**STUDENT NAME:**

# MATHEMATICAL METHODS (CAS)
## SAC 1 – EPISODE 3



## Harry Potter and the Deathly SACs

**2017**

**Reading Time: 10 minutes**
**Writing time: 50 minutes**

### QUESTION AND ANSWER BOOK

| Number of questions | Number of questions to be answered | Number of marks |
|---|---|---|
| 4 | 4 | 42 |
| | *Total* | 42 |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved CAS Calculator and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination: blank sheets of paper and/or correction fluid/tape.

**Materials supplied**
- Question and answer book of 11 pages.
- Formula sheet

**Instructions**
- Write your name in the space provided above on this page.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

**Instructions**

Answer **all** questions in the spaces provided

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.



*The Dark Lord has returned. The death of Professor Corkildore at the end of last year and the attack on the Ministry of Magic have led to dark times in the wizarding world. This past year at Hogwarts has subsequently also been quite dreary.*

*The school corridors are no longer filled with the laughter and cries of excited witches and wizards. Instead students walk from class to class in silence. Even the Quidditch matches between houses lack the energy and excitement of years past.*

*Today, your usual routine is interrupted as Harry Potter, Hermione Grainger and Ronald Weasley have returned to Hogwarts announcing that the Dark Lord himself is planning an attack on the school.*

*Now, you find yourself standing in the school courtyard alongside other members of "Corkildore's Army" and "The Order of the Phoenix", awaiting the attack from the Dark Lord and his Death Eaters.*

*Everything is quiet when suddenly, right in front of you, a Death Eater appears and the battle begins.*

*You think back to your old Defense Against the Dark Arts classes with Professor Quirrius Quermond. In these classes Professor Quermond, apart from yelling a lot, had taught you some of the basic defensive spells. The first one that comes to mind is Protego, a relativey straight forward spell that creates a temporary shield in front of you which can deflect incoming spells and curses.*

**Question 1** (12 marks)

A birds eye view of the courtyard is depicted below. You are positioned at the point *X* which has coordinates (3,1), and the Death Eater you are battling is located at point *D* which has coordinates (1,4).
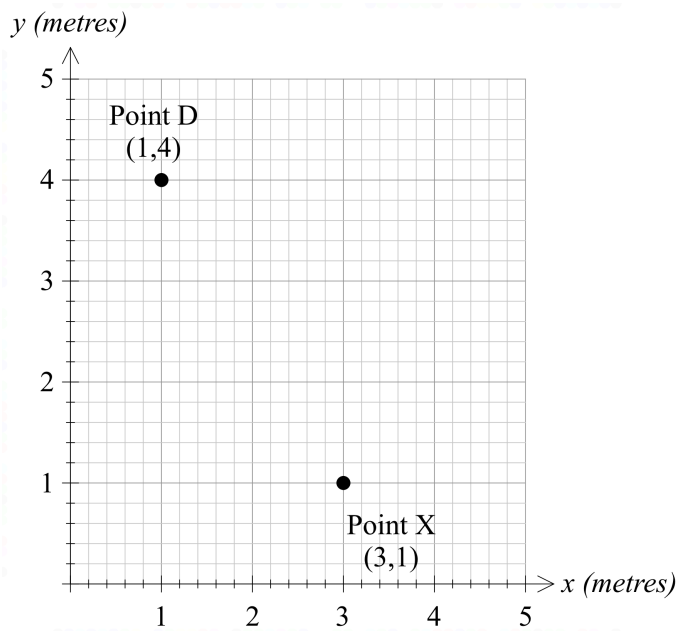


**a.**    How far away from you, in meters, is the Death Eater?                    2 marks

The Death Eater fires a Stupefy Charm along a straight line directly towards you.

