VICTORIAN CERTIFICATE OF EDUCATION

2017

STUDENT NAME:

MATHEMATICAL METHODS (CAS)

SAC 1 – EPISODE 1

Harry Potter and the Chamber of SACs

2017

Reading Time: 15 minutes
Writing time: 120 minutes

QUESTION AND ANSWER BOOK

Number of questions	*Number of questions to be answered*	*Number of marks*
5	5	57
	Total	57

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved CAS Calculator and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 16 pages.
- Formula sheet

Instructions

- Write your name in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

It's nearing the end of semester one at Hogwarts School of Witchcraft and Wizardry. It's been a long term and, as you sit in Professor McGonapizzol's last Transfiguration class for the term, listening to her drone on about the importance of doing things in the correct order when applying transformations, you start to daydream about a nice warm mug of Butterbeer from 'The Three Broomsticks' in Hogsmeade.

Suddenly everyone starts standing up and moving around. Professor McGonapizzol must have set an activity for you to do. You turn to the person next to you to find out what you are meant to be doing…

Question 1 (9 marks)

Professor McGonapizzol has written the equation $y = \frac{3}{\sqrt{x+4}} - 2$ on the board. Next to it, she has written the following set of 5 transformations.

1. Reflection in the x axis.
2. Dilation by factor $\frac{1}{2}$ from the x axis.
3. Translation of 5 units to the right.
4. Translation of 3 units down.
5. Dilation by factor 4 from the y axis.

a. Apply the 5 transformations to the equation y, in the order described above, and state the resulting equation after each transformation. 5 marks

In the meantime, that red headed kid, Ron Weasley, has managed to stuff up one of the transformations (again) and has ended up with the equation $y_{new} = 1 - \frac{12}{\sqrt{x-4}}$.

Ron insists that he did the transformations in the correct order but likely messed up one of the 5 transformations some how.

b. Identify the single mistake that Ron Weasley most likely made to end up with equation y_{new}. 1 mark

Suddenly the bell rings signalling the end of class. As you and your fellow students begin packing up your books Professor McGonapizzol waves her wand and a question appears on the board. "Please complete this question for homework everyone" Professor McGonapizzol announces, "It will be due in the first lesson back after the term break. Happy Holidays".

You've got a bit of time before your next class so you decide to try and get the homework question out of the way early so that you can enjoy your holidays.

The question asked by Professor McGonapizzol is:

c. What sequence of 3 transformations will change the equation $h = 2\sqrt{4-x}$ into the new equation $h_{new} = \sqrt{x}$? 3 marks

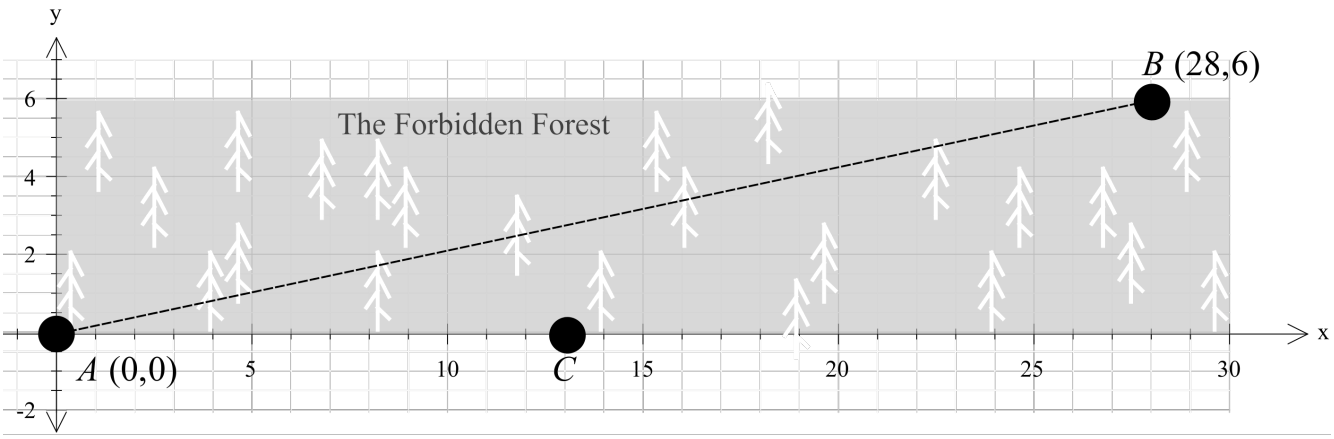
Your next class is Care of Mathemagical Creatures with Professor Alexeus Hagrilescu. As you arrive at Professor Hargilescu's cabin, you notice that the class is assembling outside meaning that you might finally be able to learn about something other than Flobberworms for a change.

