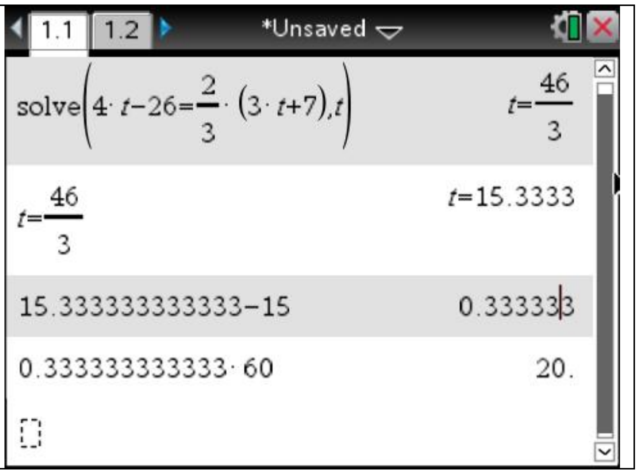
2 marks

Method 1 OR Method 2 Method 1: $f(t)=4(t-8)+d$ $f(10)=4(10-8)+d=14$ $d=6$ $f(t)=4(t-8)+6$ Method 2: $f(t)=4t+d$ $f(10)=40+d=14$ $d=-26$ *In order to get this mark, explicit use of $f(10)=14$ must be apparent.	**M1**
$f(t)=4t-26$ *Students must simplify answer to the form given to get the answer mark	**A1**

iv. Hence or otherwise, find the time, *t*, to the nearest minute, when Colin Shnierwalker will catch up to Range2Domain2.

2 marks

$4t-26=\frac{2}{3}(3t+7)$	**M1**
15 hours and 20 minutes OR 920 minutes *Units must be stated **Consequential answer from Part iii should be accepted if following correct working	**A1**



Question 2 (6 marks)

Now that their debt is overdue, Corkill the Hutt has started to charge Harnath interest on the original 10,000 credit loan.

a. Harnath learns from a bounty hunter named Greedo that Corkill the Hutt is charging a cumulative interest rate of 2.5% per day on the amount that Harnath owes.

A cumulative interest rate means that the additional interest being charged on any day is calculated on the total amount owing from the previous day. This can be expressed by the equation

$$P_n = P_0(1+I)^n$$

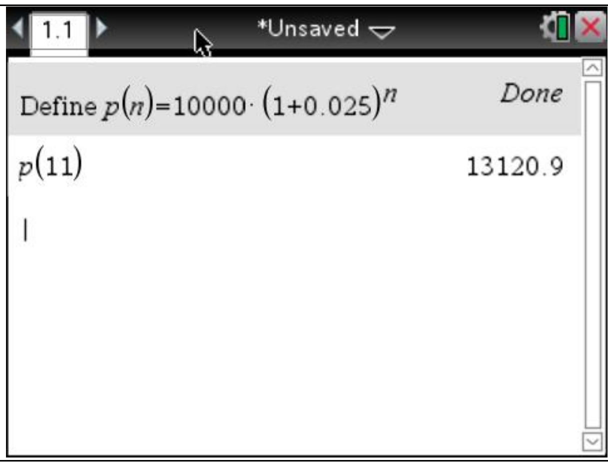
where P_n is the amount owing after n days, P_0 is the original loan amount and I is the interest rate (expressed in decimal form).

i. How much will Harnath Solo owe Corkill the Hutt at the end of the first day of being late? 1 mark

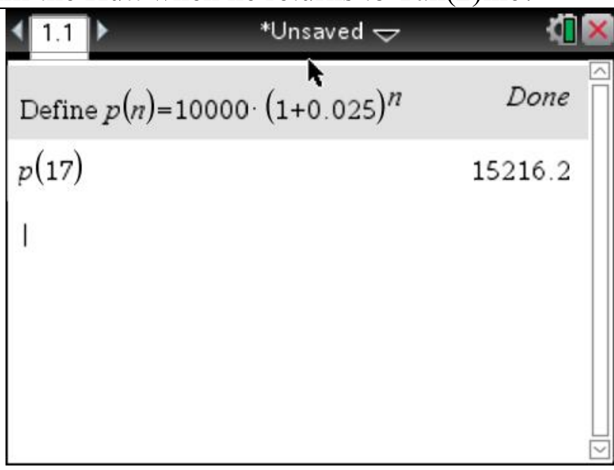
$1.025 * 10000 = 10250$ *Units are not required to get full marks	**A1**

Solo is 11 days late on his repayment to Corkill the Hutt.

ii. How many credits does he currently owe to Corkill the Hutt? 1 mark

$P_n = 1000(1+0.025)^{11}$ *an exact answer is required for this question, students who give an answer of 13120.9 (or similar with rounding) will lose this mark, but should only be penalised more than once for not giving an exact answer in question 2 **Units are not required to get full marks	**A1**	

c. Harnath Solo knows that it will take 3 days to take Obi-Wan and Colin to AldeRan and an additional 3 days to return to Tan(2)ine. If the interest continues to accumulate in the same manner, how much will he owe Corkill the Hutt when he returns to Tan(2)ine? 2 marks

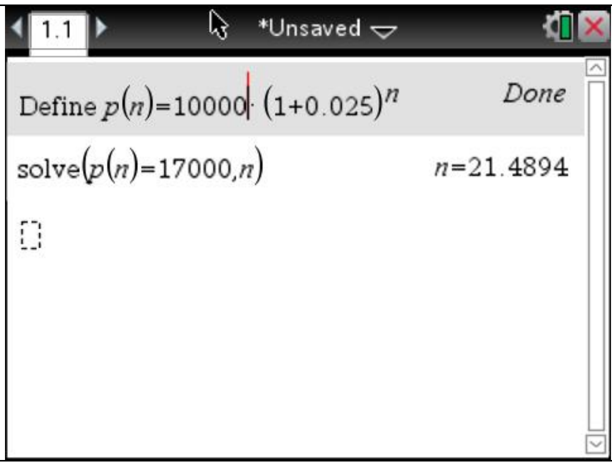
$n = 17$	**M1**	
$P_{17} = 1000(1+0.025)^{17}$ *an exact answer is again required for this question, students who give an answer of 15216.2 (or similar with rounding) will lose this mark, but should only be penalised more than once for not giving an exact answer in question 2 (So if they lost the mark in b. ii. They should not be penalised again). **Units are not required to get full marks	**A1**	

Obi-Wan offers to pay Harnath Solo 2,000 credits in advance and an additional 15,000 credits when they arrive at AlderRan. Harnath agrees and they all board Harnath's ship, the Millennium Falcon, and begin their journey.