**b.**    Show that the equation of the straight line along which the Stupefy Charm will travel can be expressed as $y = \dfrac{-3}{2}x + \dfrac{11}{2}$.                    2 marks
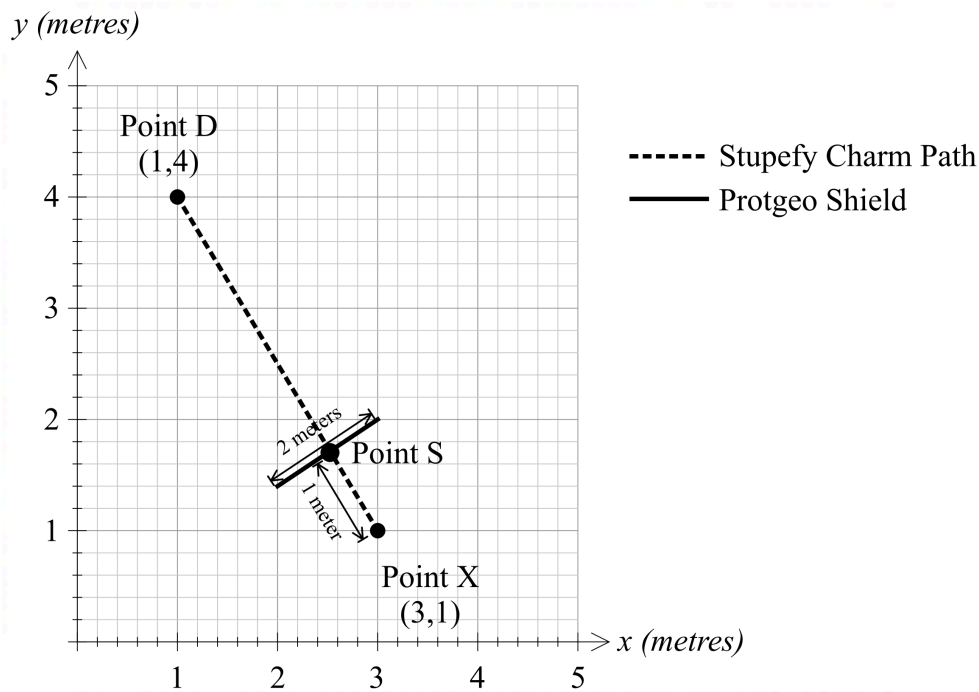
You cast the Protego Spell which produces a shield one meter directly in front of you as you face the Death Eater at Point D. This shield will deflect any incoming charm, spell or curse. The shield is 2 meters wide and is linear.



**c.    i.**    Let the *x* coordinate of Point S be given by the variable *a* which represents the middle of the Protego Shield. State the coordinates of Point S in terms of *a*.          1 mark

**ii.**    If the Point S is located one meter in front of your location at Point X, find the coordinates of Point S.          3 marks

The 2 meter long Protego Shield you produce is exactly perpendicular to the incoming Stupefy Charm.

    **iii.**   Find the equation of the line that represents the Protego Shield. State any necessary domain restrictions.    4 marks

You successfully block the incoming Stupefy Charm and it reflects back at the Death Eater who casted it. Unfortunately they are also aware of defensive spells and so manage to block the Stupefy Charm from hitting them also.

You notice that nearby is another Death Eater located at the Point E which has coordinates (4,3). This Death Eater is focussed on attacking another wizard and so you decide to manipulate your Protego Shield so that it deflects the Stupefy Charm being fired by the Death Eater at Point D towards the Death Eater at Point E.