"Alright it looks like everyone is here so let's get started" Professor Hagrilescu begins. "Today we are going to learn about capturing and returning creatures who have escaped from their enclosures. A Niffler escaped from its enclosure last night and your task is to try and locate and recapture the creature before the end of the lesson."

Opening up your book, "The Monster Book of Monsters", you do some research on Nifflers. According to your book, Nifflers are long snouted, burrowing creatures who are very attracted to shiny things.

You go to the Niffler enclosure and take a look around. Off in the distance you notice a bright shine coming from the other side of the Foridden Forest. Given Nifflers are attracted to shiny things you come to the conclusion that the Niffler is most likely trying to run to the bright shine that you see.

Rather than immediately racing off after the Niffler, you decide to try and work out the fastest route to take to get to the bright shine. You draw the following diagram to represent the scenario.



Initially you and the Niffler started at point *A*, with coordinates $(0,0)$ on the diagram, and the bright shine is located at point *B*, with coordinates $(28,6)$. All distances are measured in km.

Question 2 (23 marks)

a. The average speed of a Niffler is 1.8 km/h through the forest. How long, correct to the nearest minute, would it take for the Niffler to reach the bright shine if it goes directly from point *A* to point *B*? 2 marks

b. You can run at 5 km/h through the forest and 8km/h outside of the forest.

i. If you were to go directly from point A to point B, how long, to the nearest minute, would it take for you to reach the bright shine? 1 mark

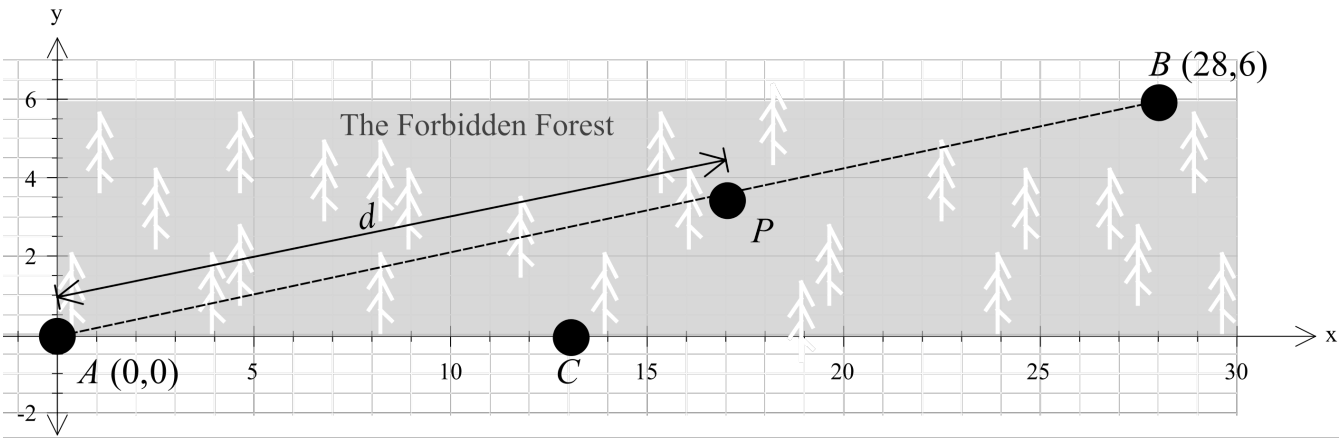
Instead you decide to run from point A to point C along the outside of the forest and then from point C to point B.

ii. If point C were at the coordinates $(28,0)$, how long, correct to the nearest minute, would it take for you to travel from A to C and then from C to B? 2 marks

iii. If point C were at the coordinates $(c,0)$, find an expression for the length of time T, in hours, that it would take to travel from A to C and then from C to B. 2 marks

iv. Hence find, correct to two decimal places, the coordinates of point C that will result in you taking the minimum amount of time to travel from A to C and then from C to B. State what this minimum amount of time is correct to the nearest minute. 3 marks

You realise that the Niffler won't have had enough time to reach the bright shine since escaping overnight and so will currently be somewhere along the line segment joining point *A* to point *B*. You decide to call the current position of the Niffler point *P* and alter your diagram to include this point.



c. You don't know how long ago the Niffler escaped the enclosure so you decide to try and work out where the Niffler would be in terms of the time, *t*, in hours since it escaped the enclosure.

