MATHEMATICAL METHODS (CAS)

SAC 1 – SOLUTIONS

Harry Potter and the Chamber of SACs

Question 1 (9 marks)

Professor McGonapizzol has written the equation $y=\frac{3}{\sqrt{x+4}}-2$ on the board. Next to it, she has written the following set of 5 transformations.

1. Reflection in the x axis.
2. Dilation by factor $\frac{1}{2}$ from the x axis.
3. Translation of 5 units to the right.
4. Translation of 3 units down.
5. Dilation by factor 4 from the y axis.

a. Apply the 5 transformations to the equation y, in the order described above, and state the resulting equation after each transformation. 5 marks

Correct reflection in the x axis $y_1=-\frac{3}{\sqrt{x+4}}+2$	**A1**
Correct dilation by factor $\frac{1}{2}$ from the x axis $y_2=-\frac{3}{2\sqrt{x+4}}+1$	**A2**
Correct translation of 5 units to the right. $y_3=-\frac{3}{2\sqrt{(x-5)+4}}+1$ OR $y_3=-\frac{3}{2\sqrt{x-1}}+1$	**A3**
Correct translation of 3 units down. $y_4=-\frac{3}{2\sqrt{x-1}}-2$	**A4**
Correct dilation by factor 4 from the y axis. $y_5=-\frac{3}{2\sqrt{\frac{1}{4}x-1}}-2$ OR $y_5=-\frac{3}{\sqrt{x-4}}-2$	**A5**

In the meantime, that red headed kid, Ron Weasley, has managed to stuff up one of the transformations (again) and has ended up with the equation $y_{new} = 1 - \frac{12}{\sqrt{x-4}}$.

Ron insists that he did the transformations in the correct order but likely messed up one of the 5 transformations some how.

b. Identify the single mistake that Ron Weasley most likely made to end up with equation y_{new}. 1 mark

Ron most likely made the mistake in his application of Transformation 2: Dilation by factor ½ from the x axis. Instead of performing $y_2 = \frac{1}{2} y_1$ he performed $y_2 = 2y_1$. Accept any reasonable explanation or identification of this error even if student does not specify the factors but only identifies that the dilation from x was incorrect.	**A1**

Suddenly the bell rings signalling the end of class. As you and your fellow students begin packing up your books Professor McGonapizzol waves her wand and a question appears on the board. “Please complete this question for homework everyone” Professor McGonapizzol announces, “It will be due in the first lesson back after the term break. Happy Holidays”.

You’ve got a bit of time before your next class so you decide to try and get the homework question out of the way early so that you can enjoy your holidays.

The question asked by Professor McGonapizzol is:

c. What sequence of 3 transformations will change the equation $h = 2\sqrt{4-x}$ into the new equation $h_{new} = \sqrt{x}$? 3 marks

Dilate by factor $\frac{1}{2}$ from the x axis. $h_1 = \sqrt{4-x}$	**A1**
Reflect in the h axis $h_2 = \sqrt{4+x}$ Students should not be penalised for saying reflect in the vertical axis but should lose a mark for incorrect variable if they state reflect in the y axis.	**A2**
Translate 4 units to the right $h_3 = \sqrt{x}$ Note that students do not need to state the equations after each transformation, only the identification of the transformation itself is needed for each mark.	**A3**

OR

Dilate by factor $\frac{1}{2}$ from the x axis. $h_1 = \sqrt{4-x}$	**A1**
Translate 4 units to the left $h_2 = \sqrt{-x}$	**A2**
Reflect in the h axis $h_3 = \sqrt{x}$	**A3**

** ACCEPT ANY OTHER CORRECT SERIES OF TRANSFORMATIONS (ie: Reflection before Dilations)

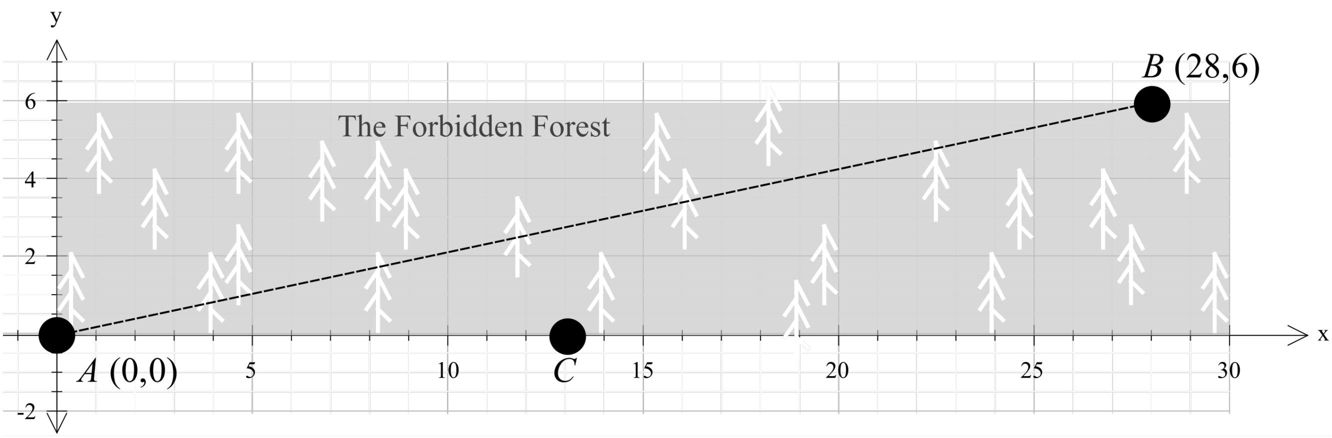
Your next class is Care of Mathemagical Creatures with Professor Alexeus Hagrilescu. As you arrive at Professor Hargilescu's cabin, you notice that the class is assembling outside meaning that you might finally be able to learn about something other than Flobberworms for a change.

"Alright it looks like everyone is here so let's get started" Professor Harilescu begins. "Today we are going to learn about capturing and returning creatures who have escaped from their enclosures. A Niffler escaped from its enclosure last night and your task is to try and locate and recapture the creature before the end of the lesson."

Opening up your book, "The Monster Book of Monsters", you do some research on Nifflers. According to your book, Nifflers are long snouted, burrowing creatures who are very attracted to shiny things.

You go to the Niffler enclosure and take a look around. Off in the distance you notice a bright shine coming from the other side of the Foridden Forest. Given Nifflers are attracted to shiny things you come to the conclusion that the Niffler is most likely trying to run to the bright shine that you see.

Rather than immediately racing off after the Niffler, you decide to try and work out the fastest route to take to get to the bright shine. You draw the following diagram to represent the scenario.



Initially you and the Niffler started at point *A*, with coordinates $(0,0)$ on the diagram, and the bright shine is located at point *B*, with coordinates $(28,6)$. All distances are measured in km.

Question 2 (24 marks)

a. The average speed of a Niffler is 1.8 km/h through the forest. How long, correct to the nearest minute, would it take for the Niffler to reach the bright shine if it goes directly from point *A* to point *B*? 2 marks

$D=\sqrt{(28-0)^2+(6-0)^2}$ $D=28.64\,km$	**M1**
$t=\dfrac{D}{1.8}=15.909$ hours $t=15$ hours, 55 minutes OR 955 minutes	**A1**

b. You can run at 5 km/h through the forest and 8km/h outside of the forest.

i. If you were to go directly from point A to point B, how long, to the nearest minute, would it take for you to reach the bright shine? 1 mark

$t=\frac{D}{5}=\frac{28.64}{5}$ =5.73 hours * accept consequential answers from part a $t=5$ hours, 44 minutes OR 344 minutes	**A1**

Instead you decide to run from point A to point C along the outside of the forest and then from point C to point B.

ii. If point C were at the coordinates $(28,0)$, how long, correct to the nearest minute, would it take for you to travel from A to C and then from C to B? 2 marks

$t=\frac{D_{AC}}{8}+\frac{D_{CB}}{5}$ $t=\frac{28}{8}+\frac{6}{5}$ accept either line of working as evidence of this mark	**M1**
$t=4.7$ hours $t=4$ hours, 42 minutes OR 282 minutes	**A1**

iii. If point C were at the coordinates $(c,0)$, find an expression for the length of time T, in hours, that it would take to travel from A to C and then from C to B. 2 marks

