

STUDENT NAME:

MATHEMATICAL METHODS (CAS)

SAC 3



MATHENGERS - PROBABILITY WAR

2018

Reading Time: 15 minutes

Writing time: 120 minutes

QUESTION AND ANSWER BOOK

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
7	7	68
	<i>Total</i>	68

- Students are permitted to bring into the assessment room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved CAS Calculator and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the assessment: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 17 pages.
- Formula sheet

Instructions

- Write your name in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the assessment room.

Instructions

Answer **all** questions in the spaces provided

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.



Our SAC begins with the superhero Colin Shnor, Asgardian God of Thunder, floating in the vacuum of space. Shnor's space ship was just attacked by the Mad Titan of JMSS John "Thanos" Kermond who is intent upon wiping out half of all of the students and teachers at JMSS in a supposedly 'random' way. He attacked Shnor's space ship in order to steal one of the six Infinity Stones. Once John "Thanos" Kermond possess all six Infinity Stones, he will be able to eradicate half of all students and teachers at JMSS with a single click of his fingers.

The Guardians of the Galaxy, led by Peter "Starlord" CorQuill, Sally Gamora-Johnson and Andrew "Rocket" Raccoon, arrive on the scene and promptly rescue Shnor.

Question 1 (13 marks)

A regular human can survive for a few minutes in the absolute vacuum of space without serious, long-term damage. The probability that a regular human dies when exposed to the vacuum of space within t minutes can be described by the continuous random variable D , with probability density function given by:

$$D(t) = \begin{cases} \frac{1}{5} - kt & , 0 \leq t \leq 10 \\ 0 & , \text{elsewhere} \end{cases}$$

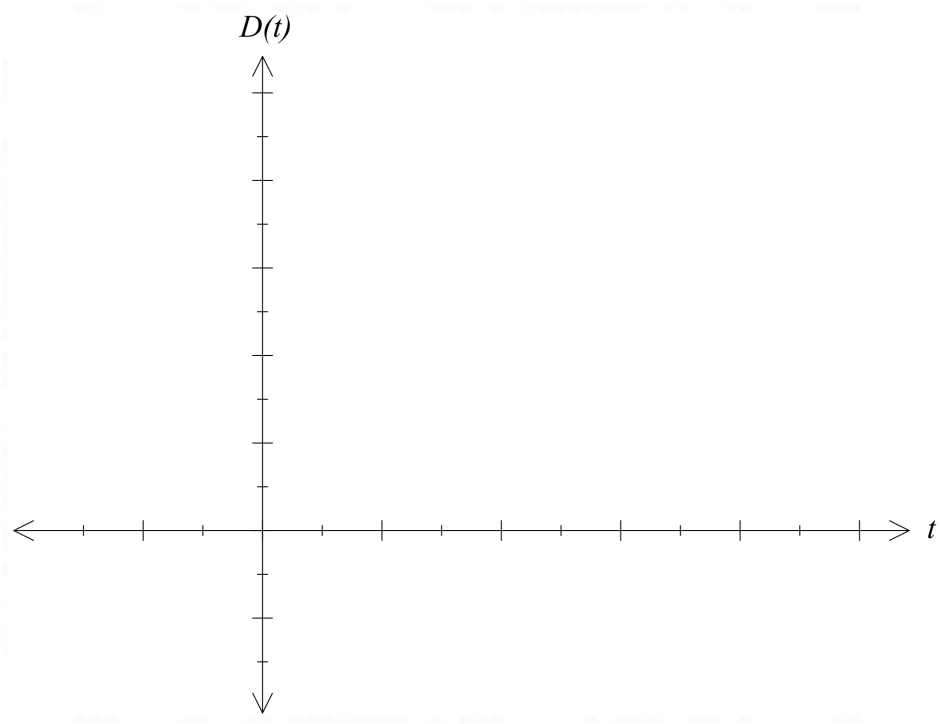
where k is a real number.

a. Show that $k = \frac{1}{50}$.

2 marks

- b.** Sketch the graph of the probability density function $D(t)$ on the axes provided below. Label all endpoints and axis intercepts.

2 marks



- c.** Find the expected length of time that a regular human can survive in the vacuum of space.