Let *x* represent the horizontal distance travelled from Point A and *y* represent the vertical distance travelled from Point A.

i. Find the equation of the straight line that passes through the point *A* and point *B* in terms of *y* and *x*. 1 mark

ii. Hence, find an equation for the distance, *d,* between Points *A* and *P*, in terms of *x*. 2 marks

iii. Recall that the Niffler can travel at 1.8 km/h through the forest. Show that at time t, the coordinates of point P are given by $\left(\frac{126t}{5\sqrt{205}}, \frac{27t}{5\sqrt{205}}\right)$. 2 marks

iv. If point C were at the coordinates $(a, 0)$, find an expression in terms of a and t for the length of time T, in hours, that it would take you to travel from A to C and then from C to P. 2 marks

v. Hence find, correct to two decimal places, the value of a that will result in you taking the minimum length of time, T, to travel from A to C and then from C to P in terms of t. 3 marks

Armed with the results you have just found in **part c.v.**, you now rush over to the Niffler enclosure to try and determine how long ago the Niffler escaped. You notice some Niffler footprints in the mud outside the enclosure leading away towards the bright shine. You know that it only started raining earlier this morning and so you know that the Niffler must have escaped after the rain started.

d. According to your estimates, the Niffler escaped the enclosure 3 hours ago.

i. Using the coordinates of point P you found in **part c. iii.**, find the coordinates of the Niffler at $t = 3$ hours. 1 mark

ii. Find the minimum amount of time, correct to the nearest minute, that it would take for you to reach the location of the Niffler at $t = 3$ hours. 2 marks

You arrive at the location of the Niffler just in time to see Hermione Grainger picking up the creature and heading off to return it to its enclosure. "Damn!" you think to yourself. If only you'd been a bit quicker working out those sums.

As you return to class Professor Hagrilescu pats you on the back and says "You know you almost beat her to it… and she found it through sheer good luck she did. She asked for help from them there Centaurs. Didn' use any fancy mathemagics like you did. Almost discovered my Aragog in the process she did." Hargilescu suddenly straightens up and sticks his hand in his belt before muttering to himself "Probably shouldn't have told yeh tha'!" and then walks away.

The class now being over, you collect your books and start to make your way back up to the castle. As you are making your way back, however, a strong breeze kicks up and blows your scarf right off your neck. You begin to chase after the scarf when it gets caught around the base of a nearby tree. You begin to walk towards the scarf when "WHOOMP!" A massive tree branch slams into the ground right in front of you. You take a step back and realise that this isn't just any tree… it's the Whomping Willow.

The Whomping Willow is a very violent form of mathemagical plant which will attempt to attack anyone or anything that comes within range of its branches.

You can't just abandon your favourite scarf though. Your parents would stupefy you if they found out that you'd lost it. So you decide to figure out a way to get past the Whomping Willow's branches and retrieve your scarf.

You sit down to observe the seemingly random movements of the Whomping Willow's branches and notice something amazing… they aren't random at all but instead follow what appears to be a repeated pattern.

Question 3 (8 marks)

The first branch you watch has a length of 5.3 meters. At its lowest point it is 1.2 meters above the ground and at its highest point it is 11.8 meters above the ground. It completes a full revolution every 30 seconds before then repeating the same movement again.

You decide that you can model the height, B, of the end of this branch by the equation $B(t) = a\cos(bt) + c$ where a, b and c are positive real numbers and t is the time in seconds after the end of this branch has reached the highest point.

a. Using the information that you have collected, find the values of a, b and c for the function $B(t)$. 3 marks

You decide that this particular branch looks a bit dangerous and so you move around to a different side of the tree. After a few minutes of observation, you come up with another two models for the ends of the two branches that cover the path you wish to follow towards your scarf.

