**STUDENT NAME:**

**TEACHER NAME:**   BOH   CVE   HRN   KER   PAI   PIZ   SHN

# MATHEMATICAL METHODS (CAS)

# MATH
# WARS

## Episode V – The Exponential Strikes Back

**2016**

**Reading Time: 10 minutes**
**Writing time: 60 minutes**

### QUESTION AND ANSWER BOOK

| Number of questions | Number of questions to be answered | Number of marks |
|---|---|---|
| 4 | 4 | 34 |
| | *Total* | 34 |

- Students are permitted to bring into the test room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved CAS Calcuator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination: blank sheets of paper and/or correction fluid/tape.

**Materials supplied**
- Question and answer book of 15 pages.

**Instructions**
- Write your name in the space provided above on this page and circle your teacher's initials.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the test room.**

**This page was left blank in honour of all the characters who**

**lost a limb whilst making this SAC**

**Instructions**

Answer **all** questions in the spaces provided

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

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**Episode V**

**THE EXPONENTIAL STRIKES BACK**

*It is a dark time for the Quadratics. Although the Polynomial Star has been destroyed, Exponential troops have driven the Quadratic forces from their hidden base and pursued them across the galaxy.*

*Evading the dreaded Exponential Starfleet, a group of freedom fighters led by Colin Shnierwalker has established a new secret base on the remote ice world of $H_0th$.*

The evil lord Darth Kermond, obsessed with finding young Shnierwalker, has dispatched thousands of remote probes into the far reaches of space….

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Due to the large number of asteroids and meteorites in the $H \circ th$ system, the Quadratics are forced to do regular patrols around their hidden base to check for signs of any Exponential probes that might have found them.

The planet $H \circ th$ is an incredibly cold world, covered in ice and snow. The Quadratics have enlisted the use of Tan Tans, a species of animal that is native to $H \circ th$ and who have developed a tolerance for the extreme temperatures.

On a recent patrol, Colin Shnierwalker was attacked by another native creature from $H \circ th$ called a Wampa, who had knocked him unconscious and dragged him back to its cave to be eaten later. Colin, however, used the Maths and managed to kill the Wampa and escape, but is now stranded in the cold and forced to try and walk back to the Quadratic base.

As night begins to descend, it gets colder and colder and Colin begins to display the symptoms of mild hypothermia. As he tumbles down a snowy slope, a vision of Obi-Wan Kebohni appears to him. "Colin. You must go to the Dagobah system." Obi-Wan says. "There you will learn from $d(y(o))/da$, the Jedpi master who instructed me."

Just as Colin Shnierwalker is about to pass out, Harnath Solo, who had stayed out in the cold looking for his missing friend, arrives to save Colin.

**Question 1** (11 marks)

**a.**  The temperature, $H$, in $^{\circ}C$, over a 24 hour cycle on the planet $H \circ th$ can be modelled by the function

$$H:[0,24] \rightarrow R, \ H(t) = 35\cos\left(\frac{\pi t}{12} - 4\right) - 25$$

where $t$ is the number of hours after midnight.

    **i.**  What is the minimum temperature that is reached on the planet $H \circ th$?          1 mark

    **ii.**  At what time of day, to the nearest minute, does this minimum occur?          2 marks

**b.**    Harnath Solo finds Colin Shnierwalker wandering through the snow at $t = 20$ hours. What
is the temperature of Planet $H \circ th$ at this time? Express your answer correct to 2 decimal
places.                                                                                                      1 mark

Whilst checking on Colin who appears to be suffering from a mild case of hypothermia,
Harnath Solo's Tan-Tan dies from exhaustion. With no way to transport Colin back to the
Quadratic's Base, Harnath sets to work constructing a shelter for him and Colin to spend the
night in.

**c.**    Harnath Solo needs to finish building the shelter before the temperature drops below
$-35°C$ or else both he and Colin will freeze to death. How long, to the nearest minute,
does Harnath Solo have after starting at time ($t = 20$ hours) to build the shelter?          2 marks

**d.**    Colin's body temperature is dropping rapidly. Harnath measures Colin's body temperature
to be $34.9°C$ at $t = 20$ hours.