$D_{AC}=c$ $D_{CB}=\sqrt{(28-c)^2+(6-0)^2}=\sqrt{820-56c+c^2}$ This mark is for correct identification of the distance from C to B. Students do not need to expand the expression or simplify to get the mark.	**M1**
$t=\frac{D_{AC}}{8}+\frac{D_{CB}}{5}$ $t=\frac{c}{8}+\frac{\sqrt{820-56c+c^2}}{5}$ This mark should be consequential on the distance found in M1. Final expression can be in un-expanded form but 6^2 should be simplified to 36.	**A1**

iv. Hence find, correct to two decimal places, the coordinates of point C that will result in you taking the minimum amount of time to travel from A to C and then from C to B. State what this minimum amount of time is correct to the nearest minute. 3 marks

$\frac{dt}{dc} = \frac{c-28}{5\sqrt{c^2-56c+820}} + \frac{1}{8} = 0$ Award consequential answers from part iii.	**M1**
$c = 23.20$ $C \rightarrow (23.20, 0)$ Students must state coordinates of point C to get this mark	**A1**
$t = 4.4367$ hours $t = 4$ hours 26 minutes OR $t = 266$ minutes	**A2**

You realise that the Niffler won't have had enough time to reach the bright shine since escaping overnight and so will currently be somewhere along the line segment joining point A to point B. You decide to call the current position of the Niffler point P and alter your diagram to include this point.

c. You don't know how long ago the Niffler escaped the enclosure so you decide to try and work out where the Niffler would be in terms of the time, t, in hours since it escaped the enclosure.

Let x represent the horizontal distance travelled from Point A.

i. Find the equation of the straight line that passes through point A to point B in terms of y and x. 1 mark

$y_{AB} = \frac{3}{14}x$ Fraction must be in simplified form	**A1**

ii. Hence, find an equation for the distance, d, between Points A and P, in terms of x. 2 marks

This mark should be awarded for substitution of (0,0) and (x,3x/14) into the expression	**M1**
$d = \sqrt{x^2 + \left(\frac{3}{14}x\right)^2}$ OR $d = \frac{\sqrt{205}}{14}x$ Students do not need to simplify the answer to get this mark	**A1**

iii. Recall that the Niffler can travel at 1.8 km/h through the forest. Show that at time t, the coordinates of point P are given by $\left(\frac{126t}{5\sqrt{205}}, \frac{27t}{5\sqrt{205}}\right)$. 2 marks

$\frac{\sqrt{205}}{14}x = 1.8t$ Rearrange to make x the subject	**M1**
Substitute this value of x into the equation from part c i. $y = \frac{3}{14}\left(\frac{126}{5\sqrt{205}}t\right) = \frac{27}{5\sqrt{205}}t$ Marks are awarded for the generation of the equation and substitution of x into y. If students don't show sufficient working, including the final answer, they should lose a mark.	**M2**

iv. If point C were at the coordinates $(a, 0)$, find an expression in terms of a and t for the length of time T, in hours, that it would take you to travel from A to C and then from C to P. 2 marks

$T = \frac{D_{AC}}{8} + \frac{D_{CP}}{5}$ $T = \frac{a}{8} + \frac{\sqrt{(x-a)^2 + (y-0)^2}}{5}$ $T = \frac{a}{8} + \frac{\sqrt{(x-a)^2 + \left(\frac{3}{14}x\right)^2}}{5}$ Accept either line as evidence of correct working This mark is for identification of the distance CP	**M1**
$T = \frac{a}{8} + \frac{\sqrt{\left(\frac{126t}{5\sqrt{205}} - a\right)^2 + \left(\frac{27t}{5\sqrt{205}}\right)^2}}{5}$ OR $T = \frac{a}{8} + \frac{\sqrt{41\left(3321t^2 - 252\sqrt{205}at + 1025a^2\right)}}{1025}$ Second answer is the answer that will be obtained from use of the CAS If students find an incorrect distance CP then this mark should be consequential on their answer to M1.	**A1**

v. Hence find, correct to two decimal places, the value of a that will result in you taking the minimum length of time, T, to travel from A to C and then from C to P in terms of t. 3 marks

$\frac{dT}{da}=\frac{1}{8}-\frac{126\sqrt{5}t-25\sqrt{41}t}{25\sqrt{3321t^2-252\sqrt{205}at+1025a^2}}=0$ Students do not need to write the full expanded derivative of T to obtain this mark.	**M1**
$a=1.45808t$ OR $a=2.06201t$	**A1**
Second solution is for values of a greater than the x coordinates of the point P. $\therefore a=1.46t$ Rejection of second value must be explicitly stated or omitted from a stated domain restriction	**A2**

Armed with the results you have just found in **part c.v.**, you now rush over to the Niffler enclosure to try and determine how long ago the Niffler escaped. You notice some Niffler footprints in the mud outside the enclosure leading away towards the bright shine. You know that it only started raining earlier this morning and so you know that the Niffler must have escaped after the rain started.

d. According to your estimates, the Niffler escaped the enclosure 3 hours ago.

i. Using the coordinates of point P you found in **part c. iii.**, find the coordinates of the Niffler at $t=3$ hours. 1 mark

Sub $t=3$ into $\left(\frac{126t}{5\sqrt{205}},\frac{27t}{5\sqrt{205}}\right)$ $\left(\frac{378}{5\sqrt{205}},\frac{81}{5\sqrt{205}}\right)$	**A1**

ii. Find the minimum amount of time, correct to the nearest minute, that it would take for you to reach the location of the Niffler at $t=3$ hours. 2 marks

$a=1.45808\times3=4.37424...$ award consequential marks from part c iv.	**M1**
$T=\frac{4.37}{8}+\frac{\sqrt{\left(\frac{126\times3}{5\sqrt{205}}-4.37\right)^2+\left(\frac{27\times3}{5\sqrt{205}}\right)^2}}{5}=0.83666$ OR $T=\frac{4.37}{8}+\frac{\sqrt{41\left(3321\times3^2-252\sqrt{205}\times4.37\times3+1025(4.37)^2\right)}}{1025}=0.83666$ $T=50$ minutes award consequential marks from part d i.	**A1**

You sit down to observe the seemingly random movements of the Whomping Willow's branches and notice something amazing… they aren't random at all but instead follow what appears to be a repeated pattern.

Question 3 (8 marks)

The first branch you watch has a length of 5.3 meters. At its lowest point it is 1.2 meters above the ground and at its highest point it is 11.8 meters above the ground. It completes a full revolution every 30 seconds before then repeating the same movement again.

You decide that you can model the height, B, of the end of this branch by the equation $B(t) = a\cos(bt) + c$ where a, b and c are positive real numbers and t is the time in seconds after the end of this branch has reached the highest point. .

a. Using the information that you have collected, find the values of a, b and c for the function $B(t)$. 3 marks

$a = 5.3$	**A1**
$b = \dfrac{\pi}{15}$	**A2**
$c = 6.5$	**A3**

You decide that this particular branch looks a bit dangerous and so you move around to a different side of the tree. After a few minutes of observation, you come up with another two models for the ends of the two branches that cover the path you wish to follow towards your scarf.