You model Branch 1 with the equation:

$$B_1(t) = -4.1\sin\left(\frac{\pi t}{45}\right) + 4.3$$

You model Branch 2 with the equation:

$$B_2(t) = 6.2\cos\left(\frac{\pi t}{50}\right) + 7$$

b. You estimate that you will need at least 30 seconds to be able to get to the trunk of the tree and back again with your scarf. To be safe, you decide that you will only run to the trunk of the tree when both branches are at least 2 meters above the ground.

i. Find the time(s) during the first 2 minutes when B_1 is greater than 2 meters above the ground. Give all answers correct to the nearest second. 2 marks

ii. Find the times during the first 2 minutes when B_2 is greater than 2 meters above the ground. Give all answers correct to the nearest second. 2 marks

iii. Remembering that you require 30 seconds to reach the scarf and get back out again, during what time period is it possible for you to run in and grab your scarf in the first 2 minutes? 1 mark

Success! You manage to get past the Whomping Willow's branches and retrieve your scarf. Unfortunately, you were a bit slower than you thought and on your way out, the Whomping Willow gave you a good wallop to the side of the head. The last thing you see before you pass out is a gigantic spider down by the edge of the Forbidden Forest.

You awake to find yourself in one of Madam Palissefrey's hospital beds. Nearby are a few other Hogwarts students who are recovering from being petrified. The rumour, so your friends tell you, is that Harry Potter, one of the students in your class, is actually the heir of Slytherin and that he has opened the Chamber of SACs and has been pertifying students all over the school. You also learn that Professor Corkilldore has been removed as school headmaster and that the Ministry of Magic has arrested Professor Hargilescu.

Your head hurts thinking about all of these events and probably also from the wack to the head that the Whomping Willow gave you. As you rest, Madam Palissefrey arrives with some medicine. "We're afraid that you have been infected with some parasitic mites my dear" she explains to you. "It's a very easy thing to fix up but I'm afraid I've been a bit swamped lately with all of the students coming in being petrified. Would you mind administering the dosage of medicine by yourself?" You agree and Madam Palissefrey leaves you the bottle of medicine along with a set of instructions on how to administer it.

The concentration of parasitic mites in your bloodstream can be modelled as a function of time. The concentration, C, of parasitic mites per mL of blood, is given by

$$C:[0,\infty) \to R,\ C(t) = 9\left(3^{t-2} + \frac{2}{3}\right)$$

where t is the number of hours after initial infection.

Question 4 (9 marks)

You estimate that you were infected 4.5 hours ago.

a. i. What was the initial concentration of parasitic mites, in mites per mL, in your bloodstream correct to the nearest whole number? 1 mark

ii. What is the current concentration of parasitic mites, in mites per mL, in your bloodstream correct to the nearest whole number? 1 mark

The label on the bottle of medicine says that, once taken, the concentration of parasitic mites in your bloodstream will decrease according to the rule

$$P(x) = \frac{10a}{3}\log_e(23 - x)$$

where x is the number of hours after taking the medicine and a is a positive, real number that represents the amount, in mL, of medicine taken.

Assume that the concentration of parasitic mites in your blood when you drink the medicine is 150 mites per mL;

b. **i.** What is the correct dosage, a, required to treat your infection correctly? 2 marks

You decide that an appropriate amount of medicine to drink is 14mL.

ii. At this dosage, how many hours will the medicine take to remove all parasitic mites from your bloodstream? 1 mark

You look at the time and realise that if the potion takes this long, you won't be able to attend the upcoming Quidditch match between Hufflepuff and Ravenclaw. In the hope of being able to make the game, you make the rather rash decision to double the dosage of the medicine to 28mL instead.

iii. Briefly explain why doubling the value of a doesn't change the amount of time taken to remove all parasitic mites from your bloodstream. 1 mark

Being determined to watch the upcoming Quidditch Match, you decide to enlist the help of the school librarian, Madam Braden. She brings you some books about making potions work faster and you discover there is a special dye known as "*Layshon*" which can decrease or increase the time a potion takes to work.

When the dye "*Layshon*" is mixed with the medicine, the concentration of parasitic mites in your bloodstream decreases according to the rule $P_D(x) = \frac{140}{3}\log_e(23 - bx)$, where b is a positive, real number.

c. **i.** What value of b will result in the time taken to remove all of the parasitic mites being halved? 1 mark

The Quidditch game starts in 3 hours.

