**VICTORIAN CERTIFICATE OF EDUCATION**

**2016**

**STUDENT NAME:**

**TEACHER NAME:**

# MATHEMATICAL METHODS (CAS)

# MATH WARS

## Episode VI – Return of the Jedpi

**2016**

**Reading Time: 10 minutes**
**Writing time: 60 minutes**

**QUESTION AND ANSWER BOOK**

| *Number of questions* | *Number of questions to be answered* | *Number of marks* |
|---|---|---|
| 4 | 4 | 35 |
| | *Total* | 35 |

- Students are permitted to bring into the test room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved CAS Calcuator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination: blank sheets of paper and/or correction fluid/tape.

**Materials supplied**

- Question and answer book of pages.

**Instructions**

- Write your name in the space provided above on this page and circle your teacher's initials.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the test room.**

**May the Maths be with you… always**

**Instructions**

Answer **all** questions in the spaces provided

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**Episode VI**

**RETURN OF THE JEDPI**

*Colin Shnierwalker* has returned to his home *planet of Tan(2)ine* in an attempt to rescue his *friend Harnath* Solo from the clutches of the *vile gangster* Corkill the Hutt.

*Little does* Colin know that the GALACTIC *EXPONENTIAL* has secretly begun *construction* on a new armored space station *even more* powerful than the first dreaded *Polynomial* Star.

*When completed*, this ultimate weapon will *spell* certain doom for the small band of *Quadratics* struggling to restore freedom to *the galaxy...*

*At the end of the last episode, Harnath Solo was frozen in carbonite by Darth Kermond and sent to Corkill the Hutt with bounty hunter Chisholm Fett. Determined to rescue their friend, Colin Shnierwalker and Princess Lisa devise an elaborate plan to break into Corkill the Hutt's palace and free Harnath.*

*Initially, Emilando Breslinian gets a job as one of Corkill's guards. Next, C3B4MePO and Range2Domain2 deliver a message to Corkill the Hutt on behalf of Colin Shnierwalker, requesting that Corkill release Harnath Solo. Corkill the Hutt laughs at this request and instead takes the droids as slaves. Princess Lisa then pretends to be a bounty hunter who has captured Cvetkovskbacca and she manages to unfreeze Harnath Solo before being captured herself.*

*Finally, Colin Shnierwalker, claiming to now be a Jedpi Knight, arrives at the palace and demands to speak to Corkill. During his talk to Corkill the Hutt, Colin gets thrown into a pit where he is forced to battle a fearsome creature known as the Rangecor who is intent on eating Colin.*

*Colin, however, has the maths on his side…*

**Question 1** (6 marks)

Whilst trying to evade the Rangecor's giant claws, Colin manages to find a large bone of length 1.5 meters. Eventually the Rangecor catches Colin in its large claw and raises him up to eat him. The height of the Rangecor's upper jaw, in meters, as it opens and closes over a 5 second interval, can be modelled by the equation:

$$U:[0,5]\to R,\ \mathrm{U}(t)=9+2\sin\left(\frac{\pi t}{5}\right)$$

The height of the Rangecor's lower jaw, in meters, over this same time can be modelled by the equation:

$$L:[0,5]\to R,\ L(t)=d-5\cos\left(\frac{\pi t}{5}-\frac{\pi}{2}\right)$$

where $c$ and $d$ are positive, real numbers.

**a.** Given that the Rangecor's mouth is closed (lower jaw and upper jaw are at the same height) at $t=0$ seconds and at $t=5$ seconds, show that $d$ =9. 2 marks

**b.** Colin intends to use the 1.5 meter long bone that he had picked up to wedge the Rangecor's mouth open.

**i.** Find an equation, $d(t)$, for the vertical distance between the upper and lower jaw at time, $t$ seconds. 1 mark

**ii.** Hence, find the time(s), to 3 decimal places, at which the vertical distance between the Rangecor's jaws is equal to the length of the bone. 2 marks

**iii.** What is the last moment in time that Colin could jam the bone into the Rangecor's jaws in order to wedge them open? Justify your answer with a reason. 1 mark

*Colin successfully wedges the bone into the Rangecor's jaws and whilst the Rangecor roars in frustration as it tries to dislodge the bone, he drops to the ground and runs to the escape door. Unfortunately the door is locked and the Rangecor comes after him. As it crouches down to try and reach Colin, Colin throws a rock at a switch on the wall and a large portcullis gate closes down and crushes the Rangecor.*

*Corkill the Hutt is furious and he summons the young Jedpi Knight as well as Harnath and Cvetkovskbacca before him. Corkill the Hutt decides that he will punish Colin and his friends by throwing them into a large sand monster known as the Sarlacc.*

