

VICTORIAN CERTIFICATE OF EDUCATION

2017

STUDENT NAME:

MATHEMATICAL METHODS (CAS)

SAC 3



SACVIVOR

2017

Reading Time: 10 minutes

Writing time: 110 minutes

QUESTION AND ANSWER BOOK

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
8	8	62
	<i>Total</i>	62

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved CAS Calculator and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 15 pages.
- Formula sheet

Instructions

- Write your name in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Let X = the number of blank pages
 $Pr(X=1)=1$

Instructions

Answer **all** questions in the spaces provided

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

It is one of the most stunning places in the world, JMSS Maths Island. Crystal clear water, diverse and dangerous wildlife, and hundreds of breathtaking islands, this will be the battleground for a Sacvivor culture war. 16 JMSS teachers are divided into two tribes.

The Millennials ranging in age from 22 years old to 35 years old. They share a world view of:

“My generation is about doing what you want to do and I have done a lot. I have swam with sharks and gotten a degree. I am definitely young at heart and will never grow up.”
- Elise Breslin

“I am a gamer. As an adult I am consistently told to grow up and stop playing it’s not a game. But I want to make my entire life about playing. Sacvivor is the game of games and I am going to win.”
- Liam Bohni

“The stereotypes of young people are that they are always on social media. They are always on twitter. But I don’t like being tied to my phone. I genuinely resent technology. I think twitter is the worst thing ever invented.”
- Lily Pizzol

Generation X ranging in age from 40 years old to 60 years old.

“I know the older generation they take information and they actually listen and they use those life experiences. If you have ever been knocked down, then you know how to get up and fight.”
- Jim Kermond

“I have kids and I am surprised by the 25 year-old kids who sit at home and play video games all day. And so I just feel like they are a little bit more Que Sera Sera. If it takes me seven years to do school who cares my parents are paying for it.” - Kat Cvetkosvska

“I am an assistant principal and it took me 14 years to finally become one. And I don’t think younger people would put that kind of time into whatever their dream is. They would just try to find a new dream.”
- Alex Chisholm

These two generations have two very different approaches to life while being abandoned and left to fend for themselves. They must learn to adapt or they will be voted out. In the end only one will remain to claim the million dollar prize. 39 days, 18 people, one survivor, who will outwit outlast, outplay.



Question 1 (15 marks)

Two tribes, ‘Millennials’ and ‘Generation X’, are about to be stranded on two separate remote beaches. Before they leave they are asked to choose some survival supplies, which will help them survive for the entire 39 days. They will determine how many items they can choose by spinning a wheel. The wheel contains the numbers 2 through 7 on it. The survivors will spin the wheel and this will determine the number of survival items they will be allowed to take with them to their beach. Let X be the random variable ‘the number that the wheel stops on’. The probability that the wheel will stop on number x , is shown in the following table.

x	2	3	4	5	6	7
$\Pr(X = x)$	0.05	0.20	k^2	$3k$	0.1	0.05

a. Find the value of k for this distribution.

3 marks

b. i. Find the mean of the random variable X .

1 mark

ii. Find the median of the random variable X .

1 mark

iii. Find the standard deviation of the random variable X , correct to 2 decimal places.

2 marks

The tribes are allowed to take 2 spins of the wheel to determine how many survival items they can collect in total.

- c. i.** Find the probability, correct to 2 decimal places, that the tribe ‘Generation X’ is allowed to chose 8 survival items. 2 marks

- ii.** In the first spin the tribe ‘Millennials’ obtain 3 survival items. Find the probability that they end up with more then 8 survival items. 1 mark

Both tribes use the wheel and are allowed to take 10 survival items to their new beaches. Before they leave for their new beaches they are given the opportunity to win the most important survival tool, flint. Flint will help the sacvivors create fire and they will be able to stay warm during the long nights on Maths Island. Both tribes are required to solve an individual maths question.

The ‘Millennial’ tribe are asked the following question about the wheel.

- d.** The wheel is spun 8 times and the number of survival items obtained each time is recorded. What is the probability, correct to 4 decimal places, that 3 survival items are recorded no more than 3 times? 2 marks

The ‘Millennial’ tribe are successful in winning flint. They will be able to start a fire on their beach and cook their food. The ‘Generation X’ tribe are asked the following question relating to the wheel.

- e. What is the least number of spins required in order to be at least 80% sure of getting 6 survival items at least 10 times?

3 marks

Unfortunately, the ‘Generation X’ tribe does not answer the question correctly. As a result, they will be required to start their fire without the aide of a flint. Hopefully, their hard work and ‘can do’ attitude will help them instead.



Both tribes begin to set up camp on their new beach. At ‘Generation X’s’ beach they are focussed on getting a fire started, so that by night-time they have some warmth.