Harnath knows that Colin's body temperature has been and will continue to decrease

according to the rule $C(t) = \begin{cases} 37 & , \quad 0 \le t < a \\ 385e^{-0.12t} & , \quad a \le t \le 24 \end{cases}$.

   **i.**    If Colin's body temperature when he was first exposed to the cold was a normal
$37°C$, find the value of $a$, to the nearest minute.                                          3 marks

Moderate hypothermia occurs when a person's body temperature falls below 32$^\circ$C. Severe hypothermia occurs when a person's body temperature falls below 28$^\circ$C.

**ii.**   How long, to the nearest minute, will Colin experience moderate hypothermia before he starts to experience severe hypothermia?                                           2 marks

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*In order to try and keep Colin warm, Harnath Solo takes Colin's Lightsaber and slices open the dead Tan Tan. He then places Colin inside the Tan Tan to try and keep him from reaching severe hypothermia. This works and Harnath is able to build a shelter to keep them both warm through the cold  H $\circ$ th  night.*

*The next morning, Harnath and Colin are rescued and returned to the Quadratic base. Colin is found to have caught a  H $\circ$ th  cold, a nasty viral infection that can only be treated by taking a Kaldunphloo tablet that combats the  H $\circ$ th  cold.*

**Question 2** (13 marks)

The  $H \circ th$  cold virus is a particularly dangerous virus and the Quadratic Medical Department are keen to return Colin to full health as quickly as possible. The number of  $H \circ th$  cold viruses in Colin's body,  $V$ , will grow according to the rule  $V(t) = V_0 \times 2^{kt}$  where  $t$  is the time in minutes after the initial infection.

**a.**   Initially, Colin was infected by a single  $H \circ th$  cold virus.

**i.**   Show that  $V_0 = 1$                                                                                             1 mark

Upon first contact with the Quadratic Medical Department, a scan reveals that Colin has 250,000 viruses in his system. At this time, the number of viruses in his system are increasing at a rate of 2,406 viruses per minute.

**ii.**    Determine the value of $k$ and the length of time, in hours, that Colin has been infected with the $H \circ th$ cold virus at the time of this scan. Express both your answers correct to 3 decimal places.                                    4 marks

**b.**    The Kaldunphloo tablet contains an anti-virus that prevents all viruses from replicating and that kills off viruses in Colin's system. Unfortunately, the antivirus stops working exactly after 3 hours and in between then and taking the next tablet, the number of viruses in Colin's system will double.

The graph of the number of viruses, $N$, remaining in Colin's system at time, $t$, hours after taking the first Kaldunphloo tablet looks like this. Each section of the curve has exactly the same shape as the curve $AB$.
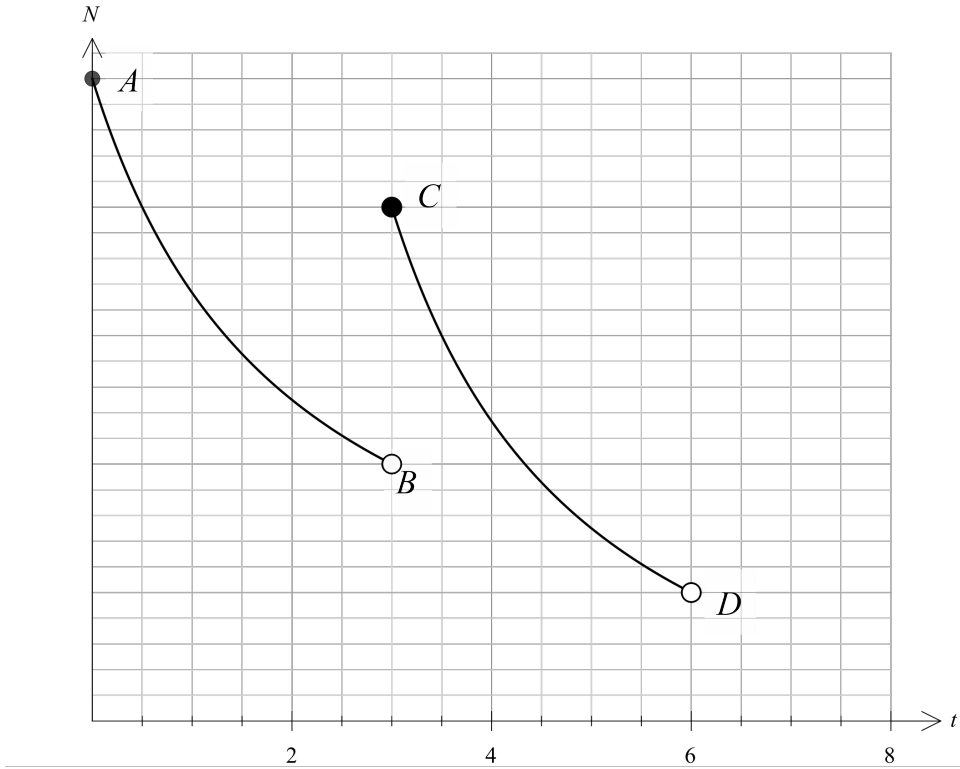