Once on their way, Cvetkovskbacca decides to work out how much time 17,000 credits will buy them, in case something unexpected happens on their trip and they are held up.

d. Given they are already 11 days late in payment, how many more days can they afford to lose before Obi-Wan's 17,000 credits no longer covers their debt to Corkill the Hutt?

2 marks

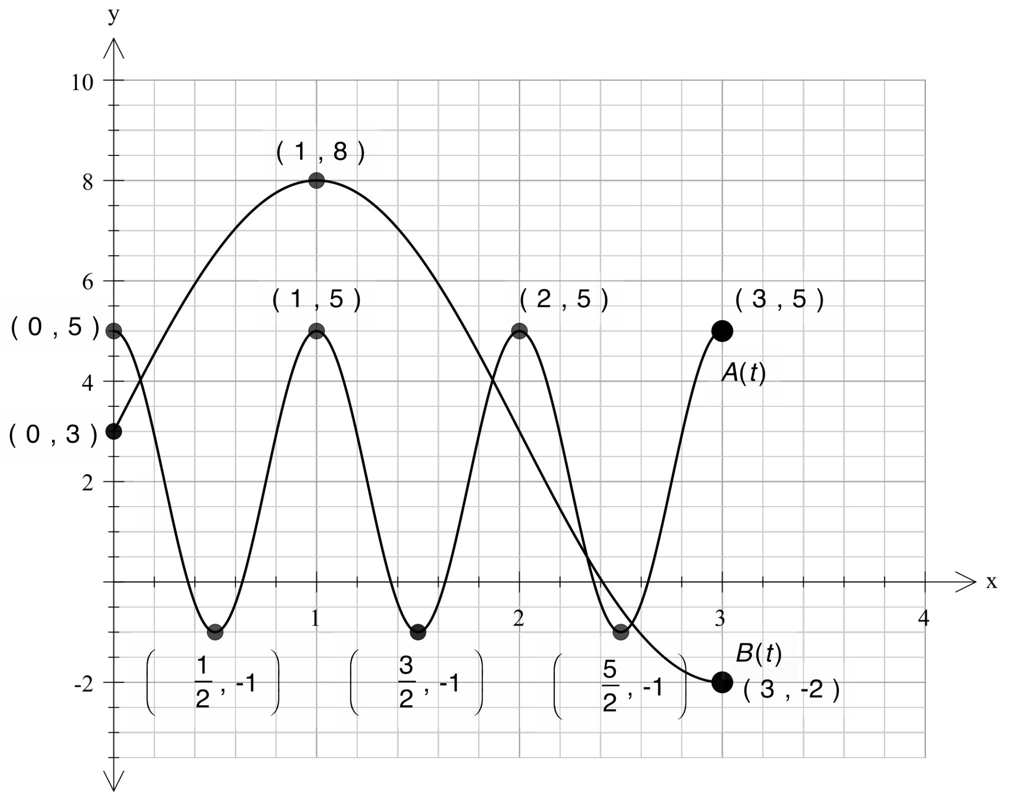
$10000(1+0.025)^n = 17000$	**M1**	
10 more days *students who answer 21 days should not get this mark	**A1**	

Question 3 (19 marks)
The two graphs that Colin senses in his mind are described by the following functions:

$A:[0,3] \to \mathbb{R},\ A(t) = 3\cos(2\pi t) + 2$ and $B:[0,3] \to \mathbb{R},\ B(t) = 5\sin\left(\frac{\pi t}{2}\right) + 3$.

where t is the time in seconds after the training remote begins moving.

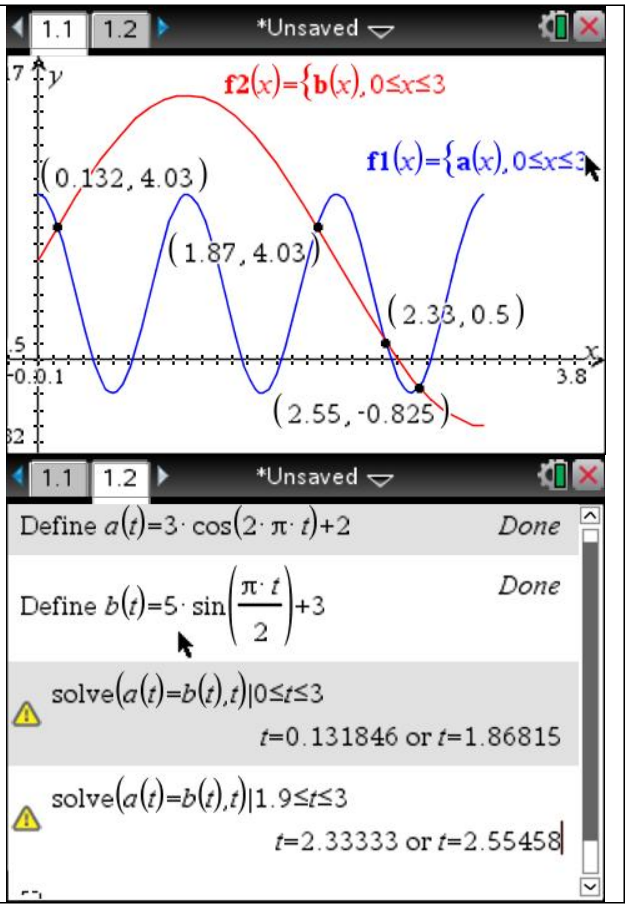
a. Sketch the graphs of $y = A(t)$ and $y = B(t)$ on the same set of axis provided below, labelling all endpoints and turning points. 2 marks