Question 2 (7 marks)

The ‘Generation X’ tribe know that the probability of starting a successful fire within t hours of their first attempt is a continuous random variable, T , with probability density function given by:

$$f(t) = \begin{cases} a(t^2 + b) & 0 \leq t \leq 12 \\ 0 & \text{elsewhere} \end{cases}$$

where a and b are real numbers.

- a.** If the mean of T is 7, find the value of a and b .

3 marks

‘Generation X’ were given the incorrect mean for T , as a result, they are told the values of a and b are $a = \frac{1}{1296}$ and $b = 60$. Use these values for parts **b.** and **c.** below.

- b.** The sun sets eight hours after ‘Generation X’ starts trying to make a fire. What is the probability that they do not get their fire started until after the sun sets?

2 marks

- c.** Find, correct to 3 decimal places, the median number of hours taken to create a fire.

2 marks

Meanwhile, on the beach at the ‘Millennial’ tribe’s camp there is a much more relaxed atmosphere. Due to the large number of survival items they have won and their ability to make fire with flint the ‘Millennial’s’ camp is set up in under an hour. As a result the tribe go exploring. They find a dam full of fish, not far from their camp.

Question 3 (10 marks)

On their first day using a fishing net the ‘Millennials’ capture 10 carp, 50 sturgeon and 40 algae eaters. The ‘Millennials’ mark these fish with red paint and return them to the dam.

Assume that on their second day, the tribe capture 20 carp, of which 1 has red paint on it, 16 sturgeon of which 4 have red paint on them, and 7 algae eaters of which 5 have red paint on them.

- a. Estimate the number of carp, sturgeon and algae eaters in the dam. Assuming that the 43 fish captured are representative of the entire population.

3 marks

On further investigation of the algae eaters and carp, the contestant Liam Bohni notices that the lengths of the fish are normally distributed with means of 25 cm and 40 cm respectively and standard deviations of 2 cm and 4cm respectively.

- b. i. Find the probability, correct to 5 decimal places, that an algae eater has a length greater than 30 cm.

2 marks

- ii. If the probability that a carp has a length of less than c cm is equal to the probability that an algae eater has a length greater than 30 cm, what is the value of c ?

2 marks

One of the other contestants in the ‘Millennials’ tribe, Elise Breslin, reminds Liam Bohni that algae eater fish cannot be removed from the dam if they are smaller than some value c and larger than some value d . This is because fish need time to grow and breed.

- c. Determine the value of c and d , correct to the nearest centimetre, if 2% of the algae eater fish are below minimum length and 6% are above maximum length.

3 marks

Both tribes received tree mail outlining the imminent arrival of their first immunity challenge, where one tribe will lose and go to tribal council. The tree mail read as follows:

‘You’ve been playing so well, and now I want to know just how far in this game you are willing to go. In this game, I can’t force you to do maths, but this challenge rewards those who strive for their goal.

Your algebra is tested! How long will you spend before quitting and saying that you’ve reached the end? You control your own fate, and there’s no end in sight, so get comfortable! You’re gonna be here all night!’



Both tribes arrive at the immunity challenge ready to use their strong will and determination to solve a series of both physical and mental problems.

In the first part of the challenge, three people from each tribe must work together to solve a mathematical problem. Once they have solved the problem their tribe can move onto the second part of the immunity challenge. The problem posed to the tribes is as follows:

Question 4 (2 marks)

Three people work independently at deciphering a message in a code. The probability that they will each individually decipher the code are $\frac{1}{5}$, $\frac{1}{4}$, and $\frac{1}{3}$. What is the probability that the code will be deciphered by at least one of them?



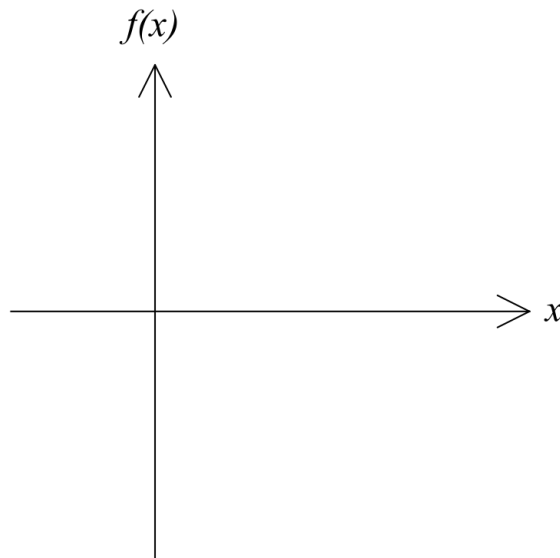
The ‘Millennials’ take an early lead in the fight for immunity by solving the stage one question the fastest. The second part of the immunity challenge involves only one member of the tribe. This tribe member has to solve the following question as fast as possible.