You model Branch 1 with the equation:

$$B_1(t) = -4.1\sin\left(\frac{\pi t}{45}\right) + 4.3$$

You model Branch 2 with the equation:

$$B_2(t) = 6.2\cos\left(\frac{\pi t}{50}\right) + 7$$

b. You estimate that you will need at least 30 seconds to be able to get to the trunk of the tree and back again with your scarf. To be safe, you decide that you will only run to the trunk of the tree when both branches are at least 2 meters above the ground.

i. Find the time(s) during the first 2 minutes when B_1 is greater than 2 meters above the ground. Give all answers correct to the nearest second. 2 marks

$B_1(t)=2,\ t\in[0,120]$ explicit statement of the domain restriction not required for this method mark and the mark should still be awarded if students use the correct inequality rather than =.	**A1**
$t=8.53,\ 36.47,\ 98.53$ $\therefore 0\le t<9 \cup 36<t<99$ Take note of the inclusion of 0 as at this time $B(t)$ is clearly greater than 2. Note that students can also give the correct values with $0\le t\le 8 \cup 37\le t\le 98$ so long as they also justify or explain that they are doing real life rounding.	**A2**

ii. Find the times during the first 2 minutes when B_2 is greater than 2 meters above the ground. Give all answers correct to the nearest second. 2 marks

$B_2(t)=2,\ t\in[0,120]$ explicit statement of the domain restriction not required for this method mark	**A1**
$t=39.93,\ 60.07,$ $\therefore 0\le t<40 \cup 60<t\le 120$ Take note of the inclusion of 0 and 120 as at this time $B(t)$ is clearly greater than 2. Students can also state $0\le t\le 39 \cup 61\le t\le 120$ as long as they state justification of real life rounding.	**A2**

iii. Remembering that you require 30 seconds to reach the scarf and get back out again, during what time period is it possible for you to run in and grab your scarf in the first 2 minutes? 1 mark

$t\in(60,99)$ The only period of time, in the first 2 minutes, that coincides with at least 30 seconds of continuous time where both branches are at least 2 meters above the ground is between 60 seconds and 99 seconds. award consequential marks from parts i and ii as long as these answers are reasonable (within domain of 0 and 120).	**A1**

The concentration of parasitic mites in your bloodstream can be modelled as a function of time. The concentration, C, of parasitic mites per mL of blood, is given by

$$C:[0,\infty)\to R,\ C(t)=9\left(3^{t-2}+\frac{2}{3}\right)$$

where t is the number of hours after initial infection.

Question 4 (9 marks)
You estimate that you were infected 4.5 hours ago.

a. **i.** What was the initial concentration of parasitic mites, in mites per mL, in your bloodstream correct to the nearest whole number? 1 mark

$C(0)=7$ mites per mL Units are not required for mark	**A1**

ii. What is the current concentration of parasitic mites, in mites per mL, in your bloodstream correct to the nearest whole number? 1 mark

$C(4.5)=146$ mites per mL Units are not required for mark	**A1**

The label on the bottle of medicine says that, once taken, the concentration of parasitic mites in your bloodstream will decrease according to the rule

$$P(x)=\frac{10a}{3}\log_e(23-x)$$

where x is the number of hours after taking the medicine and a is a positive, real number that represents the amount, in mL, of medicine taken.

Assume that the concentration of parasitic mites in your blood when you drink the medicine is 150 mites per mL;

b. **i.** What is the correct dosage, a, required to treat your infection correctly? 2 marks

$P(0)=150$ $\frac{10a}{3}\log_e(23)=150$	**M1**
$a=\frac{45}{\log_e 23}$ Exact answer is required.	**A1**

You decide that an appropriate amount of medicine to drink is 14mL.

ii. At this dosage, how many hours will the medicine take to remove all parasitic mites from your bloodstream? 1 mark

$P(x)=0$ $x=22$ hours units are not required for mark	**A1**

You look at the time and realise that if the potion takes this long, you won't be able to attend the upcoming Quidditch match between Hufflepuff and Ravenclaw. In the hope of being able to make the game, you make the rather rash decision to double the dosage for the medicine to 28mL instead.

iii. Briefly explain why doubling the value of a doesn't change the amount of time taken to remove all parasitic mites from your bloodstream. 1 mark

Doubling the dosage is the same as dilating the graph in the vertical direction by a factor of 2. While this increases the height of the graph at each point, it does nothing to the horizontal features of the graph, in this case 'time', and so the time at which	**A1**

the concentration reaches 0 will remain unchanged. Or any similar statement or valid statement	

Being determined to watch the upcoming Quidditch Match, you decide to enlist the help of the school librarian, Madam Braden. She brings you some books about making potions work faster and you discover there is a special dye known as "*Layshon*" which can decrease or increase the time a medicine takes to work.

When the dye "*Layshon*" is mixed with the medicine, the concentration of parasitic mites in your bloodstream decreases according to the rule $P_D(x)=\frac{140}{3}\log_e(23-bx)$, where b is a positive, real number.

c. **i.** What value of b will result in the time taken to remove all of the parasitic mites being halved? 1 mark

$b=2$	**A1**

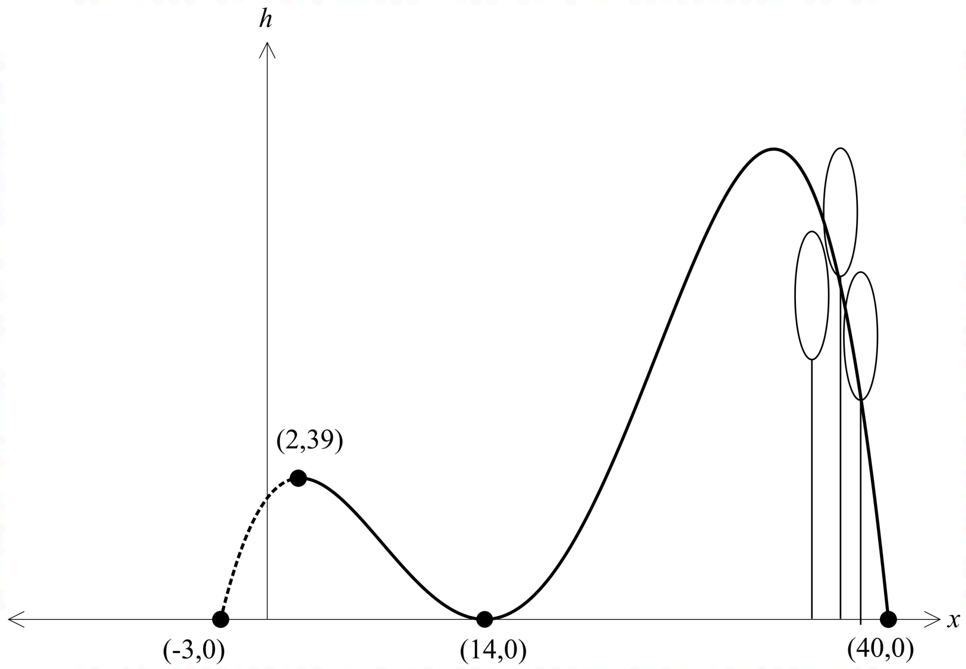
The Quidditch game starts in 3 hours.

ii. What value of b will result in the time taken to remove all of the parasitic mites being reduced to 3 hours? 2 marks

$\frac{140}{3}\log_e(23-3b)=0$	**M1**
$b=\frac{22}{3}$	**A1**

Suddenly a bludger slams into the stand you are sitting in. You assume that one of the Ravenclaw beaters must have miss-hit but then you notice one of the quaffles flying in a very erratic fashion.

On a piece of scrap paper you happened to have in your pocket, you draw a quick graph of the path of the quaffle as it travels through the air and through one of the goal hoops.



You first started observing the quaffle when it was located at the point (2,39) on your diagram but your best friend tells you that it actually started moving from the ground at (-3,0). As you watch the quaffle, you see that it hits the ground once at (14,0) before flying upwards and then passing through the middle right most goal hoop. It then proceeds to smash into the ground at (40,0) and stop moving.

Looking at your finished sketch, you identify that the path of the quaffle appears to have followed the path of a quartic function.