*As Colin is forced to walk off a plank suspended above the Sarlacc's open mouth, he twists and uses the plank to launch himself into the air. On a nearby sail barge, Range2Domain2 waits patiently. As Colin flies through the air, Range2Domain2 throws Colin his lightsaber. Colin catches it and quickly frees Harnath and Cvetkoskbacca. With the help of Emilando Breslinian, they then proceed to rescue Princess Lisa and the droids from Corkill the Hutt, ending Corkill's life in the process of escaping.*

*Having rescued Haranth Solo, the heroes leave Tan(2)ine to return to the Quadratic Fleet. Colin Shnierwalker, however, makes a detour in his x-wing fighter to visit* $d(y(o))/da$ *on Dagobah and seek confirmation that his father is in fact Darth Kermond.*

*When Colin arrives on Dagobah he finds a very old and tired* $d(y(o))/da$ *on his deathbed.* $d(y(o))/da$ *confirms that Darth Kermond is Colin's father and that Colin's final test to become a Jedpi Knight is to face Darth Kermond again. As* $d(y(o))/da$ *takes his final breaths, he tells Colin that there is another Shnierwalker before finally passing away and becoming one with the maths.*

*As Colin returns to his* x*-wing, the spirit of Colin's old mentor, Obi-Wan Kebohni, appears to Colin to discuss how Colin is feeling about Darth Kermond being his father. Obi-Wan explains why he didn't tell Colin the truth and then reveals that Colin does have a sister as* $d(y(o))/da$ *suggested and that his sister is in fact Princess Lisa. Obi-Wan warns Colin to be careful when facing Darth Kermond and to be wary of his master, Emperor Palissitine, who is actually a powerful Sith Lord known as Darth Sin(uous).*

*Colin then returns to the Quadratic Fleet who are making plans to attack the Exponential's partially completed, second Polynomial Star. The plan is for a small team of quadratic heroes led by Harnath Solo, to sneak onto the forest moon of Endor and destroy a shield generator that is currently creating a force field around the Polynomial Star. The Quadratic Fleet will then attack and destroy the Polynomial Star.*

*Harnath's team consists of Cvetkovskbacca, Princess Lisa, C3B4MePO, Range2Domain2 and Colin Shnierwalker. They all board a stolen Exponential Shuttle and take off to begin their journey to Endor.*

**Question 2** (10 marks)

Harnath's team arrive in their stolen Exponential Shuttle near the forest moon of Endor. In order to get safe passage to the moon's surface however, they need to pass an Exponential authentication test that involves them solving 3 different problems correctly.

**a.** Given that $g:[1,a)\to R,\ g(x)=x^2-3x+4$ and $h:\left[\frac{7}{4},4\right)\to R,\ h(x)=3-\sqrt{4-x}$, find the maximal value of $a$ for which the function $h(g(x))$ is defined. 2 marks

**b.** State the set of values of $b$, **correct to 2 decimal places**, for which the expression $2x^3+3x^2-5x+b=0$ has 3 solutions. 3 marks

**c.** Consider the function $f:(c,5] \to R, f(x) = 3x - 6x^2$

**i.** Find the minimum value of $c$ for which the inverse function $f^{-1}(x)$ exists. 2 marks

**i.** Hence find the rule for $f^{-1}(x)$ and state its domain. 3 marks

*The team successfully manages to answer the problems correctly and they are allowed to proceed to the surface of the moon. Once on the surface, they make their way on foot towards the location of the shield generator. Unfortunately they stumble upon some Sin Trooper Scouts and Princess Lisa gets separated from the group. She is just about to be captured when a cute and cuddly creature known as an* $e^{wok}$*, rescues her and brings her back to his tree-top village.*

*Meanwhile, Harnath and the rest of the team go in search of Princess Lisa and end up getting caught in one of the $e^{wok}$'s traps. The $e^{wok}$'s don't realise that the Quadratics are the good guys and so tie them up and bring them back to their village as prisoners.*

*Once at the village, they find Princess Lisa and, through some clever use of the Maths by Colin, they are able to convince the $e^{wok}$'s that they are on the same side and so are released. With their new allies, Harnath and his team make plans to attack the shield generator.*

*Colin, however, can sense the presence of Darth Kermond nearby and so decides to leave the team and face him. Before he leaves, Colin tells Princess Lisa that they are brother and sister and that Darth Kermond is their father. She begs Colin not to go but Colin insists that he has sensed good mathematics in Darth Kermond and he thinks that he can bring him back from the dark side.*

*Colin turns himself in to the Exponential Forces and is brought before Darth Kermond. Darth Kermond in turn brings Colin before Emperor Palissitine who is overseeing the final stages of construction of the second Polynomial Star.*