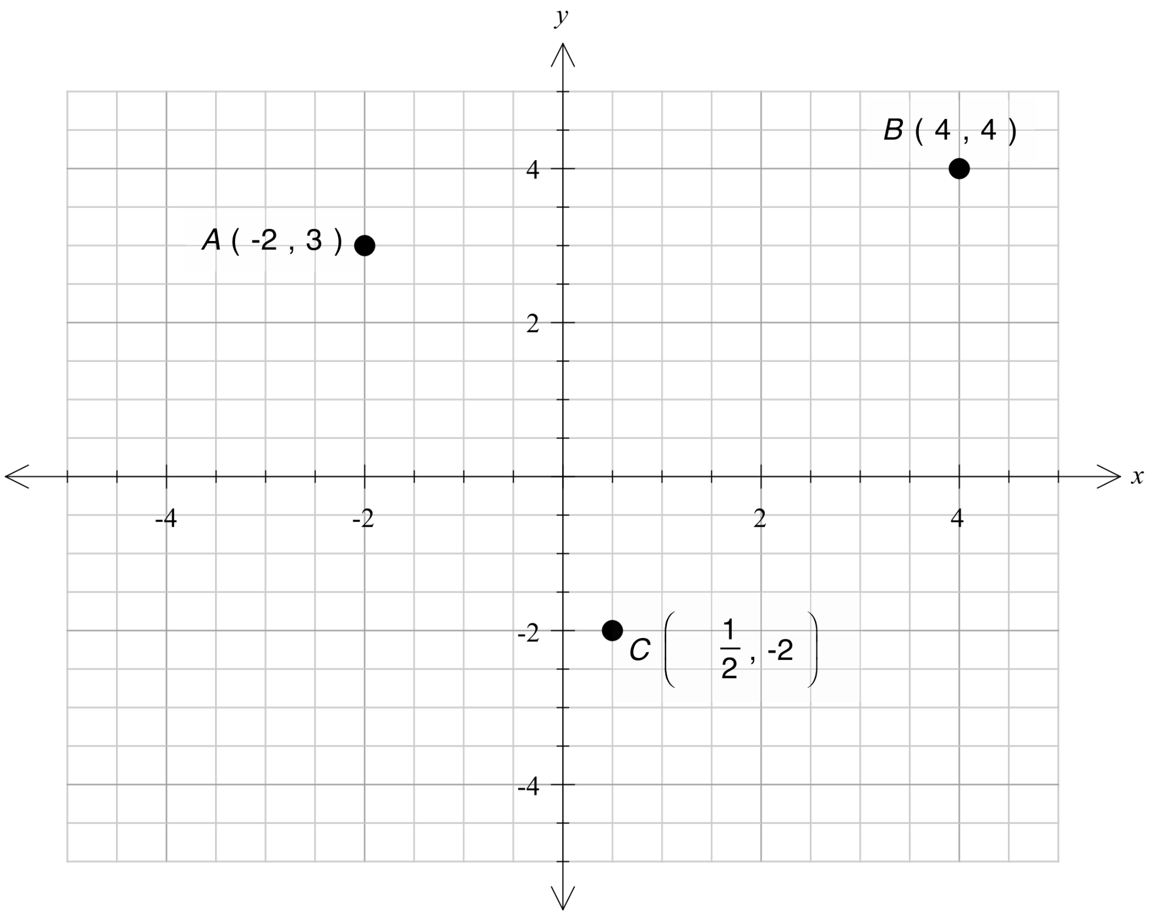


Accurate sketch of $A(t)$ including all labels	**A1**
Accurate sketch of $B(t)$ including all labels *For every mistake a student makes, they should lose ½ a mark and this is then rounded down	**A2**

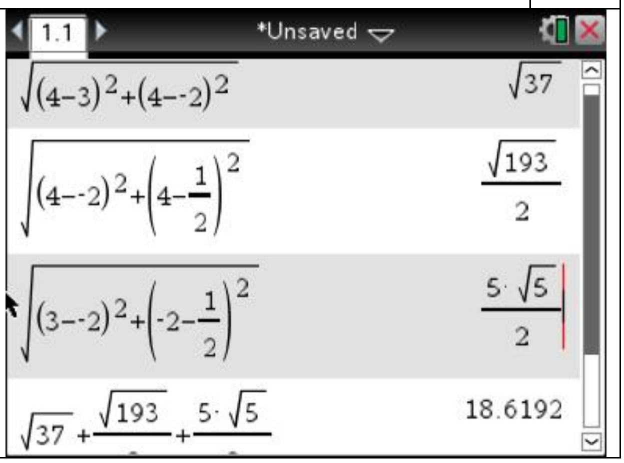
b. Find the times of all points of intersection between $A(t)$ and $B(t)$. Express your answer **correct to 2 decimal places**. 2 marks

$A(t) = B(t)$ $t = 0.13$ or 1.87 * students can get this mark for finding the first two values of the intersections points	**A1**
$t = 2.33$ or 2.55 seconds *Students must give all 4 solutions in order to obtain both answer marks.	**A2**

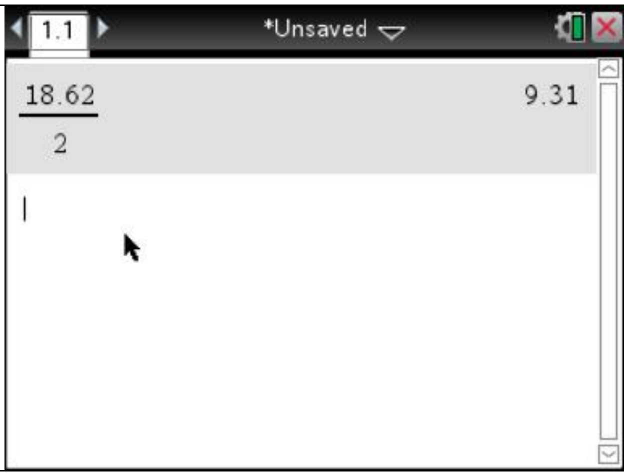




c. What is the total distance, correct to 2 decimal places, travelled by the remote as it moves from A to B to C and back to A again? 2 marks

$d = AB + BC + CA$ $d = \sqrt{(4-3)^2 + (4-(-2))^2} + \sqrt{(4-(-2))^2 + \left(4 - \left(\frac{1}{2}\right)\right)^2} + \sqrt{(3-(-2))^2 + \left((-2) - \left(\frac{1}{2}\right)\right)^2}$	**M1**	
$d = 18.62$ meters *Units are required for final answer mark	**A1**	

d. It takes 2 seconds for the remote to travel completely around this path. Assuming that it moves with constant speed, show that it is travelling at $9.31 ms^{-1}$. 1 mark

$\text{speed} = \dfrac{\text{distance}}{\text{time}} = \dfrac{18.62}{2}$ $\text{speed} = 9.31\ ms^{-1}$	**M1**	

e. If the remote leaves at time $t = 0$, at what time, t, **correct to 2 decimal places**, during the first 2 seconds will it be located at point B?
(Use the value for the speed given in part d.)