Question 5 (8 marks)

You think back to your charms classes a few years ago with Professor Flitwhitty. She had drilled into you and your classmates that there were a variety of different models that could be used to represent a quartic function.

a. Consider the quartic models given below. State the features of a quartic graph that would make each of these models the most appropriate one to use in finding the equation of a quartic graph.

i. $f_1(x) = a(x-h)^4 + k$ 1 mark

The graph has a single turning point located at (h,k)	**A1**

ii. $f_2(x) = ax^4 + bx^3 + cx^2 + dx + e$ 1 mark

The graph has a clear y intercept	**A1**

iii. $f_3(x) = a(x-b)(x-c)(x-d)(x-e)$ 1 mark

The graph has four x intercepts located at $(b,0)$, $(c,0)$, $(d,0)$, and $(e,0)$ Labelling of intercepts is not required	**A1**

iv. $f_4(x) = a(x-b)^2(x-c)(x-d)$ 1 mark

The graph has three x intercepts, one of which, $(b,0)$, is also a turning point	**A1**

v. $f_5(x) = a(x-b)^3(x-c)$ 1 mark

The graph has two x intercepts, one of which, $(b,0)$, corresponds to a stationary point of inflection	**A1**

b. Use one of the five models from **Question 5, part a** to find the equation for the path of the quaffle that was sketched. 3 marks

$f_4(x)=a(x-b)^2(x-c)(x-d)$ $f_4(x)=a(x-14)^2(x+3)(x-40)$ Correct form identified and points substituted in	**M1**
$f_4(2)=a(2-14)^2(2+3)(2-40)=39$	**M2**
$f_4(x)=\dfrac{-13}{9120}(x-14)^2(x+3)(x-40)$	**A1**

OR

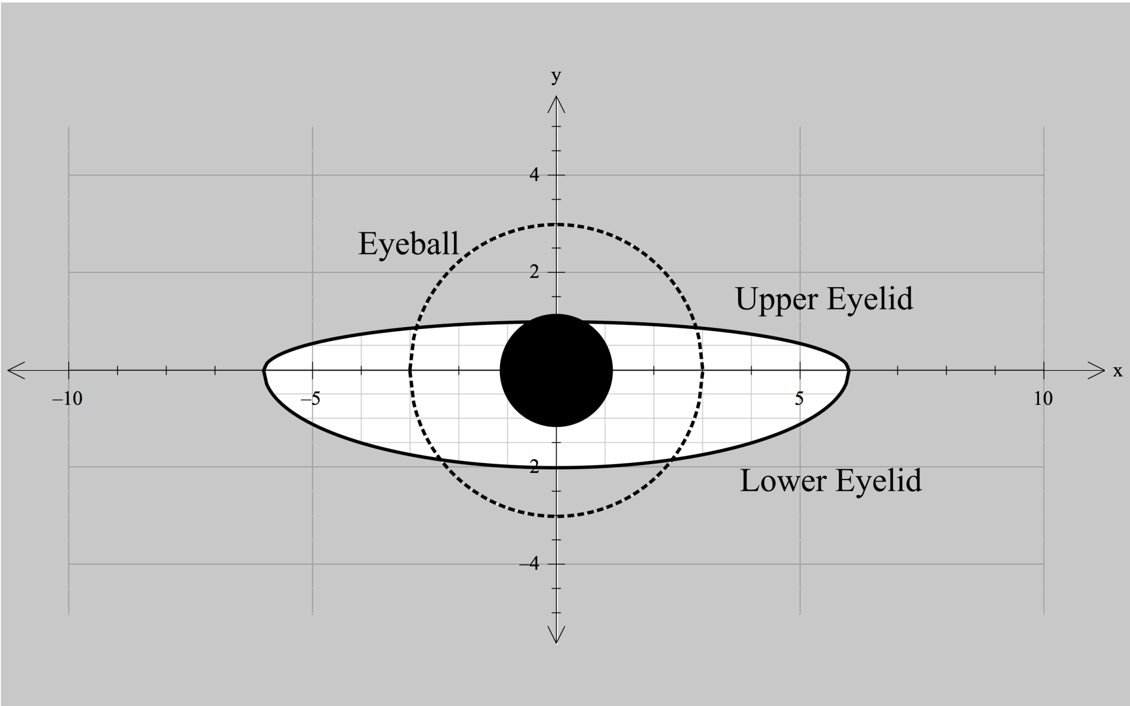
$f_4(x)=a(x-b)^2(x-c)(x-d)$ Correct form identified and points substituted in	**M1**
$f_4(-3)=0$ $f_4(14)=0$ $f_4'(14)=0$ $f_4(40)=0$ $f_4(2)=39$ Substitution of all points must be stated	**M2**
$f_4(x)=\dfrac{-13}{9120}(x-14)^2(x+3)(x-40)$	**A1**

END OF SAC 1 - EPISODE 1

Harry Potter and the SAC of Fire

Question 1 (6 marks)

As you read up about the Wideye Potion, you discover that it works by keeping a person's eyes wide open so that they don't fall asleep. The amount of ingredients required is determined by how wide you need the lids of the eyes to open. Glancing at your reflection in your brand new, very shiny copper cauldron you come up with the following model for your eyes.



The Eyeball is described by the equation $E(x) = \pm\sqrt{9 - x^2}$.

The Upper Eyelid is described by the equation $U(x) = \frac{1}{2}\sqrt{4 - \left(\frac{x}{3}\right)^2}$.

The Lower Eyelid is described by the equation $L(x) = -\sqrt{4 - \left(\frac{x}{3}\right)^2}$.

a. What is the domain and range of the Eyeball in this model? 2 marks

Domain $[-3,3]$	**A1**
Range $[-3,3]$	**A2**

The first ingredient that you need for the Wideye Potion is some snake fangs. Snake fangs are directly responsible for moving the Upper Eyelid.

b. What dilation factor is required so that the Upper Eyelid touches the top of the Eyeball? State the new equation of the Upper Eyelid, U_2, after this transformation has been applied 2 marks

$U(0)=1$ So the graph needs to be dilated by a factor of 3 from the x axis.	**A1**
$U_2(x)=\frac{3}{2}\sqrt{4-\left(\frac{x}{3}\right)^2}$ *OR* $U_2(x)=\frac{1}{2}\sqrt{36-x^2}$	**A2**

You look up the conversion of dilation factors into snake fangs and determine that you will need 6 snake fangs to achieve sufficient widening of the Upper Eyelid.

You work out that the Lower Eyelid will require a dilation factor of $\frac{3}{2}$ from the x axis.

c. State the new equation of the Lower Eyelid, L_2, after this transformation has been applied 1 mark

$L_2(x)=-\frac{3}{2}\sqrt{4-\left(\frac{x}{3}\right)^2}$ If students don't write L_2 they should lose a notation mark (max of 1 per paper)	**A1**

You work out that this will equate to 4 measures of standard ingredient being added to the cauldron. Unfortunately, you realise that standard ingredient also results in a dilation of the Lower Eyelid from the y axis.

The new equation of the Lower Eyelid, L_3, can be expressed as $L_3 = L_2\left(\frac{x}{5}\right)$.

Fortunately, you know that dried Billywig Stings can be used to apply a dilation from the y axis as well.

d. What dilation factor would be required in order to undo the dilation from the y axis caused by the standard ingredient added in **part c** to turn L_3 into L_2? 1 mark

A dilation factor of $\frac{1}{5}$ from the y axis is required to undo the transformation.	**A1**

This dilation factor means that you will need to include 6 dried Billywig Stings in your potion as well. You put all of your ingredients aside and begin to heat up the water in your cauldron.

You know that in order for your potion to work effectively, you need to ensure that the rate at which the temperature increases over time remains constant. You also know that every time you add an ingredient to the mixture, it will drastically effect the rate at which the temperature changes. You will have to use twigs of wolfsbane that have been imbued with mathematical equations to try and counteract the effect of each ingredient as you add it.

Question 2 (11 marks)

The initial temperature of the water in your cauldron is 17° Celsius. You adjust the fire heating your cauldron so that the temperature is increasing at a steady 1° Celsius every minute.

a. **i.** Find an equation for the temperature, T, in ° Celsius, t minutes after you begin heating up the cauldron. 1 mark

$T = 17 + t$	**A1**

ii. You need to add the snake fangs when the temperature reaches 25° Celsius. After how many minutes will this occur? 1 mark

$t = 8$ minutes	**A1**

The addition of snake fangs alters the temperature of the potion so that it now follows the equation $T_1(t) = T(t) + S(t)$, where $S(t)$ is the temperature change caused by the addition of the snake fangs and is described by the equation:

$$S(t) = 7 - 3e^{5t+2}$$

In order to counteract the addition of snake fangs, you need to imbue a twig of wolfsbane with the equation of the inverse of $S(t)$ and add it to the potion.

b. Find the equation of the inverse, S^{-1}, of $S(t)$. 2 marks

Let $y = S^{-1}(t)$ $\therefore S(y) = t$ $7 - 3e^{5y+2} = t$ Logical set up of equation to enable swapping of y and t to take place must be shown Penalise students who use x instead of t – Maximum of once per paper.	**M1**
$S^{-1}(t) = \frac{1}{5}\log_e\left(\frac{7-t}{3}\right) - \frac{2}{5}$ *OR* $S^{-1}(t) = \frac{\log_e(7-t) - \log_e(3) - 2}{5}$ accept y= as long as it has been defined as being the inverse function	**A1**

You find the equation of the inverse of S and perform an incantation to imbue the wolfsbane with the mathematical equation. You then proceed to add the wolfsbane and snake fangs at the same time. You are very pleased when the potion turns a nice shade of green and the rate at which the temperature is increasing remains constant.