2 marks

If a regular human survives for longer than 4 minutes in the vacuum of space without dying, then they must have some Asgardian blood in their veins.

- d. i.** Find the probability that a regular human has some Asgardian blood in their veins.

2 marks

There are twelve mathematics teachers in the JMSS maths faculty. Assume a mathematics teacher to be a regular human.

- ii.** Find the probability, correct to 4 decimal places, that there is exactly one mathematics teacher with Asgardian blood in their veins. 2 marks

- iii.** Find the probability, correct to 4 decimal places, that there are at least two mathematics teachers with Asgardian blood in their veins. 1 mark

It is known that Colin Shnor has Asgardian blood in his veins and is also a mathematics teacher in the JMSS maths faculty.

- iv.** Given that Shnor has Asgardian blood in his veins, find the probability that there are at least two mathematics teachers with Asgardian blood in their veins. Give your answer correct to 4 decimal places. 2 marks



Once revived, Shnor informs the Guardians of John “Thanos” Kermond’s plan and recruits the help of Andrew “Rocket” Raccoon and the living tree, Groot, to take him to Nidavellir, a forge that uses the heat from a Neutron Star to create some of the Galaxy’s most powerful weapons.

Question 2 (9 marks)

When operating at full capacity, the Nidavellir forge was producing a large number of weapons every day. Each weapon that is produced is given a power rating. Any weapon that has a power rating less than 100 is considered to be too weak and is rejected. Any weapon with a power rating of over 850 is considered powerful enough to kill a Titan. On one particular day 16% of the weapons that are produced are rejected due to being too weak and only 4% are found to be powerful enough to kill a Titan.

Assume that the power rating of weapons is normally distributed.

- a. Find the mean and standard deviation of the power ratings, correct to one decimal place. 3 marks

The length, in cm, of a weapon produced by the Nidavellir forge is normally distributed with a mean of 67cm and a standard deviation of 14cm.

- b. i.** Find the probability, correct to 4 decimal places, that the Nidavellir forge produces a weapon that is longer than 1 metre.

1 mark

- ii.** Find the probability, correct to 4 decimal places, that a weapon that is found to be greater than 80cm in length will not be greater than 90cm in length.

2 marks

- c.** 20 weapons are produced on a particular day. Find the probability, correct to 3 decimal places, that at least 7 of these weapons are longer than 75cm.

3 marks



Shnor works with Andrew “Rocket” Raccoon, Groot and the King of the Dwarves, King Alex Gavreitrilescu, to create a weapon that is capable of defeating John “Thanos” Kermond.

Meanwhile, back at JMSS, the Black Order (John “Thanos” Kermond’s band of supervillains), led by The Emily Maw, is trying to steal the Infinity Stone known as the Time stone from Doctor Schmidt, A.K.A. Doctor Strange.

Doctor Strange, in conjunction with Bohni “Iron Man” Stark, Rowen “The Hulk” May and Sonny “Spiderman” Bingham battle fiercely with The Emily Maw but to no avail. The Emily Maw manages to successfully capture Doctor Strange and takes him aboard her space ship to return him, and the Time Stone, to John “Thanos” Kermond.

Unbeknownst to The Emily Maw, Bohni “Iron Man” Stark and Sonny “Spiderman” Bingham both managed to stow away on the space ship and proceed to make a plan that will enable them to rescue Doctor Strange.

Question 3 (8 marks)

When Bohni Stark first arrives on board The Emily Maw’s space ship, his Iron Man suit scans the environment and identifies that there are some harmful bacteria present in the air. The Iron Man suit can’t determine the exact concentration of the harmful bacteria but it does know that the amount of harmful bacteria in the air, in parts per million (ppm), is normally distributed with a mean of 342ppm and a standard deviation of 43ppm. The Iron Man suit also knows that the amount of harmful bacteria in the air that will result in the death of a person is normally distributed with a mean of 461ppm and a standard deviation of 63ppm.

- a. State the expected value, in ppm, for the amount of harmful bacteria in the air.

1 mark

The Iron Man suit will only allow Bohni Stark to remove his helmet if it considers the air to be safe. The air is considered safe when the probability of death caused by the amount of harmful bacteria in the air is less than 0.3.