2 marks

$\text{time} = \dfrac{\text{distance}}{\text{speed}} = \dfrac{\sqrt{37}}{9.31}$	**M1**	
$t = 0.65\ \text{seconds}$ *Units are required for final answer mark	**A1**	

f. The first shot from the remote is fired at $t = 0.13$ seconds.

i. How far, correct to 2 decimal places, will the remote have moved from point A when it fires its first shot? 1 mark

distance $= 9.31 \times 0.13 = 1.21$ meters *Answer must be rounded correctly to two decimal places **Units are required for final answer mark	**A1**

ii. Find the equation of the line that joins point A and point B. 2 marks

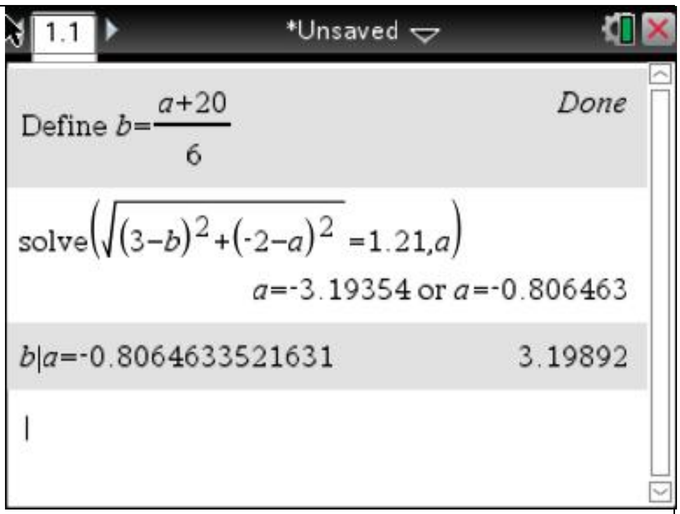
$m = \dfrac{4-3}{4-(-2)} = \dfrac{1}{6}$	**M1**
$y - 4 = \dfrac{1}{6}(x-4)$ $y = \dfrac{1}{6}x + \dfrac{10}{3}$	**A1**

iii. The coordinates of the point Q, where this first shot is fired will be somewhere along the line joining point A and point B. If the coordinates of Q are (a,b), use your answer to part **f. ii.** to find an expression for b in terms of a. 1 mark

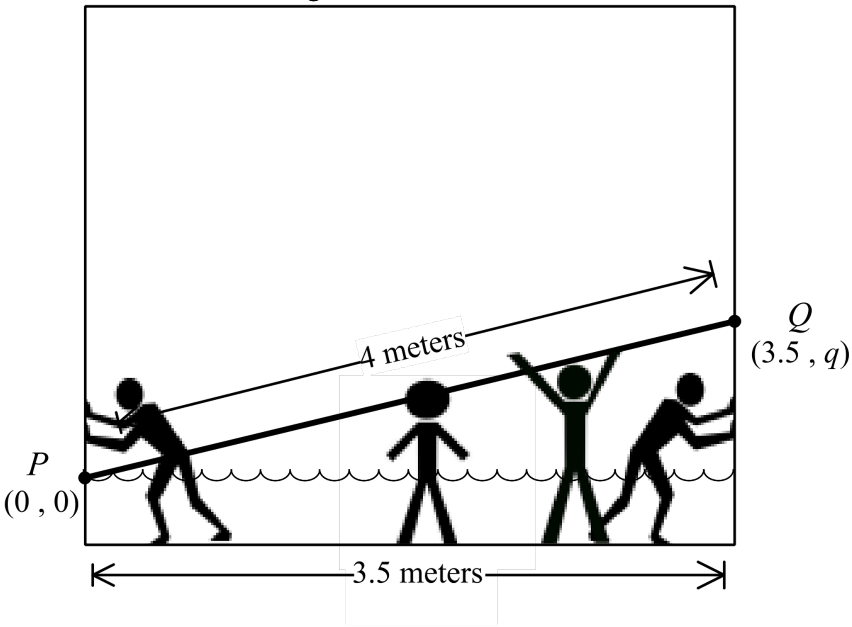
$b = \dfrac{1}{6}a + \dfrac{10}{3}$ *or* $b = \dfrac{a+20}{6}$ *accept either answer	**A1**

iv. **Hence** find the coordinates of the point Q correct to 2 decimal places. 4 marks

distance $= \sqrt{(3-b)^2 + (-2-a)^2} = 1.21$	**M1**
$\sqrt{\left(3 - \dfrac{a+20}{6}\right)^2 + (-2-a)^2} = 1.21$	**M2**
$a = -0.81$ *rejection of a=-3.19 is not explicitly required	**A1**
$b = 3.20$ $\therefore$ coordinates are $(-0.81, 3.20)$ *coordinates do not need to be stated in coordinate form as long as it is clear what a and b are. **There is an alternate approach that students might use involving similar triangles, if they carry this out correctly with an appropriate diagram, they can still achieve full marks.	**A2**

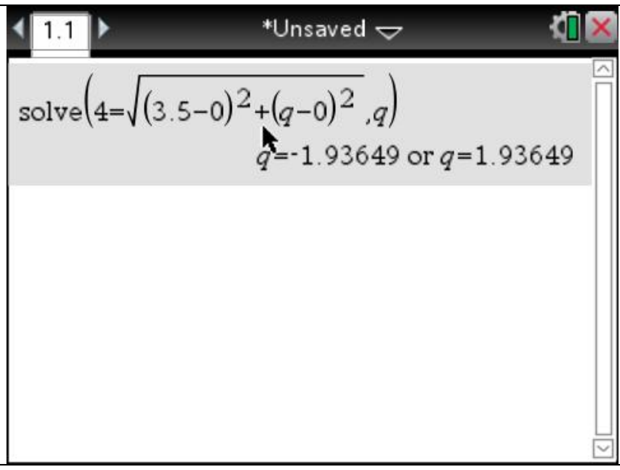


Question 4 (4 marks)
As the garbage compactor walls close in upon our heroes, Colin attempts to use the Maths to help him find a solution to stop them from being squished. He closes his eyes and visualises the garbage compactor as shown in the diagram below.