*Emperor Palissitine attempts to corrupt Colin Shnierwalker to the dark side of the Maths but Colin refuses to give in and a fearsome lightsaber battle ensues between he and Darth Kermond. Colin ends up winning the battle and Emperor Palissitine, angered by Colin's refusal to join the dark side, begins to electrocute Colin with lightning bolts that she fires from her hands (you can do that when you master the dark side of the Maths).*

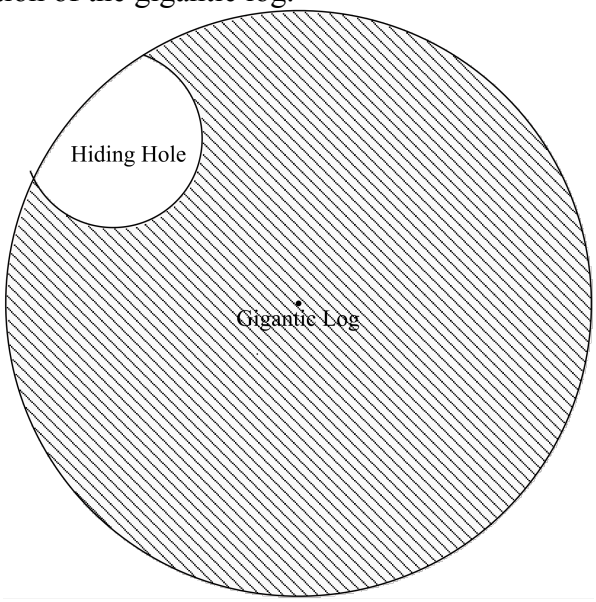
*Watching Colin being electrocuted is too much for Darth Kermond though and he turns against the dark side of the Maths and picks up Emperor Palissitine and throws her off the balcony before she can kill Colin. In the process, Darth Kermond is mortally wounded and he only has a few moments to tell Colin that he was right and that there was still good maths inside him.*

*Colin Shnierwalker, having now faced Darth Kermond and survived, is a true Jedpi Knight. He proceeds to make his escape from the Polynomial Star before it is blown up by the Quadratic Fleet.*

*Meanwhile, on the surface of Endor, Harnath Solo's team are engaged in a battle with Exponential Sin Troopers for control of the shield generator. It isn't looking good for the Quadratics until suddenly they are joined by the $e^{wok}$ army who help to turn the tide of the battle.*

**Questino 3** (8 marks)

The $e^{wok}$'s are excellent trap builders and they make use of lots of traps in the forest to fight the Exponential troops. One such trap that the $e^{wok}$'s have built involves rolling a gigantic log down a slope to squash any Exponential troops. Carved into the log are small holes that allow an $e^{wok}$ that is caught in the path of the log, to hide in and avoid being squashed. The diagram below shows a cross-section of the gigantic log.




The height, *H*, in meters, of the hiding hole above the ground as the gigantic log rolls along the slope can be modelled by the equation:

$$H(t) = 15 + 15\sin(nt)$$

where *t* is the time in seconds since the log starts rolling.

**a.** It is known that the log completes 5 full revolutions in 3 seconds.

**i.** Show that the value of *n* is $\frac{10\pi}{3}$ 2 marks

**ii.** According to the model, at what times in the first 2 seconds is the hiding hole on the ground? 2 marks

An $e^{wok}$ trap designer notices that the actual times at which the hiding hole first reaches the ground is at $t = 0.5$ seconds. The $e^{wok}$ changes the model to the form:

$$H_2(t) = 15 + 15\sin\left(\frac{10\pi t}{3} - p\right)$$

where $p$ is a positive, real number.

**iii.** Find the smallest value of $p$ that will allow the model to accurately reflect the time at which the hiding hole first reaches the ground. 2 marks

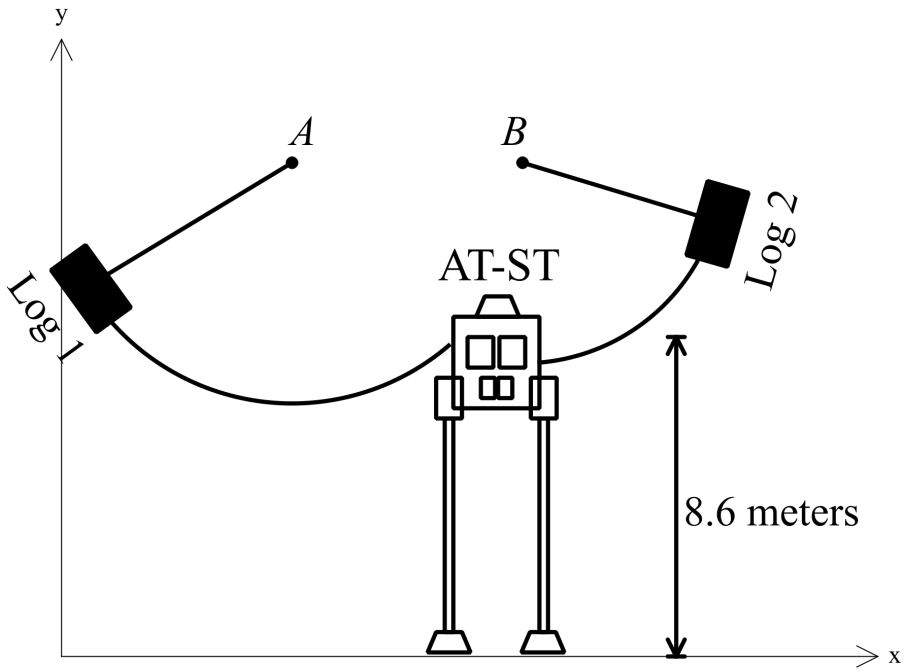
**iv.** What is the radius of the gigantic log? 1 mark