The next ingredient, the standard ingredient, already has crushed-up wolfsbane incorporated into it and so it won't alter the rate of change of temperature of the potion. The dried Billywig Stings are a different story however.

Dried Billywig Stings are quite volatile and will alter the temperature according to the equation:

$$B(x) = -2x^2 + 8x - 2$$

where x is the time, in minutes, after the Billywig Stings have been added to the potion.

c. **i.** Explain why the function $B(x)$ does not have an inverse function, B^{-1}. 1 mark

$B(x)$ is not a one-to-one function and hence can not have an inverse function.	**A1**

You realise that you can still use the wolfsbane to counteract the effect of the Billywig Stings but that you will need to use two separate pieces, each with a different imbued equation that you will use at different times. You let the alteration caused by the Wolfsbane be described by the equation $B^{-1}(x)$.

ii. Find the maximum value of a such that the domain restriction $[0, a]$, when applied to $B(x)$, allows the inverse function, $B^{-1}(x)$ to exist. 1 mark

The turning point of $B(x)$ occurs at $(2,6)$ $\therefore a = 2$	**A1**

iii. Find the equation of $B^{-1}(x)$ over the domain $[0, a]$. 3 marks

Let $y = B^{-1}(x)$ $\therefore B(y) = x$ $x = -2y^2 + 8y - 2$	**M1**
$y = 2 \pm \frac{\sqrt{12-2x}}{2}$	**A1**
Must reject the positive square root answer as positive square root only exists outside of the range [0,2]. $y = 2 - \frac{\sqrt{12-2x}}{2}$ Explicit rejection of the positive square root is not required and is implied by students who get the correct final answer	**A2**

You imbue the first piece of wolfsbane with this equation and add it to the potion with the Billywig Stings so that the Temperature is now described by $B(B^{-1}(x))$.

iv. At what time, x minutes, after adding the Billywig Stings should the first piece of wolfsbane be removed and swapped with your next piece? 1 mark

after 2 minutes	**A1**

v. The next piece of wolfsbane is added after the time found in **part iv.** What should $B^{-1}(x)$ be now to counteract the Billywig Stings after this time? 1 mark

$y = 2 + \frac{\sqrt{12-2x}}{2}$	**A1**

Suddenly, Professor "Mad Eye" Bohni turns towards you and fires a spell. "Solve this!" he cries maniacally.

Question 3 (1 mark)

The function $f(x)=x^{\frac{5}{13}}$ is greater than the function $g(x)=x^{\frac{2}{3}}$ for what values of x?

$x\in(0,1)$	**A1**

You decide to use a technique that you had been studying over the holidays involving matrices.

Question 4 (8 marks)

a. Let T_1 be the transformation that translates the unconscious student's head from (0,0) to (0,2). Write $T_1\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)$ as a matrix equation. 1 mark

$T_1\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)=\begin{bmatrix} x \\ y \end{bmatrix}+\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ *OR* $T_1\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}+\begin{bmatrix} 0 \\ 2 \end{bmatrix}$	**A1**

"Very good" remarks the Professor. "Now watch carefully what I do…"

Professor "Mad Eye" Bohni, starting with the unconscious student's head located at (0,2), flips the body upside down, moves it 1 unit the right and then proceeds to move the body upwards by 3 units.

Let T_2 be the transformations applied by "Mad Eye" Bohni. You jot down the transformations as a matrix equation.

$$T_2\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)=\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}+\begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

b. i. What are the coordinates of the head of the unconscious student after these transformations have taken place? 1 mark

(1,1)	**A1**

"Now, try and undo what I just did" shouts Professor Bohni enthusiastically.

You decide that the easiest way to undo what Professor "Mad Eye" Bohni did was to apply a negative sign to everything that the professor did. Doing so, you get the following transformation matrix equation:

$$T_3\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = -T_2\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)$$

ii. Starting from coordinates (1,1), what are the coordinates of the location of the head of the unconscious student after these transformations have taken place? 1 mark

$(-2,-2)$	**A1**

"Hmmm, that isn't right" muses Professor "Mad Eye" Bohni. He quickly undoes your transformations and places the head of the unconscious student back at coordinates (1,1). "Try again!" he commands.

iii. What is a correct sequence of transformations, expressed as a matrix equation, that will move the upside down unconscious student's head from (1,1) to the original position of (0,2) whilst also flipping the unconscious students body the right way up again? 2 marks

$T_2\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\left(\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 1 \\ 3 \end{bmatrix}\right)$ *OR* $T_2\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ Deduct ½ a mark for each error to a maximum of 2 marks and round down.	**A1 +** **A2**

Question 5 (17 marks)

You do some research in your textbook "One Thousand Magical Herbs and Fungi" by Phyllida Spore and discover that the activity level, A, of Devil's Snare plants can be modelled by an equation of the form:

$$A(t) = a\cos(bt + c) + d$$

where a, b, c and d are real numbers and t represents the time of day after midnight.

Phyllida Spore, in her book, mentions that Devil's Snare plants do not like bright lights and instead prefer cool, damp locations. The book provides the following estimates for the values of a, b, c and d.

$$a = 3$$

$$b = \frac{\pi}{12}$$

$$c = -\frac{1}{12}$$

$$d = 5$$

The plant is most docile when the activity level, A, is a minimum.

a. **i.** Using the values of a, b, c and d provided by Phyllida Spore's book, state what the minimum activity level, A, of the Devil's Snare plant is. 1 mark

The minimum activity level of A, is 2. No units are required for this mark. If a unit is given, mark should still be leniently awarded unless the given unit is clearly incorrect.	**A1**

ii. Find all times during a day (from midnight to 11:59pm) is the Devil's Snare plant most docile? Give your answer correct to the nearest minute 2 marks

Minimum value of $A(x)$ is 2 $A(t) = 3\cos\left(\frac{\pi}{12}t - \frac{1}{12}\right) + 5 = 2$	**M1**
$t = \frac{12\pi + 1}{\pi} = 12:19\,pm$ pm is not required	**A1**

According to the book, it is considered safe to approach the Devil's Snare plant when its activity level is below 2.6

iii. At what times of the day is it safe to approach the Devil's Snare plant? Give your answer correct to the nearest minute. 2 marks

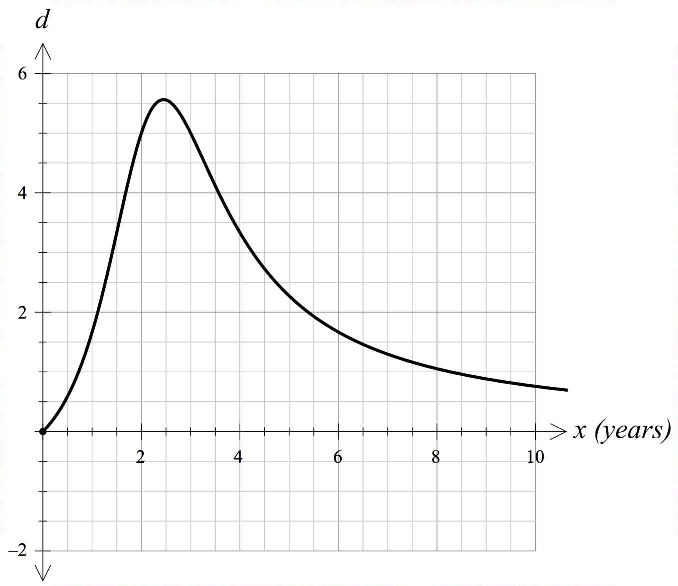
$A(t) = 3\cos\left(\frac{\pi}{12}t - \frac{1}{12}\right) + 5 = 2.6$ $t = 9.86$ or $t = 14.78$	**M1**
$9:52am \le t \le 2:47\,pm$ accept 14:47 as an answer as well	**A1**

Unfortunately, you quickly realise that the Devil's Snare plant directly in front of you is not an average plant and that these estimated values will not suffice. Upon further reading of Phyllida Spore's book, you discover that each of the terms, a, b, c, and d are affected by various environmental factors:

a is related to the humidity level of the environment in which the Devil's Snare plant resides.

b and c are both related to the intensity of the light around the plant and d is a function of the age of the plant in years.