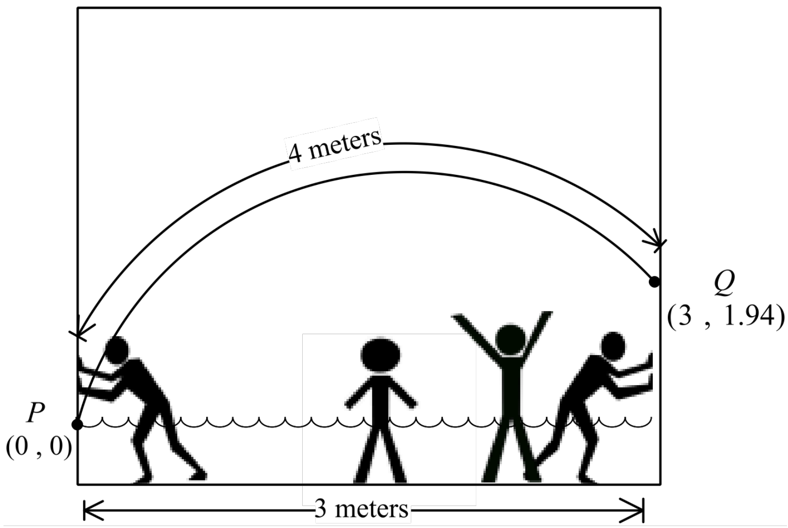


The initial width of the garbage compactor is 3.5 meters and there is a 4 meter long metal pole that can be used as a wedge between the two walls.

a. The left end of the metal pole is fixed at the coordinates (0,0). Show, correct to two decimal places, that the value of q, the vertical component of the coordinate Q where the right end of the metal pole touches the other wall, is 1.94 meters. 1 mark

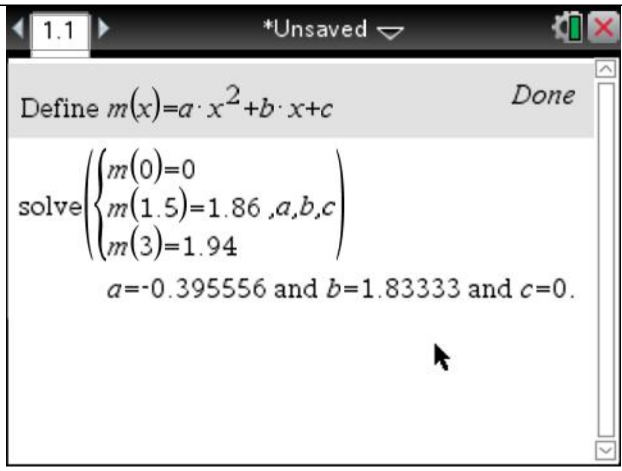
$4 = \sqrt{(3.5-0)^2 + (q-o)^2}$ q = 1.94 meters *If students do not state the final value of q then they should lose the final answer mark.	**M1**	

The walls proceed to move inwards, causing the metal pole to bend upwards as shown below. The metal pole remains fixed at (0,0) and at point Q which remains at the same height but whose horizontal coordinate value changes as the wall moves together.



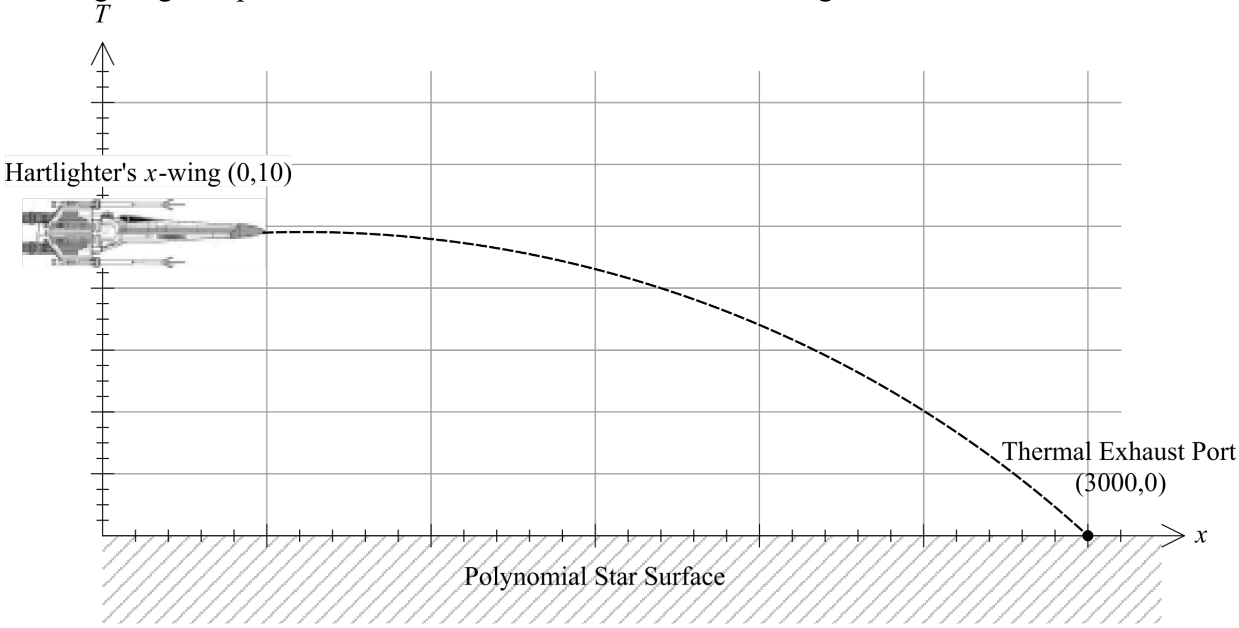
Colin realises that the shape of the curved rod can be modelled as a parabola with equation $M(x) = ax^2 + bx + c$.

b. Colin is standing in the exact middle of the room. He estimates that the metal pole is 1.86 meters above the water at this point. Find the values of a, b and c, correct to 2 decimal places, and hence state the equation for $M(x)$ when the room is 3 meters wide. 3 marks

$M(0) = 0$ $\therefore c = 0$	**M1**	
$M(3) = a(3)^2 + b(3) + 0 = 1.94$ $M(1.5) = a(1.5)^2 + b(1.5) + 0 = 1.86$ *Students do not need to show the substitution of values into the equation in order to get method marks, M(3)=1.94 and M(1.5)=1.86 is sufficient.	**M2**	
$a = -0.395556$ $b = 1.8333$ $M(x) = -0.40x^2 + 1.83x$ *Students must state the equation in order to get the final mark	**A1**	

Question 5 (25 marks)

The targeting computer models the scenario as shown in the diagram below.