The Diagram below shows the scenario.
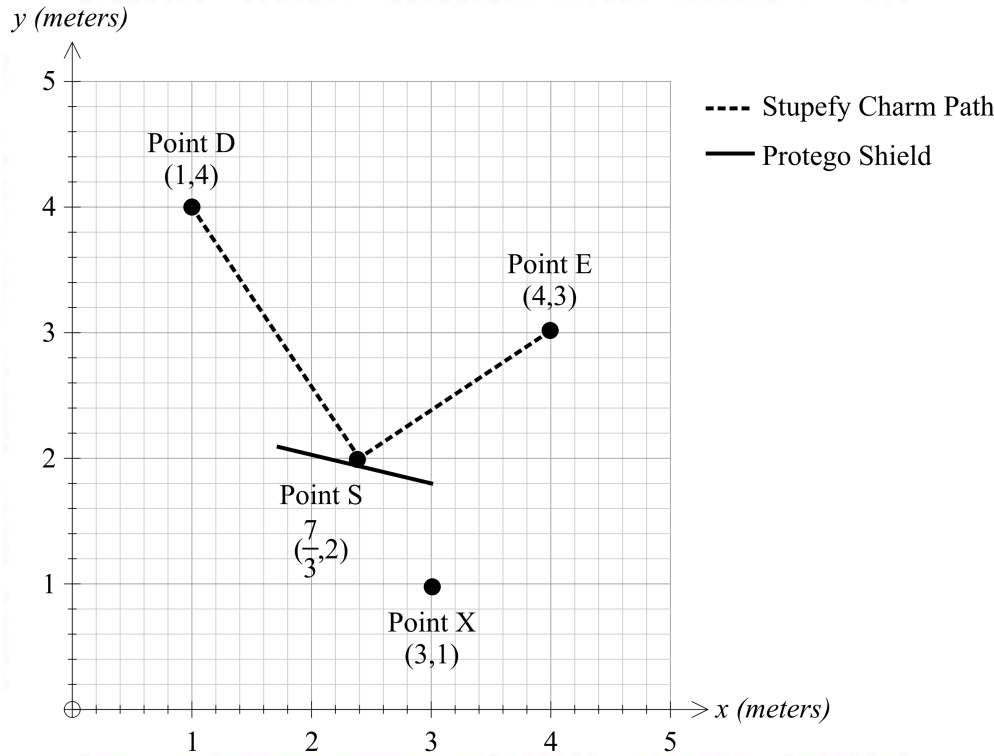


**Question 2** (8 marks)

You cast your Spell so that the centre of the 2 meter long Protego Shield is located at Point S with coordinates $\left( \dfrac{7}{3}, 2 \right)$.

**a.**  Find the gradient of the line joining Point D and Point S and hence, determine the acute angle, to the nearest degree, that this line makes from the *x* axis.                2 marks
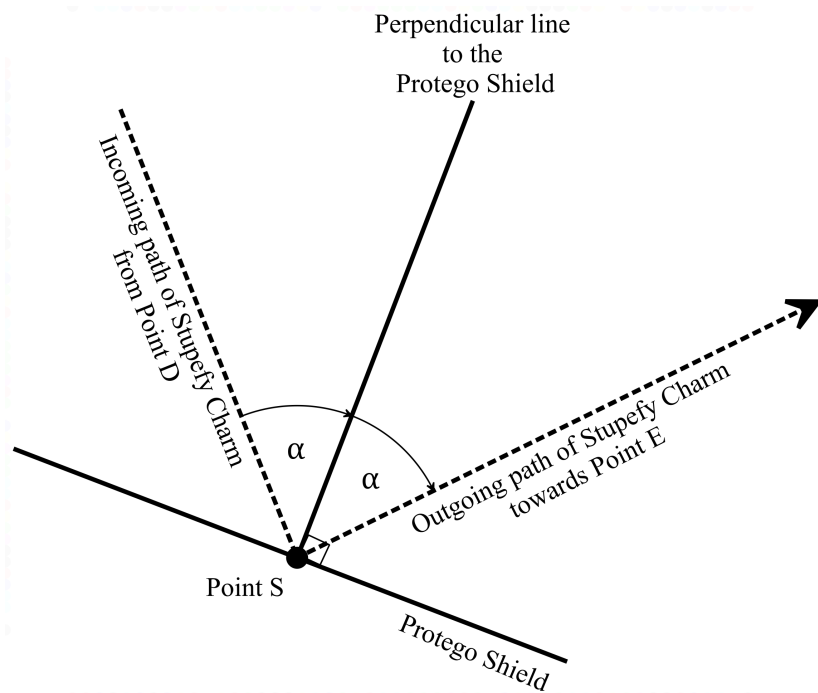
The angle between the incoming path of the Stupefy Curse and the line that is perpendicular to the Protego Shield will be the same as the angle between the line that is perpendicular to the Protego Shield and the outgoing path of the Stupefy Curse.



**b.**    Find the gradient of the line joining Point E and Point S and hence, determine the acute angle, to the nearest degree, that this line makes from the *x* axis.                    2 marks

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**c.**    Hence show that the total angle between the incoming and outgoing paths of the Stupefy Charm is $93°$.                    1 mark

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You know that the angle of the perpendicular line to the Protego Shield must be $\dfrac{93^\circ}{2}$ in a clockwise direction from the incoming path of the Stupefy Charm.

**d.**  **i.**  Use this information as well as your answer to **Question 2 part b**, to determine the gradient of the perpendicular line to the Protego Shield, correct to 2 decimal places.      2 marks

**ii.**  **Hence** determine the equation of the line that represents the required location of the Protego Shield so that the Stupefy Charm will be deflected at the Death Eater located at Point E. Express all constants correct to two decimal places.      2 marks

Just as you successfully cast your Protego Shield Spell as calculated, you end up deflecting the Stupefy Charm at the Death Eater located at Point E and thus render the Death Eater unconscious. You pump your fist in the air and cry out with an enthusiastic 'whoop!' when another wave of Death Eaters suddenly appear randomly, all around the courtyard.