The value of d is a function of the age of the plant as described by the graph shown below.



The equation of this graph is:

$$d(x) = \frac{5x}{x^2 - 4x + 6}$$

where x is a positive real number that represents the number of years that have passed since the Devil's Snare plant was potted.

You ask Professor Sproutkovska when these Devil's Snare plants were potted and she tells you that they were potted exactly 1 year and 2 months ago by the 3rd year Herbology students.

b. According to this information, what value of d will the Devil's Snare plant in front of you have? 1 mark

1 year and 2 months $= \frac{14}{12} = \frac{7}{6}$ years $d\left(\frac{7}{6}\right) = \frac{210}{97}$	**A1**

Next you plan to determine the humidity level of the atmosphere around the plant. Every 2 hours, a sprinkler comes on to water the Devil's Snare plants. Whilst the sprinkler is on, the humidity level of the air around the plant can be considered to be 1. It then decreases according to the rule:

$$a: [0,2) \to R, a(w) = e^{\frac{-w}{3}}$$

where w is the time, in hours, after the sprinkler was turned off.

The sprinkler was last turned off at 1:30pm.

c. **i.** The time is currently 1:43pm. Find the value of a at this time, correct to two decimal places. 1 mark

$a\left(\frac{13}{60}\right) = 0.93$	**A1**

It is possible to rewrite the equation for a from above, in terms of the time, t, in hours since midnight in the form

$$a: [13.5, 15.5) \to R, a(t) = e^{-\frac{(t-h)}{3}}$$

where h is a positive, real number.

ii. State the value of h in this case. 1 mark

$h = 13.5$	**A1**

Finally, you tackle the task of determining the values of b and c that are related to the light intensity around the Devil's Snare plant. You read that the terms b and c can in fact be replaced with a function that represents the light intensity level, L.

The light intensity level, L, in the Herbology Greenhouse varies over the course of a day according to a hybrid function shown below:

$$L(t) = \begin{cases} -\frac{1}{18}t^2 + \frac{4}{3}t - 6 & , \quad 6 \le t < 7 \cup 17 \le t < 18 \\ \frac{1}{4}\sin\left(\frac{3\pi t}{5}\right) + H & , \quad 7 \le t < 17 \\ 0 & , \quad \text{elsewhere} \end{cases}$$

where t is the time, in hours, after midnight and H is a positive, real number.

d. **i.** Assuming $L(t)$ is a continuous function, find the value of H. 2 marks

In order for the function to be continuous, $-\frac{1}{18}(7)^2 + \frac{4}{3}(7) - 6 = \frac{1}{4}\sin\left(\frac{3\pi(7)}{5}\right) + H$	**M1**
$H = \frac{11}{18} - \frac{\sqrt{-2\left(\sqrt{5}-5\right)}}{16}$	**A1**

ii. State the values of t for which the light intensity, L, a maximum. 2 marks

Max value occurs when $L(t)=\frac{1}{4}+\frac{11}{8}-\frac{\sqrt{-2\left(\sqrt{5}-5\right)}}{16}$ accept consequential from part d.i.	**M1**
$t=\frac{15}{2},\ \frac{65}{6},$ and $\frac{85}{6}$	**A1**

iii. What is the maximum light intensity, L, correct to three decimal places? 1 mark

$L\left(\frac{15}{2}\right)=0.714$ accept consequential from part d.ii.	**A1**

Using your answers to **part c, ii.** and **part d, i.** as well as the approximate value of 2 for d, a new equation for the activity level, A_1, of the Devil's Snare plant at time, t hours after midnight can be written in the form:

$$A_1\colon [13.5, 15.5) \to R, A_1(t) = a(t)\cos(L(t)\times t) + d.$$

e. i. According to A_1, what is the activity level of the Devil's Snare plant in front of you at the current time of 1:43pm? Express your answer correct to two decimal places. 2 marks

$a(t)$ at 1:43 =0.93 $1:43=\frac{823}{60}$ $L\left(\frac{823}{60}\right)=0.629$ $A_1\left(\frac{823}{60}\right)=0.93\cos\left(0.629\times\left(\frac{823}{60}\right)\right)+2$ accept consequential from part c.i. for value of a, from part c.iii. for value of L.	**M1**
$A_1\left(\frac{823}{60}\right)=1.35$	**A1**

To be safe, you decide to only approach the Devil's Snare plant when its activity level is below 1.6.

ii. At what time(s) between 1:30pm and 3:30pm is it safe to approach the Devil's Snare plant? Give your answer correct to the nearest minute. 2 marks

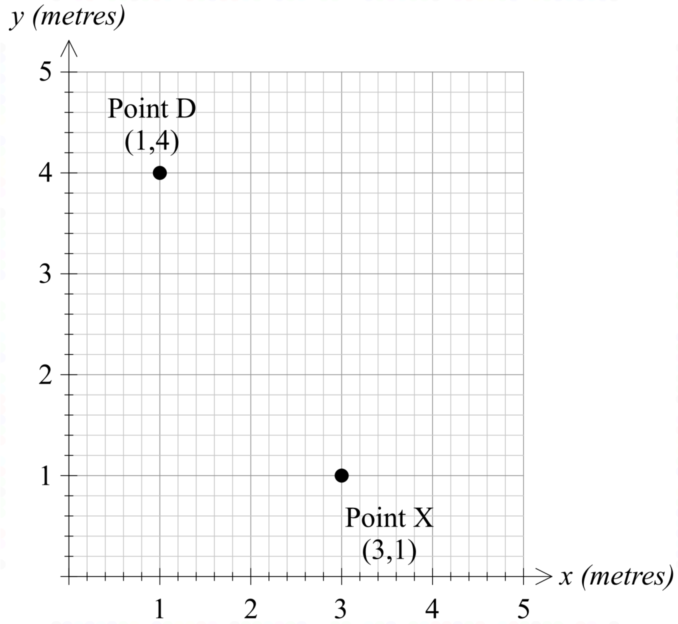
$A_1(t)=e^{-\frac{(t-13.5)}{3}}\cos\left(t\left(\frac{1}{4}\sin\left(\frac{3\pi t}{5}\right)+\frac{11}{18}-\frac{\sqrt{-2\left(\sqrt{5}-5\right)}}{16}\right)\right)+2$ $A(t)=1.6$ accept consequential from part c.i. for value of a, from part c.iii. for value of L.	**M1**
$13.66<t<14.75$ $1:39\text{pm}<t<2:45\text{pm}$ Accept either answer	**A1**

END OF SAC 1 - EPISODE 2

Harry Potter and the Deathly SACs

Question 1 (12 marks)

A birds eye view of the courtyard is depicted below. You are positioned at the point X which has coordinates (3,1), and the Death Eater you are battling is located at point D which has coordinates (1,4).