The equation of the cuve $AB$ is $N_{AB}(t) = \dfrac{500000}{t+2}$ .

**i.**    Find the coordinates of points $A$ and $B$.                                              2 marks

**ii.** After 3 hours, the effect of the Kaldunphloo tablet wears off and the number of viruses double before Colin takes another tablet to start the process again. Find the equation of the curve *CD*.                    2 marks

**iii.** If the pattern continues, and after each tablet wears off, the number of viruses double before the next tablet starts to work, at what time, to the nearest minute, will all of the viruses be removed from Colin's system?                    2 marks

While Colin is recovering from $H \circ th$ cold virus he begins to think about the pattern made by the starting point of each of the curves of $N$ against $t$ when a patient is being treated with Kaldunphloo tablet (see the diagram below).

He begins to think that there might be a way of determining the coordinates of each of the starting points of the curves $N(t)$ for any starting number of viruses, $k$.



Assume that $k$ is a very large, positive integer value and that the graph of $N(t)$ terminates when $N = 0$.

**c.**   The coordinates of the starting points can be expressed as $(3n, y)$ where $n$ is an integer that represents which number curve is being referenced.
Find the general rule that will give the value of $y$, the starting points of the $n^{th}$ curve given that the initial number of viruses in the system is some number, $k$.          2 marks

*Colin makes a full recovery and everyone in the Quadratic base is very happy about it. Unfortunately, while all this was happening, one of the Exponential probe droids launched by Darth Kermond had found the Quadratic Base and the Quadratics are now forced to flee the frozen planet of $H \circ th$.*

*Harnath Solo and Cvetkovskbacca take Princess Lisa and C3B4MePO on board the Millennium Falcon to escape. Unfortunately the Millennium Falcon's hyperbolic-drive is not fully repaired and so they can't escape to the rendezvous point. As a result, they are forced to travel to a nearby system. In this nearby system is a mining colony called 'cloud city' that is run by an old friend of Harnath's called Emilando Breslinian. Harnath is sure that Emilando will help them hide from the Exponentials.*

*While Harnath Solo, Cvetkovskbacca, C3B4MePO and Princess Lisa make their escape, Colin Shnierwalker and Range2Domain2 head off to Dagobah in search of the Jedpi master $d(y(o))/da$.*

*When Colin and Range2Domain2 arrive at Dagobah, they meet with $d(y(o))/da$ who reluctantly agrees to train Colin in the ways of the Maths.*

**Question 3** (6 marks)

$d(y(o))/da$ is a tough master. He begins training Colin by making him run through the swamps of Dagobah whilst carrying $d(y(o))/da$ on his back. After a few days of this intense training, $d(y(o))/da$ begins asking Colin mathematical questions of varying difficulty as part of the training.