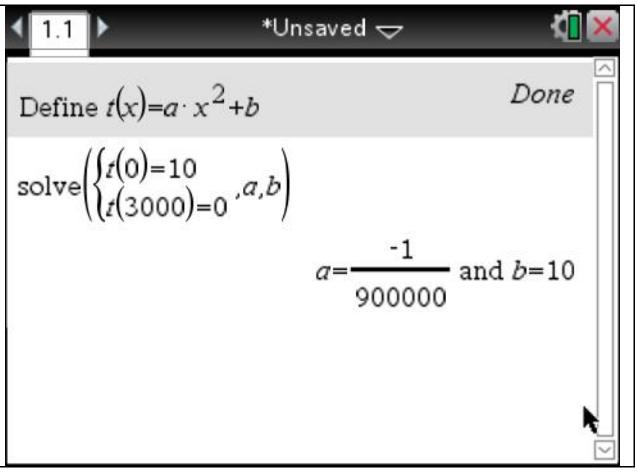


Hartelighter flies 10 meters above the surface of the polynomial star and initially Hartelighter's x-Wing is a horizontal distance of $d = 3000$ meters from the thermal exhaust port.

The targeting computer needs to work out the equation of the dashed line in the diagram that will represent the path of the proton torpedo. The computer models this path as a quadratic function of the form $T_1(x) = ax^2 + b$.

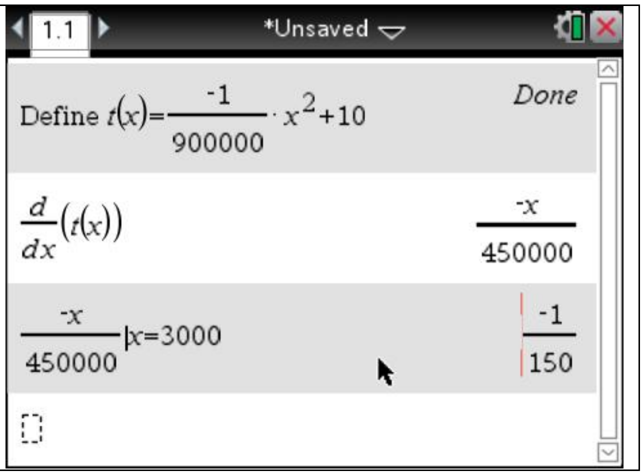
a. At this initial moment in time, the targeting computer places Hartelighter's X-Wing at coordinates (0,10) and the thermal exhaust port at (3000,0).

i. Determine the values of a and b in $T(x)$ that the targeting computer would assign for the path of the proton torpedo. 2 marks

$T(0) = a(0)^2 + b = 10$ $T(3000) = a(3000)^2 + b = 0$	**M1**	
$b = 10$ $a = \dfrac{-1}{900000}$ *Students do not need to state the equation to get this mark, clear statement of a and b is enough.	**A1**	

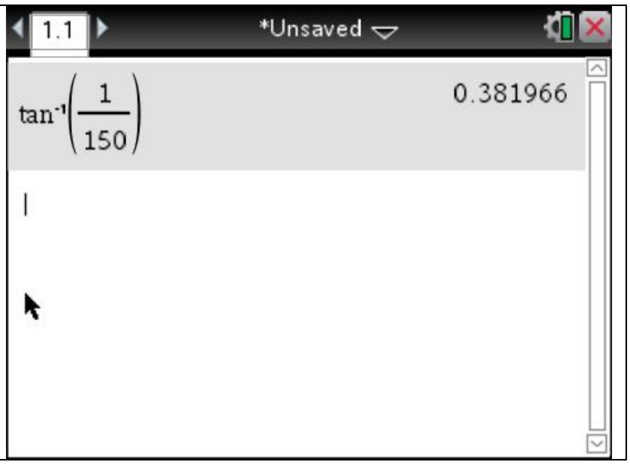
ii. What is the gradient of the path when the proton torpedo reaches the thermal exhaust port? 2 marks

$T'(x) = \frac{-x}{450000}$ *Can be consequential based on a. i.	**M1**
$T'(3000) = \frac{-3000}{450000} = \frac{-1}{150}$	**A1**



iii. In order for the proton torpedo to successfully enter the thermal exhaust port, it must enter at an angle of 45^{o} to the horizontal. If fired from 3000 meters away, what is the angle at which the proton torpedo will enter the thermal exhaust port, correct to 3 decimal places? 2 marks

$\tan(\theta) = m = \frac{-1}{150}$	**M1**
$\theta = \tan^{-1}\left(\frac{-1}{150}\right) = -0.382^{\circ}$ So the torpedo will enter the thermal exhaust port at an angle of 0.382^{o}. Should also accept 179.618^{o}. *Final answer given by students must be positive.	**H1**

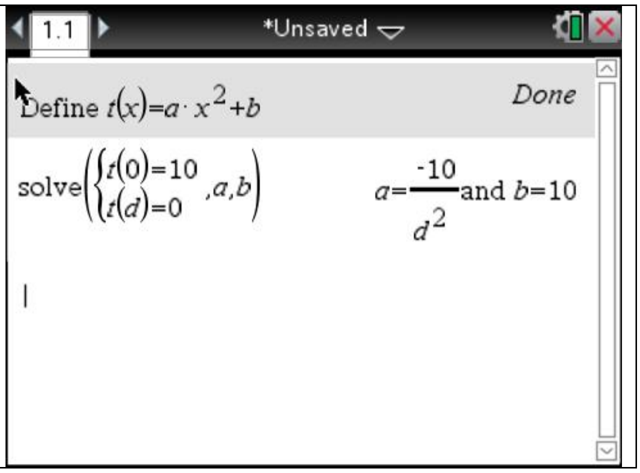


In order to accommodate for this problem. The targeting computer works out the equation of the line in terms of d the horizontal distance of the X-Wing from the thermal exhaust port, where $0 < d < 3000$.

b. Assuming that the coordinates of the thermal exhaust port are at $(d, 0)$;

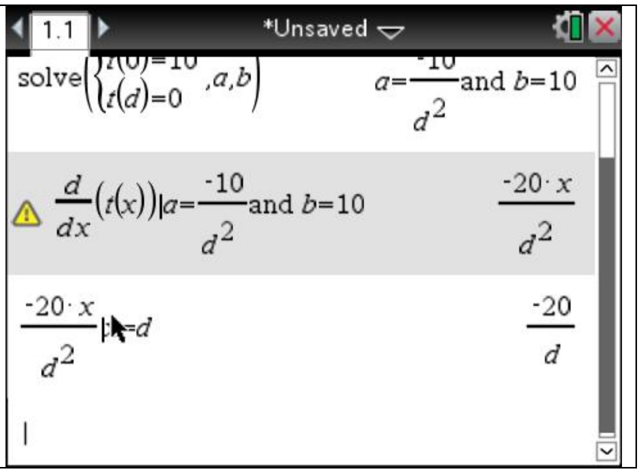
i. Find the values of a and b in $T_2(x) = ax^2 + b$ that the targeting computer would find for the path of the proton torpedo in terms of d, the distance of the X-Wing from the exhaust port. 2 marks