a. How far away from you, in meters, is the Death Eater? 2 marks

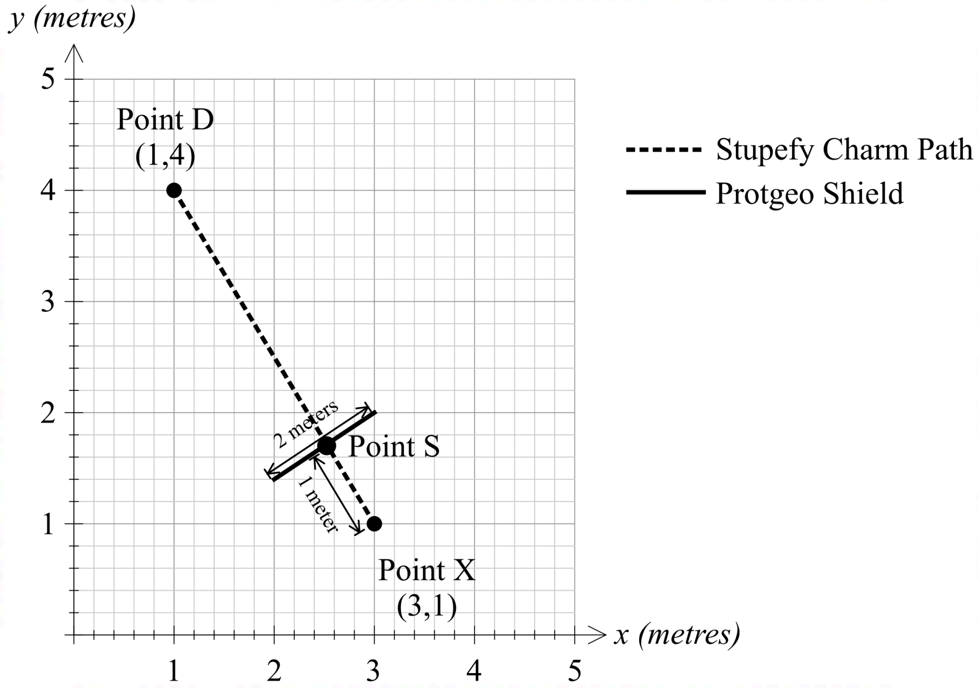
$D=\sqrt{(3-1)^2+(1-4)^2}$	**M1**
$D=\sqrt{13}$ meters	**A1**

The Death Eater fires a Stupefy Charm along a straight line directly towards you.

b. Show that the equation of the straight line along which the Stupefy Charm will travel can be expressed as $y=\frac{-3}{2}x+\frac{11}{2}$. 2 marks

$m=\frac{4-1}{1-3}=\frac{-3}{2}$	**M1**
$y-4=\frac{-3}{2}(x-1)$ Substitution of point must be shown If final answer is not shown also, students should lose the mark for not completing the show.	**M2**

You cast the Protego Spell which produces a shield one meter directly in front of you. This shield will deflect any incoming charm, spell or curse. The shield is 2 meters wide and is linear.



c. **i.** Let the x coordinate of Point S be given by the variable a. State the coordinates of Point S in terms of a. 1 mark

$\left(a, \frac{-3}{2}a + \frac{11}{2}\right)$	**A1**

ii. If the Point S is located one meter in front of your location at Point X, find the coordinates of Point S. 3 marks

$1 = \sqrt{(3-a)^2 + \left(1 - \left(\frac{-3}{2}a + \frac{11}{2}\right)\right)^2}$	**M1**
$a = 3 - \frac{2\sqrt{13}}{13}$ Any rejection of the other solution to a is not required	**A1**
$\left(3 - \frac{2\sqrt{13}}{13}, \frac{3\sqrt{13}}{13} + 1\right)$	**A2**

The 2 meter long Protego Shield you produce is exactly perpendicular to the incoming Stupefy Charm.

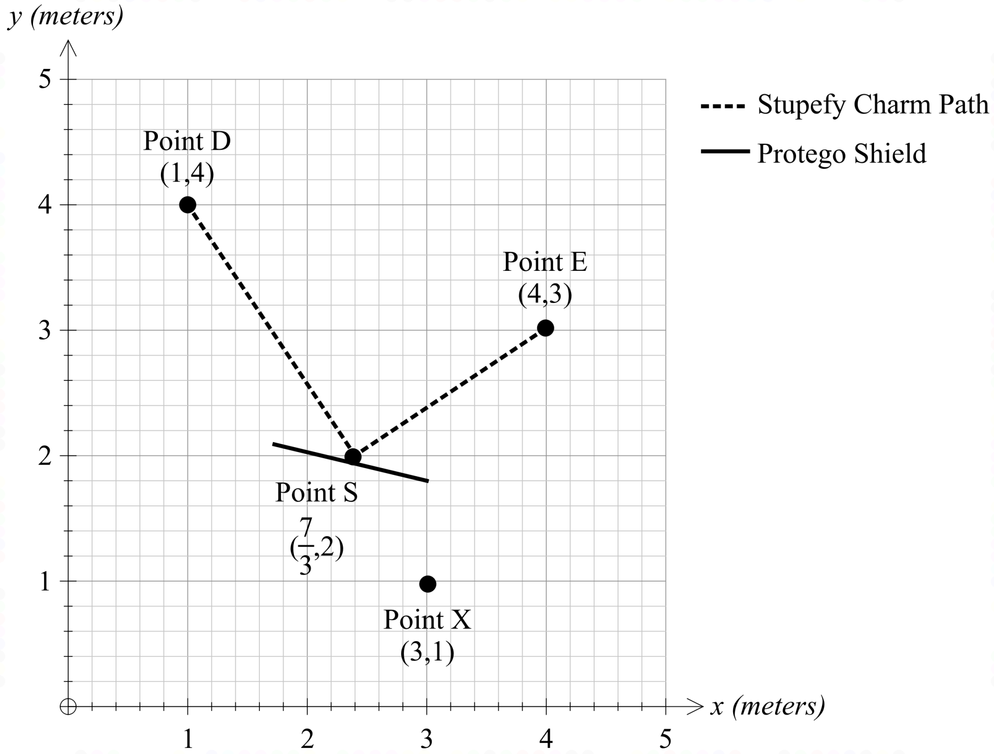
iii. Find the equation of the line that represents the Protego Shield. State any necessary domain restrictions. 4 marks

$m_1 \times m_2 = -1$ $m_1 = \frac{-1}{-3/2} = \frac{2}{3}$	**M1**
$y - \left(\frac{3\sqrt{13}}{13} + 1\right) = \frac{2}{3}\left(x - \left(3 - \frac{2\sqrt{13}}{13}\right)\right)$ $y = \frac{2}{3}x + \frac{\sqrt{13} - 3}{3}$ Accept any other correct form of the answer. This answer should be consequential on answer from **part c i.**	**A1**
Let endpoints of the Protego Shield have coordinates $\left(b, \frac{2}{3}b + \frac{\sqrt{13} - 3}{3}\right)$ $1 = \sqrt{\left(\left(3 - \frac{2\sqrt{13}}{13}\right) - b\right)^2 + \left(\left(\frac{3\sqrt{13}}{13} + 1\right) - \left(\frac{2}{3}b + \frac{\sqrt{13} - 3}{3}\right)\right)^2}$ $b = \frac{-5\sqrt{13} + 39}{13}$ OR $b = 3$	**M2**
$y : \left[\frac{-5\sqrt{13} + 39}{13}, 3\right] \to R, y = \frac{2}{3}x + \frac{\sqrt{13} - 3}{3}$	**A2**

You successfully block the incoming Stupefy Charm and it reflects back at the Death Eater who casted it. Unfortunately they are also aware of defensive spells and so manage to block the Stupefy Charm from hitting them also.

You notice that nearby is another Death Eater located at the Point E which has coordinates (4,3). This Death Eater is focussed on attacking another wizard and so you decide to manipulate your Protego Shield so that it deflects the Stupefy Charm being fired by the Death Eater at Point D towards the Death Eater at Point E.

The Diagram below shows the scenario.



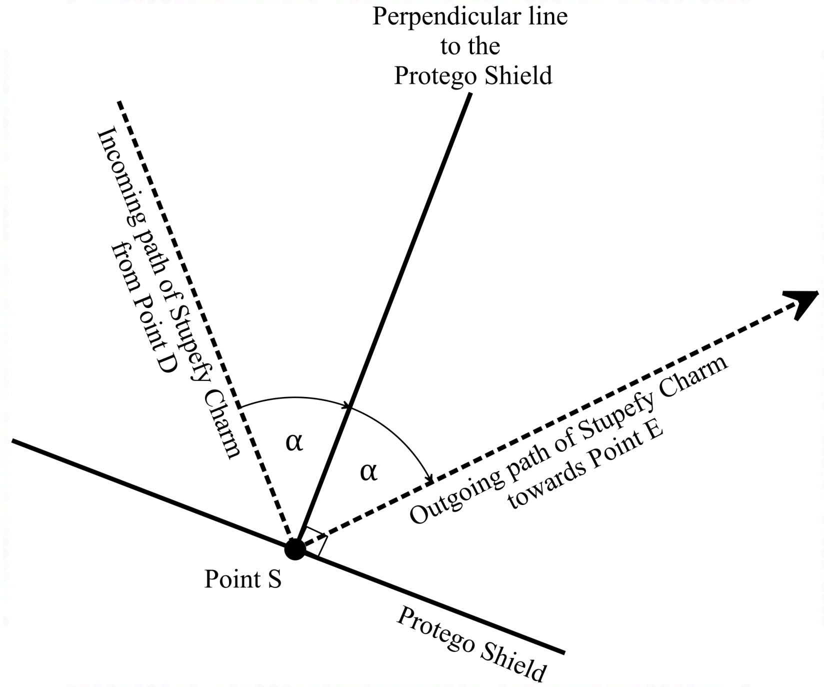
Question 2 (9 marks)