**a.** If $\sin(x) = 0.3$ where $0 \le x \le \dfrac{\pi}{2}$, find $\sin\left(\dfrac{\pi}{2} + x\right)$ to 3 decimal places.          1 mark

**b.** If $\tan(x) = \dfrac{7}{10}$ and $0 \le x \le \dfrac{\pi}{2}$, find $\cos(x)$.          1 mark

Due to the nature of $d(y(o))/da$'s peculiar speech pattern, when he describes a series of multiple transformations to Colin, it can become quite confusing for Colin to determine the correct order in which to apply them.

**c.**  Starting with the equation $y = \sqrt{x}$, Colin is asked to apply the following sequence of transformations to find the final equation, $y_f$.

    **1.** Dilation by factor 4 from the $x$ axis.

    **2.** Reflection in the $x$ axis.

    **3.** Vertical translation of $+3$ units.

    **4.** Horizontal translation of $-6$ units.

    **5.** Dilation by factor $\dfrac{1}{2}$ from the $y$ axis

    **i.**  What is the final equation, $y_f$, according to the described sequence of transformations                                                                                           2 marks

"Noooo!" says $d(y(o))/da$, "The answer should have been $y_f = 12 - 4\sqrt{2x+12}$."

    **ii.** Assuming that the transformations remain the same, what should the order of the transformations above have been in order to result in the answer that $d(y(o))/da$ expected?                                                                                                           2 marks

✦

*"Sometimes, through the math, predict the future, you will." $d(y(o))/da$ tells Colin. As Colin mediates upon a logarithm, he experiences a vision of his friends Harnath Solo, Princess Lisa and Cvetkovskbacca being held prisoner by the evil Darth Kermond in a city amongst the clouds. Unable to shake the feeling that this is the future that he has seen, Colin insists that he must leave Dagobah and go to rescue his friends.*

*What Colin had seen in his vision was true. Darth Kermond had hired a group of bounty hunters to track down the Millennium Falcon. One of these bounty hunters, Chisholm Fett, managed to track the Millennium Falcon to Cloud City and alert the Exponentials.*

*Now, Darth Kermond and the Exponential Forces had taken over Cloud City and were holding our heroes prisoner in an attempt to lure Colin Shnierwalker into a trap. Darth Kermond's plan was to freeze Colin Shnierwalker in carbonite so that he could transport the aspiring Jedpi to his master, the Emperor Palissitine. In order to test the freezing facility, Darth Kermond tests it on Harnath Solo, freezing him in carbonite. He then gives the frozen Harnath Solo to Chisholm Fett who intends to give Harnath Solo to Corkill the Hutt who has a huge bounty on Harnath's head due to Harnath having still not payed him back for that loan.*

*Colin arrives at Cloud City and is lured in to the freezing facility. Colin is not so easy to capture though and he manages to foil Darth Kermond's trap. Darth Kermond and Colin engage in a fierce lightsaber duel where they each utilise their knowledge of the Math to combat one another.*

*Colin does surprisingly well against Darth Kermond's mastery of the Math but Darth Kermond eventually forces Colin out onto a platform above the lower levels of Cloud City and there, cuts his hand off, ending the duel. Colin, cradling his now handless arm, climbs out as far as he can on the platform to stay away from Darth Kermond. It is then that Darth Kermond makes an offer for Colin to join him as his apprentice. Colin refuses saying "I'll never join you! You killed my father!"*