$T(0) = a(0)^2 + b = 10$ $T(d) = a(d)^2 + b = 0$	**M1**
$b = 10$ $a = \frac{-10}{d^2}$ *Students do not need to state the equation to get this mark, clear statement of a and b is enough.	**A1**



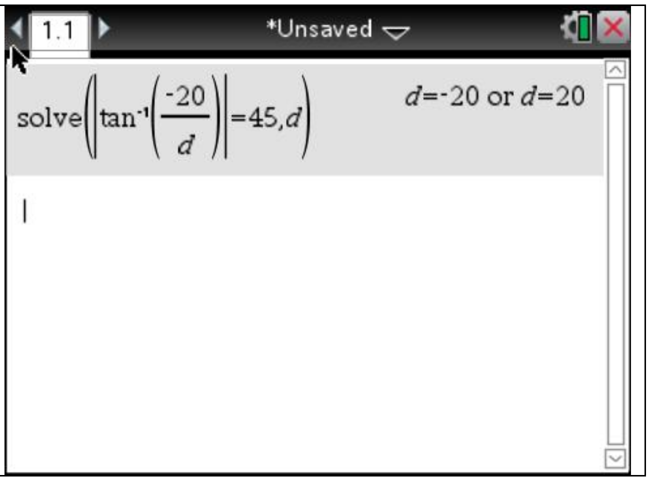
ii. Hence show that the gradient of the proton torpedo's path when it hits the thermal exhaust port can be given by $T_2'(d) = \frac{-20}{d}$. 2 marks

$T(x) = \frac{-10}{d^2}x^2 + 10$ $T'(x) = \frac{-20}{d^2}x$	**M1**
$T'(d) = \frac{-20}{d}$	**A1**

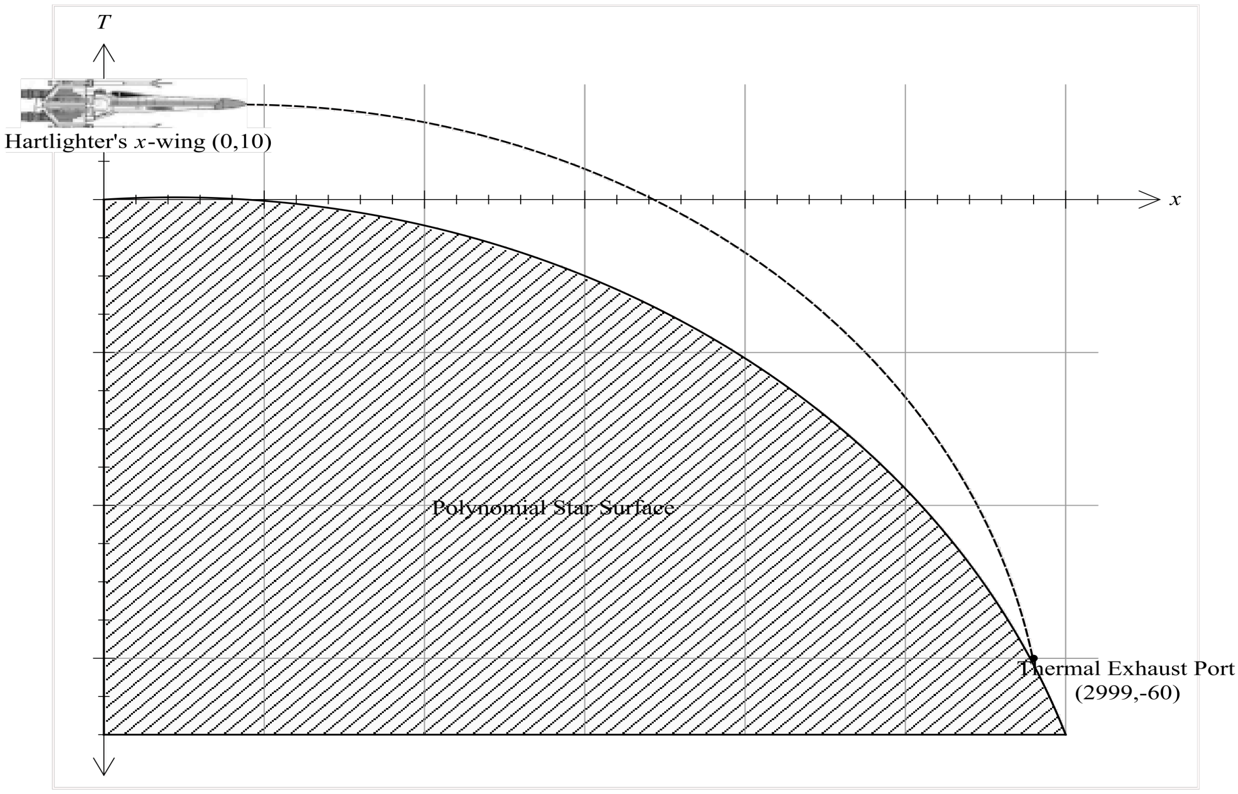


iii. How close does the X-Wing need to get to the thermal exhaust port before it is possible to fire the proton torpedo so that it will hit the thermal exhaust port at an angle of 45°? 2 marks

$T(x) = \frac{-10}{d^2}x^2 + 10$ $\tan^{-1}\left(\frac{-20}{d}\right) = 45$	**M1**
$d = 20$ meters *If students give -20 meters as an answer, they should not be awarded this mark.	**A1**



c. Colin has modelled the surface of the Polynomial Star by the equation $S(x) = -75{,}000 + \sqrt{5{,}625{,}000{,}000 - x^2}$. Through the maths he pictures the following diagram to represent the situation.



He reasons that the equation of his *x*-Wing through the trench can, therefore, be modelled by the equation $C(x) = -74{,}990 + \sqrt{5{,}625{,}000{,}000 - x^2}$.