You cast your Spell so that the centre of the 2 meter long Protego Shield is located at Point S with coordinates $\left(\frac{7}{3},2\right)$.

a. Find the gradient of the line joining Point D and Point S and hence, determine the acute angle, to the nearest degree, that this line makes from the x axis. 2 marks

$m_{DS}=\dfrac{2-4}{\frac{7}{3}-1}=\dfrac{-3}{2}$	**M1**
$\theta=\tan^{-1}\left(\dfrac{-3}{2}\right)=-56^{\circ}$ But since the angle must be positive $\theta=56^{\circ}$ Students should not receive this mark if they leave their answer as negative.	**A1**

The angle between the incoming path of the Stupefy Curse and the line that is perpendicular to the Protego Shield will be the same as the angle between the line that is perpendicular to the Protego Shield and the outgoing path of the Stupefy Curse.



b. Find the gradient of the line joining Point E and Point S and hence, determine the acute angle, to the nearest degree, that this line makes from the x axis. 2 marks

$m_{SE} = \dfrac{2-3}{\dfrac{7}{3}-4} = \dfrac{3}{5}$	**M1**
$\theta = \tan^{-1}\left(\dfrac{3}{5}\right) = 31^\circ$	**A1**

c. Hence show that the total angle between the incoming and outgoing paths of the Stupefy Charm is 93°. 1 mark

$180 - 31 - 56 = 93^\circ$	**M1**

You know that the angle of the perpendicular line to the Protego Shield must be $\frac{93^\circ}{2}$ in a clockwise direction from the incoming path of the Stupefy Charm.

d. **i.** Use this information as well as your answer to **Question 2 part a**, to determine the gradient of the perpendicular line to the Protego Shield, correct to 2 decimal places. 2 marks

$m=\tan\left(124-\frac{93}{2}\right)=\tan\left(\frac{155}{2}\right)$ Any identification of the correct angle, through diagrams or otherwise should be awarded the method mark here.	**M1**
$m=4.51$	**A1**

ii. **Hence** determine the equation of the line that represents the required location of the Protego Shield so that the Stupefy Charm will be deflected at the Death Eater located at Point E. 2 marks

$m_{\text{Protego}}=\frac{-1}{4.51}=-0.22$	**M1**
$y-2=-0.22\left(x-\frac{7}{3}\right)$ $y=2.52-0.22x$ Answer must be in y=mx+c form. Award consequential marks from part di.	**A1**

Question 3 (8 marks)

You begin by choosing a random point in the courtyard, Point P, with the general coordinates (p,q), where p and q are both real numbers and $p>\frac{7}{3}$ and $q>2$.

a. **i.** Write an expression for the gradient of the line joining Point S and Point P in terms of p and q. 1 mark

$m_{SP}=\dfrac{2-q}{\frac{7}{3}-p}$ $m_{SP}=\dfrac{6-3q}{7-3p}$ Substitution and final simplification must be shown in order to demonstrate the show	**M1**

ii. Hence, write an expression for the acute angle, θ, that this line makes with the x axis in terms of p and q. 1 mark

$\theta=\tan^{-1}(m_{SP})$ $\theta=\tan^{-1}\left(\dfrac{6-3q}{7-3p}\right)$* OR $\theta=\tan^{-1}\left(\dfrac{2-q}{\frac{7}{3}-p}\right)$* Accept either line denoted by a *	**A1**

The Death Eater who is still firing Stupefy Charms at you has moved to try and get around your Protego Shield and the incoming path of the Stupefy Charms now make an angle of 20° with the x axis.

b. Using your answer to **Question 3 part a.ii**, show that a general expression for the gradient of the perpendicular line to the Protego Shield will be:

$$m_{\perp}=\tan\left(80+\frac{1}{2}\tan^{-1}\left(\frac{6-3q}{7-3p}\right)\right)$$

3 marks

Angle between lines DS and SP $=\alpha$ $m_{\perp}=\tan\left(160-\dfrac{1}{2}\alpha\right)$	**M1**
$\alpha=180-20-\tan^{-1}\left(\dfrac{6-3q}{7-3p}\right)$	**M2**

$m_{\perp} = \tan\left(160 - \frac{1}{2}\left(160 - \tan^{-1}\left(\frac{6-3q}{7-3p}\right)\right)\right)$ $m_{\perp} = \tan\left(80 + \frac{1}{2}\tan^{-1}\left(\frac{6-3q}{7-3p}\right)\right)$	**M3**

c. Keeping the center of your Protego Shield located at Point S, with coordinates $\left(\frac{7}{3}, 2\right)$, find the generalised equation for the Protego Shield in terms of p and q. 2 marks

$m_{\text{Protego}} = \frac{-1}{m_{\perp}} = \frac{-1}{\tan\left(80 + \frac{1}{2}\tan^{-1}\left(\frac{6-3q}{7-3p}\right)\right)}$	**M 1**
$y = \frac{-1}{\tan\left(80 + \frac{1}{2}\tan^{-1}\left(\frac{6-3q}{7-3p}\right)\right)}\left(x - \frac{7}{3}\right) + 2$ *OR* $y = \frac{3\sin\left(\frac{\tan^{-1}\left(\frac{6-3q}{7-3p}\right)}{2} + 80\right)x + 6\cos\left(\frac{\tan^{-1}\left(\frac{6-3q}{7-3p}\right)}{2} + 80\right) - 7\sin\left(\frac{\tan^{-1}\left(\frac{6-3q}{7-3p}\right)}{2} + 80\right)}{3\cos\left(\frac{\tan^{-1}\left(\frac{6-3q}{7-3p}\right)}{2} + 80\right)}$	**A1**

Now that you have your generalised equation, you decide to test it out on a nearby Death Eater. The coordinates of this Death Eater are (5,5).

d. What is the equation of the Protego Shield that you must cast in order to deflect the Stupefy Charm at the Death Eater located at (5,5)? Express your answer in the form $y = mx + c$ where m and c are both real numbers expressed to an accuracy of 2 decimal places. 1 mark

$y = 0.25x + 1.41$ Do not accept consequential marks for this	**A1**

As you begin to dispatch enemy Death Eaters with ease, the original Death Eater begins to realise what you are doing and tries to come up with a way of tricking you into letting your guard down.

The first thing that this Death Eater does is send another Death Eater to a coordinate (p,q), where p and q are both real numbers but where $p<\frac{7}{3}$ and $q>2$.

Question 4 (4 marks)

a. State the effect that this change to the coordinates of point P will have on your answer to **Question 3, part b**. 1 mark

This change in the location of p will result in the calculation of the angle, α, being different. As the new Point P is now on the left of Point S, the angle between the two lines will be determined by the size of the $\tan^{-1}(m_{SP})-\tan^{-1}(m_{DS})$ Students only need to recognise that the angle will change.	**A1**

b. State the new equation that can be used instead of your answer to **Question 3, part b,** to address this change in the values of p. 3 marks

$m_{\perp}=\tan\left(\frac{1}{2}\left(20+\tan^{-1}\left(\frac{6-3q}{7-3p}\right)\right)\right)$ if $-\tan^{-1}\left(\frac{6-3q}{7-3p}\right)\le 20$ OR $m_{\perp}=\tan\left(\frac{1}{2}\left(20-\tan^{-1}\left(\frac{6-3q}{7-3p}\right)\right)\right)$ if $20\le -\tan^{-1}\left(\frac{6-3q}{7-3p}\right)$ award one mark for each case	**M1 + M2**
$y-2=\frac{-1}{m_{\perp}}\left(x-\frac{7}{3}\right)$	**A1**