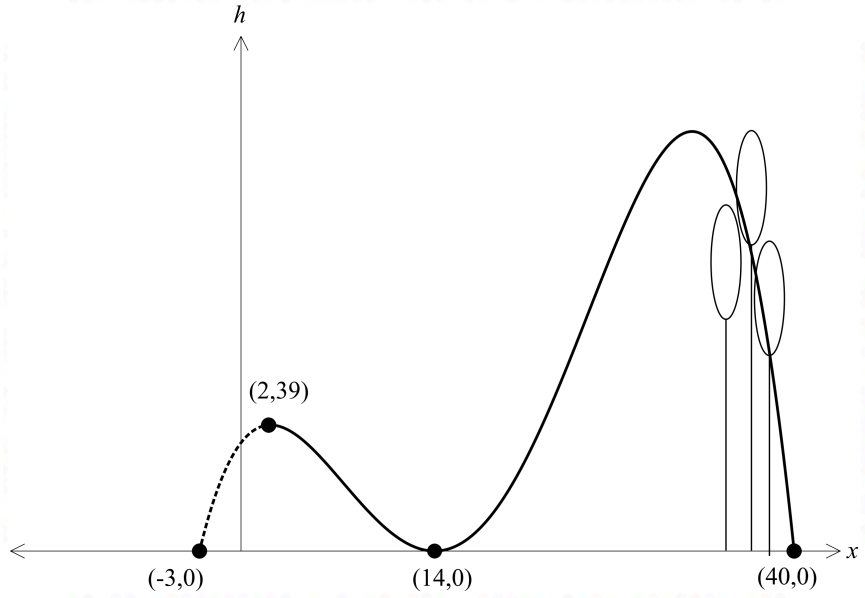
ii. What value of b will result in the time taken to remove all of the parasitic mites being reduced to 3 hours? 2 marks

Having determined the amount of the dye "Layshon" to add to the medicine, you mix them together and drink it in one gulp. 3 hours later you find yourself being released from Madam Palissefrey's hospital and you make your way to watch the final game of Quidditch before the school holidays.

Whilst watching the game, your friends fill you in on all of the details about how Harry Potter isn't actually the Heir of Syltherin and that he in fact saved Ginny Weasley from a Basalisk. They also tell you about how the charges against Professors Corkilldore and Hagrilescu have turned out to be false and that they are now reinstated in their old positions at the school. They also tell you about the new Defence Against the Dark Arts teacher, Professor Gilderoy Lockharnath, who had abruptly resigned and left the school.

Suddenly a bludger slams into the stand you are sitting in. You assume that one of the Ravenclaw beaters must have miss-hit but then you notice one of the quaffles flying in a very erratic fashion.

On a piece of scrap paper you happened to have in your pocket, you draw a quick graph of the path of the quaffle as it travels through the air and through one of the goal hoops.



You first started observing the quaffle when it was located at the point (2,39) on your diagram but your best friend tells you that it actually started moving from the ground at (-3,0). As you watch the quaffle, you see that it hits the ground once at (14,0) before flying upwards and then passing through the middle left most goal hoop which has a height of 67.5 meters above the ground. It then proceeds to smash into the ground at (40,0) and stop moving.

Looking at your finished sketch, you identify that the path of the quaffle appears to have followed the path of a quartic function.

Question 5 (8 marks)

You think back to your charms classes a few years ago with Professor Flitwhitty. She had drilled into you and your classmates that there were a variety of different models that could be used to represent a quartic function.

a. Consider the quartic models given below. State the features of a quartic graph that would make each of these models the most appropriate one to use in finding the equation of a quartic graph.

i. $f_1(x) = a(x-h)^4 + k$ 1 mark

ii. $f_2(x) = ax^4 + bx^3 + cx^2 + dx + e$ 1 mark

iii. $f_3(x) = a(x-b)(x-c)(x-d)(x-e)$ 1 mark

iv. $f_4(x) = a(x-b)^2(x-c)(x-d)$ 1 mark

v. $f_5(x) = a(x-b)^3(x-c)$ 1 mark

b. Use one of the five models from **Question 5, part a** to find the equation for the path of the quaffle that was sketched. 3 marks

You notice that the referee, Madam Zhaooch has noticed the enchanted quaffle and has removed the ball from play. Thankful that you don't have do any spells to try and restore the quaffle to its un-enchanted form, you decide to return to your bed and have a bit of a rest.

END OF SAC 1 - EPISODE 1