**v.** Using your answer to **part iv.** find the distance, **in meters and to 2 decimal places**, between successive points where an $e^{wok}$ can make use of the hiding hole being on the ground. Assume that the $e^{wok}$ will be in the exact centre of the hiding hole. 1 mark

**Question 4** (11 marks)

Another one of these traps consists of two large logs that are connected to very tall trees by long ropes. When these are released, the logs swing through the air and collide with the Exponential attack vehicles known as AT-ST's.

A diagram to show how these traps work is shown below, note that this diagram is not to scale.




In the particular example that is shown above, the point at which the logs need to hit the AT-ST is 8.6 meters above the ground. Points $A$ and $B$ are at coordinates $(7,16)$ and $(11,16)$ respectively.

**a.** The path of Log 1 can be described by the equation $L_1(x) = 16 - \sqrt{81-(x-7)^2}$ where $x$ is the horizontal distance from the point where Log 1 is released and $L_1$ is the height above ground of Log 1.

**i.** Find the height, in meters correct to **2 decimal places**, from which Log 1 is initially released. 1 mark

**ii.** Log 1 will collide with the AT-ST just after it has passed its lowest point. Show that the coordinates at which Log 1 collides with the AT-ST are $(12.1,\ 8.6)$. 1 mark

**b.** The path of Log 2 can be described by a different equation of the form:

$$L_2(x) = f - \sqrt{d^2 - (x-e)^2}$$

**i.** Explain why, when considering Log 2, $e = 11$ and $f = 16$. 1 mark

**ii.** In the equation for $L_2$ above, $d$ can be described as the length of the rope connecting Log 2 to point B. If Log 2 collides with the AT-ST at coordinates $(12.1,\ 8.6)$, find the value of $d$ to **2 decimal places**. 2 marks

Most of these traps are working well throughout the forest, but during the battle, Cvetkovskbacca comes across a group of $e^{wok}$ 's who are having trouble setting up their trap.

This particular group of $e^{wok}$ 's have had to move the location of the original point $B$ to the right, call it point $C$, due to the branches on which the ropes are tied not being in the correct location.

Cvetkovskbacca and the $e^{wok}$ 's spot an AT-ST approaching with it's top hath open and a Sin Trooper poking his head out of the top of the AT-ST. Cvetkovskbacca sees this as a perfect opportunity to steal the AT-ST and use it against the Exponential Troops.

Some quick calculations show that the $e^{wok}$ 's and Cvetkovskbacca should aim for Log 2 to hit anywhere in the target region of $(12.1,\ 8.6)$ to $(12.1,\ 9.6)$.

**c.** Consider point $C$, with general coordinates $(a,\ b)$, to be a point on the straight branch that joins the points $(13,\ 16)$ and $(20,\ 20)$.

**i.** Find all possible values for the amount of rope, $R$, needed in order for Log 2 to be able to hit the target region, in terms of $a$ only. 4 marks

**ii.** The $e^{wok}$ 's have 8 meters of rope available to them. What values of $a$ can they use? 2 marks

*With the assistance of the $e^{wok}$ army, Harnath Solo and his team defeat the Exponential troops and then successfully manage to destroy the shield generator protecting the Polynomial Star.*

*With the shield down, Admiral Ackbartlett, the commander of the Quadratic Fleet, leads the fleet in its attack of the Polynomial Star. While the large cruiser ships battle the Exponential's Star Destroyers, a small group of fighters, lead by Emilando Breslinian in the Millennium Falcon, fly into the center of the Polynomial Star and fire a proton torpedo at the core. This torpedo sets off a chain reaction that destroys the Exponential's ultimate weapon in a massive explosion.*

*With Emperor Palissitine and Darth Kermond now deceased, and Exponential forces scattered after the battle with the Quadratic Fleet, the Exponential Empire collapses and the Quadratics restore freedom to the galaxy.*

**END OF EPISODE VI – RETURN OF THE JEDPI**