Reluctant to repeat the individual calculations that you just did in **Question 2** to try and deflect Stupefy Charms at each of these new Death Eaters, you decide that it would instead be easier to create a generalised formula that you can then quickly input the coordinates of each new Target Death Eater to determine how to place your Protego Shield so that they end up being stunned.

**Question 3** (8 marks)

You begin by choosing a random point in the courtyard, Point P, with the general coordinates $(p,q)$, where $p$ and $q$ are both real numbers and $p > \dfrac{7}{3}$ and $q > 2$.

**a.**   **i.**   Show that an expression for the gradient of the line joining Point S and Point P is given by $m_{SP} = \dfrac{6-3q}{7-3p}$                                                                     1 mark

**ii.**   Hence, write an expression for the acute angle, $\theta$, that this line makes with the $x$ axis in terms of $p$ and $q$.                                                                     1 mark

The Death Eater who is still firing Stupefy Charms at you from Point D has moved to try and get around your Protego Shield and the incoming path of the Stupefy Charms now make an acute angle of $20°$ with the $x$ axis.

**b.**   Using your answer to **Question 3 part a.ii**, show that a general expression for the gradient of the perpendicular line to the Protego Shield will be:

$$m_\perp = \tan\left(80 + \frac{1}{2}\tan^{-1}\left(\frac{6-3q}{7-3p}\right)\right)$$

3 marks

**c.** Keeping the center of your Protego Shield located at Point S, with coordinates $\left(\dfrac{7}{3}, 2\right)$, find the generalised equation for the Protego Shield in terms of $p$ and $q$.          2 marks

Now that you have your generalised equation, you decide to test it out on a nearby Death Eater. The coordinates of this Death Eater are (5,5).

**d.** What is the equation of the Protego Shield that you must cast in order to deflect the Stupefy Charm at the Death Eater located at (5,5)? Express your answer in the form $y = mx + c$ where $m$ and $c$ are both real numbers expressed to an accuracy of 2 decimal places.          1 mark

As you begin to dispatch enemy Death Eaters with ease, the original Death Eater begins to realise what you are doing and tries to come up with a way of tricking you into letting your guard down.

The first thing that this Death Eater does is send another Death Eater to a coordinate $(p,q)$, where $p$ and $q$ are both real numbers but where $p < \dfrac{7}{3}$ and $q > 2$.

**Question 4** (4 marks)

**a.**    State the effect that this change to the coordinates of point P will have on your answer to **Question 3, part b**.                                                                    1 mark

**b.**    State the new gradient that can be used instead of your answer to **Question 3, part b,** to address this change in the values of $p$.                                                   3 marks

*Thwarted by your brilliance yet again, the Death Eater does something drastic and instead fires a Descendo Charm at Professor Sybill Bresliney's Divination Tower causing it to shatter apart and start falling into the courtyard.*

*Professor Bresliney, who is standing not far from where you are, cries out in dismay as she watches her Divination Tower disintegrate. Whilst she is distracted a Death Eater casts a curse directly at Professor Bresliney. Luckily, the Auror Nymphadolpha Tonks, is nearby and steps in to protect the Professor.*

*Confident that Nymphadolpha Tonks and Professor Bresliney will be safe, you drop your Protego Shield and cast the Wingardium Leviosa Spell and catch the falling rocks in mid air.*

*Your opponent Death Eater smiles wickedly at you thinking that you are now defenceless. They begin to cast a curse towards you when you turn and smile back at them…*

*…It turns out you didn't catch all of the falling stones and instead let one get through which proceeds to fall directly onto your opponent Death Eater's head.*

*With the Death Eaters in the courtyard defeated, you and the remaining wizards and witches move through Hogwarts to drive out any other Death Eaters that might remain.*

*As you search Hogwarts, you hear the Dark Lords voice boom out asking for Harry Potter to come forth so that the lives of those still in Hogwarts might be spared.*

*Later it turns out that Harry did answer the Dark Lord's call and, in fact ended up defeating the Dark Lord once and for all.*

*Glad that the battle (and the SAC for that matter) is over, you decide to go home and do some relaxing maths homework.*

**END OF SAC 1 - EPISODE 3**