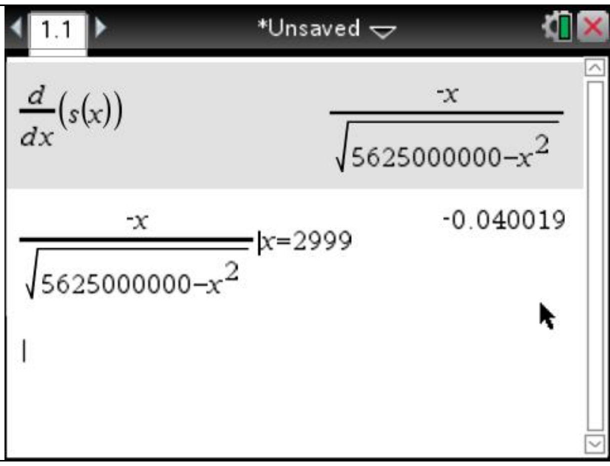
i. Give an explanation for why Colin's X-Wing can be modelled by $C(x)$. 1 mark

Colin's *x*-wing always flies 10 meters above the surface of the Polynomial Star hence, the path of Colin's *x*-wing is simply a vertical translation of 10 meters upwards or, $C(x) = S(x)+10$	**A1**

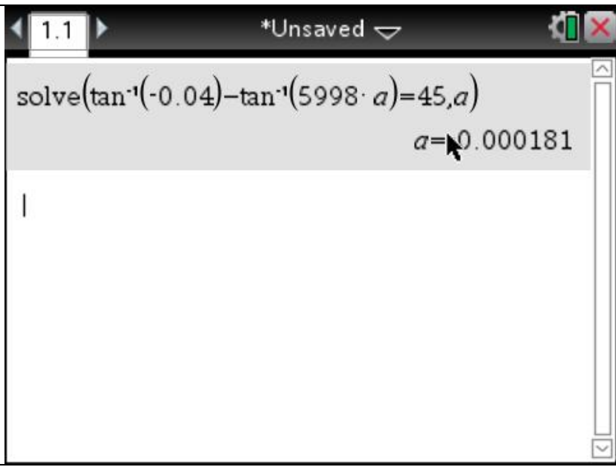
ii. Using the equation for the path of the proton torpedo, $T(x) = ax^2 + b$, find an expression for the gradient of proton torpedo at $(2999, -60)$ in terms of *a*. 2 marks

$T'(x) = 2ax$	**M1**
$T'(2999) = 5998a$	**A1**

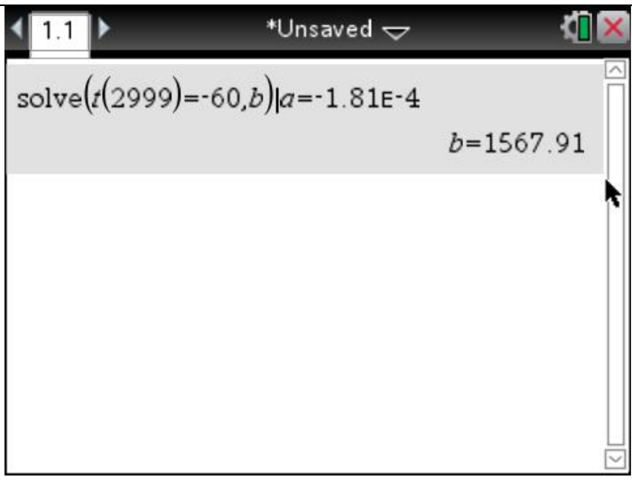
iii. What is the gradient of the surface of the Polynomial Star at $(2999,-60)$ expressed correct to 2 decimal places? 2 marks

$S'(x)=\dfrac{-x}{\sqrt{5625000000-x^2}}$	**M1**	
$S'(2999)=-0.04$	**A1**	

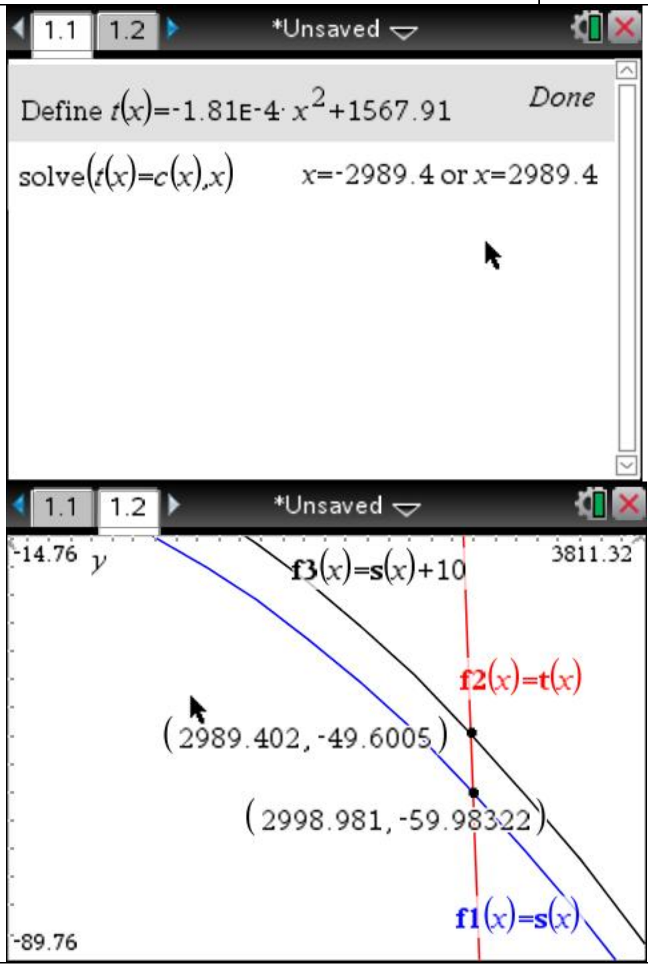
iv. Colin knows that the angle between two lines can be worked out using the gradients of the lines. In this case, he knows that $\theta=\tan^{-1}(S'(2999))-\tan^{-1}(T'(2999))$. Determine the value of a for which $\theta=45°$ correct to 6 decimal places. 1 mark

$\tan^{-1}(-0.04)-\tan^{-1}(5998a)=45$ $a=-0.000181$	**A1**	

v. Hence determine the value of b in $T(x)$, correct to 2 decimal places, for this particular proton torpedo path. 1 mark

$T(x)=-0.000181x^2+b$ $T(2999)=-0.000181(2999)^2+b=-60$ $b=1567.91$ Also accept b = 1564.52 as this is the answer that one would get when using a more exact answer than the 6 dp answer from part iv.	**A1**	

vi. Using your answers to **part iv.** and **part v.** find the x-coordinate where Colin should fire the proton torpedo from in order to hit the thermal exhaust port at the correct angle. 2 marks

$T(x)=C(x)$ $-0.000181x^2+1567.91=-74{,}990+\sqrt{5{,}625{,}000{,}000-x^2}$		**M1**
$x=2989.4$ meters	**A1**	

END OF EPISODE IV – A NEW GRAPH