*"No." Darth Kermond replies, "I am your father."*

✦

**Question 4** (6 marks)

Colin hangs suspended above the lower levels of Cloud City, horrified by Darth Kermond's revelation that he is his father. Instead of giving in to the dark lord, however, he decides to jump from the platform (point $A$ in the diagram below), aiming for the point $B$ in the diagram below, which is located on the horizontal axis.
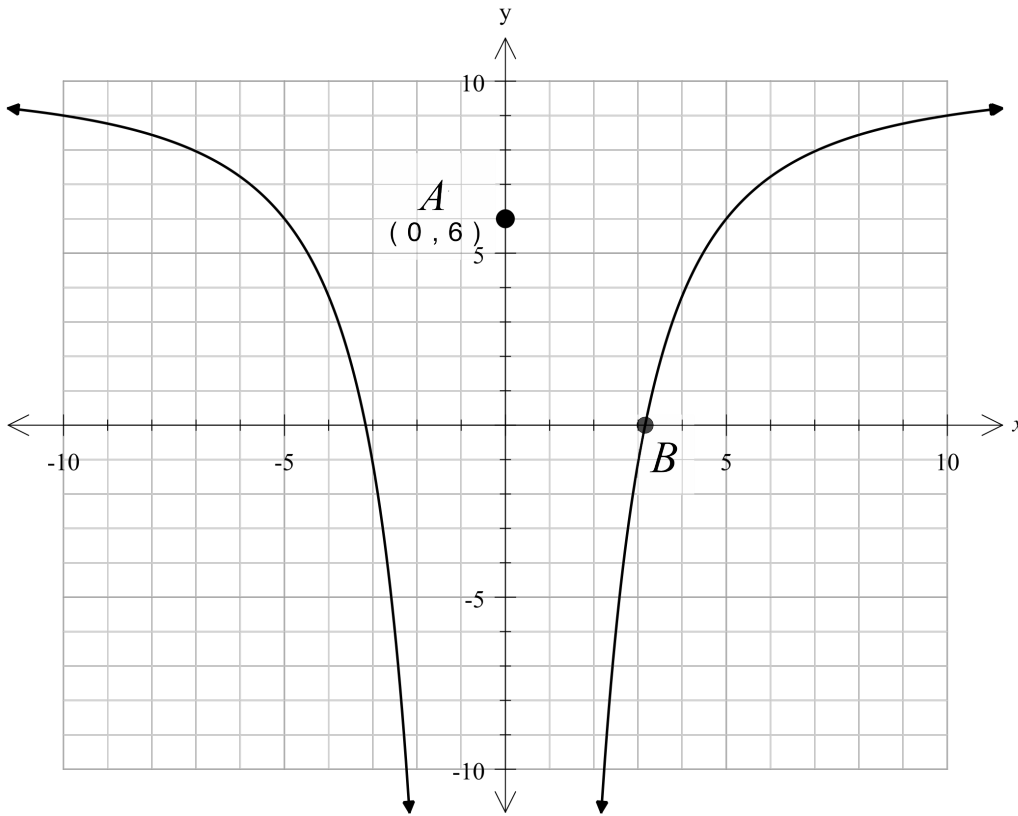


The equation of the curve upon which point $B$ can be found is, $y = \dfrac{10x^2 - 100}{x^2}$ .

**a.** Find, correct to 3 decimal places, the coordinates of point $B$.                    1 mark

**b.** Due to the pain from losing his hand, Colin can't focus and use the Math as well as he could in the past. As such, he decides to use an approximation technique to find the coordinates of point $B$.

    **i.** Find the equation of the tangent line to the curve $y$ at $x = 4$ .                    2 marks

**ii.** Hence, find the approximate values for the coordinates of point *B*, correct to 2 decimal places, that Colin would find by using 3 iterations of Newton's Method.　　3 marks

*Colin, having determined where to aim his fall, lets go of the platform and falls. He hits his target and falls straight into an open gas port which he slides down until he finds himself hanging on the underside of Cloud City with the Gas Giant Planet of Bespin below him. He uses the Maths and calls upon Princess Lisa to rescue him.*

*Princess Lisa was on board the Millennium Falcon with Emilando Breslinian, Cvetkovskbacca and the droids. Emilando, who had become increasingly frustrated by the Darth Kermond's evil ways, had the Hyperbolic-drive on the Millennium Falcon repaired and then helped our heroes escape Cloud City and the Exponential's Sin Troopers.*

*Princess Lisa hears Colin's call and directs Emilando and Cvetkovskbacca to Colin's location at the base of Cloud City. They quickly bring him on board the Millennium Falcon and make their escape.*

*Once they arrive back with the Quadratic fleet, Colin gets a new, artificial hand to replace his lost hand and our heroes begin working on a plan to rescue Harnath Solo from the clutches of the vile gangster, Corkill the Hutt.*

**END OF EPISODE V – THE EXPONENTIAL STRIKES BACK**