Question 5 (6 marks)

The continuous random variable, X , has a probability density function $f(x)$ given by

$$f(x) = \begin{cases} \frac{1}{4}x & 0 \leq x \leq 2 \\ 1 - \frac{1}{4}x & 2 < x \leq 4 \\ 0 & \text{elsewhere} \end{cases}$$

- a.** Sketch, $y = f(x)$ labelling the coordinates of the end points for each part of the function. 2 marks



- b.** Find $\Pr\left(1 \leq X \leq \frac{7}{2}\right)$. 1 mark

- c. i.** Find the mean of X . 1 mark

- ii.** Find the mode of X . 1 mark

- iii.** Find the standard deviation of X , correct to 2 decimal places. 1 mark

After the second part the ‘Millennials’ hold a small lead over ‘Generation X’ in the immunity challenge. However, this is SACvivor and anything can happen.

Question 6 (7 marks)

In stage 3 of the immunity challenge a contestant from each tribe must ride a bike around a course. Unfortunately the wheels on the bikes they are using are designed to easily puncture. The distance these bikes can cycle between punctures is modelled by the random variable, W , metres, with probability density function $g(w)$ given by:

$$g(w) = \begin{cases} 0.005e^{-0.005w} & w > 0 \\ 0 & \text{elsewhere} \end{cases}$$

- a.** Find the mean distance, in metres, the contestants can cycle for before they get a puncture. 1 mark

- b.** What is the probability, correct to 3 decimal places, that the contestants will travel at least 500 metres before getting a puncture? 2 marks

- c.** What is the probability, correct to 3 decimal places, that the contestants will travel less than 30 metres before getting a puncture? 2 marks

The contestant racing for the ‘Generation X’ team, Jim Kermond, has a puncture within 30 metres. When Jim starts racing again he has another puncture k metres further along. Assuming that punctures are independent of one another, according to this model the probability of a puncture occurring within 30 metres and then another puncture within k metres is 0.01.

- d.** What is the value of k , to the nearest metre? 2 marks

Due to Jim Kermond’s two flat tyres the ‘Generation X’ tribe started falling even further behind. It was looking like the ‘Millennials’ would take victory and not be going to tribal council to send someone home. However, the final part of the immunity challenge involved some high powered maths which gave the ‘Generation X’ tribe an advantage due to their years of experience as mathematicians.

Question 7 (10 marks)

In the final part the remaining three contestants from each tribe are required to solve the following problem.

They are presented with a large population of coloured coins. They are told that 85% of the coins have the survivor slogan ‘**Outwit, Outlast, Outplay.**’

The contestants are asked to chose a sample of 80 coins and provide answers to the following questions:

- a. What is the expected number of coins and the standard deviation of the sample with the survivor slogan written on them? 3 marks

- b. Find, correct to 5 decimal places, the probability that at least 90% of the coins in the sample have the survivor slogan written on them. 2 marks

- c. Use the normal approximation to find the probability, correct to 5 decimal places, that at least 90% of the coins in the sample have the survivor slogan written on them. 2 marks

Out of nowhere the 'Generation X' tribe answer the above three questions quickly and efficiently. As a result they are now in front of the 'Millennial' tribe. In order to win immunity all 'Generation X' need to do is determine the answer to the final question below.

- d. The contestants count 73 coins in their sample with the survivor slogan written on them. What is the 90% confidence interval, correct to 5 decimal places, for the number of coins with the survivor slogan on them?

3 marks

'Generation X' come from behind to solve the final task before the 'Millennials' this results in the 'Millennials' going to tribal council that night to vote one member of their team off. Who will be left standing and who will become the sole survivor.....?



The ‘Millennials’ attend tribal council. The tribe is divided on who to send home. Liam Bohni and Lily Pizzol both receive 4 votes each. After revote the tribe is still evenly split with 4 votes each. According to survivor rules when a tribe cannot decide who goes home the contestants who have had votes casted against them are safe and the remaining members of the tribe now draw rocks from a bag to see who goes home.

Question 8 (5 marks)

The remaining six contestants must draw rocks from a bag. In the bag are 9 rocks. Six of these rocks are red and the other rocks are black. Each contestant will take it in turns to draw a rock from the bag without replacing the rock until the first black rock is drawn. The person who draws the first black rock will go home.

- a. Calculate the probability, correct to 3 decimal places, that the third contestant draws a black rock.

2 marks

- b. Calculate the probability, correct to 3 decimal places, that none of the 6 contestants draw a black rock after they have all chosen one rock each.

2 marks

- c. As it happens, none of the remaining six contestants drew a black rock. They were then told to continue to draw rocks from the bag. What is the probability that the first contestant to redraw from the bag will go home?

1 mark

As the seventh rock is drawn a black rock appears and the contestant holding the rock in her hand is Elise Breslin. Elise Breslin is the first contestant voted out of Sacvivor. Who will eventually be the ultimate winner....?

END OF SAC 3

Next week on SACvivor....