- b.** Find the probability, correct to 4 decimal places, that the amount of harmful bacteria in the air exceeds the level considered safe by the Iron Man Suit.

3 marks

As part of the plan to defeat The Emily Maw, Bohni Stark and Sonny “Spiderman” Bingham have to distract Emily Maw. In order to do so, Bohni Stark jumps out in front of Emily Maw and says “Hey Sharkface! You *think* taking the Infinity Stone off that Sorcerer Supreme is hard? Why don’t you try this problem first?”

- c.** Let X be a normally distributed random variable with a mean of μ and a standard deviation of σ .

- i.** Find $\Pr(X < \mu - 2\sigma)$ correct to 3 decimal places.

1 mark

- ii.** Find $\Pr(\mu - \sigma < X < \mu + 2\sigma)$ correct to 3 decimal places.

1 mark

iii. Find a given $\Pr(\mu - a\sigma < X < \mu + a\sigma) = 0.8664$, correct to 1 decimal place. 2 marks



Emily Maw smiles back at Bohni Stark before remarking “Your questions are inconsequential compared to mine. Prepare to die”

*Bohni Stark looks disappointed and shakes his head before saying to Emily Maw, “The least you could have done is try to solve the problem. The good guys always give it a go.” He then blows a hole in the side of the ship and Sonny “Spiderman” Bingham swings in and knocks Emily Maw into the vacuum of space where she quickly dies.**

With Emily Maw gone, Bohni Stark and Sonny “Spiderman” Bingham quickly rescue Doctor Strange. Unfortunately they don’t know how to pilot the space ship and the autopilot ends up taking them to John “Thanos” Kermond’s homeworld of Titan where John “Thanos” Kermond is waiting for the time stone to be delivered.

**Emily Maw has no Asgardian blood in her veins and thus no resistance to the coldness of space.*

Whilst waiting to face the formidable foe that is John “Thanos” Kermond, Doctor Strange uses the Time Stone to explore the possible futures.

Question 4 (3 marks)

In any given encounter with John “Thanos” Kermond, the chance of the Mathengers winning is 0.0025.

What is the minimum number of future scenarios that Doctor Strange needs to view in order for the probability of finding at least one winning scenario to be greater than 0.95.



“I have analysed the possible futures in search of a way to defeat John “Thanos” Kermond” Doctor Strange tells Bohni Stark.

“How many do we win?” Bohni Stark asks.

Doctor Strange grimaces before answering. “One” he replies...

John “Thanos” Kermond then arrives on the planet and promptly defeats the Mathengers present before taking the Time Stone from Doctor Strange.

With a wide grin on his face, John “Thanos” Kermond prepares to make the journey to find the final Infinity Stone on Earth.



Meanwhile, back on Earth, Captain Amar-Porcica, Kristina “The Scarlet Witch” Cvetkovska and Nadine “Black Widow” Phillips have travelled to the country of Wakanda in the hope of being able to remove the Infinity Stone known as the Mind Stone from The Vision’s head.

The Black Parabola, also known as T’Challa, King of Wakanda, is happy to help but it will take some time to perform the procedure.

Whilst they get started trying to remove the Mind Stone from The Vision’s head, an army of Outriders (an alien race loyal to John “Thanos” Kermond) arrive in Wakanda ready to do battle with the Mathengers.

Question 5 (14 marks)

Proxima Midpoint leads the army of Outriders but she doesn’t know how many different types of each outrider are on each space ship. An Outrider can have either four or six arms.

Proxima Midpoint takes a random sample of 20 Outriders from one of the space ships. She counts 7 Four-Armed Outriders and 13 Six-Armed Outriders. She marks each of these Outriders with a green dye and puts them back into the space ship.

5 hours later, Proxima Midpoint takes another random sample of 20 Outriders from the same ship. This time she collects 12 Four-Armed Outriders and 8 Six-Armed Outriders. Of the 12 Four-Armed Outriders, only 2 have green dye on them and of the 8 Six-Armed Outriders, 4 have green dye on them.

- a. Estimate the number of Four-Armed and Six-Armed Outriders there are in the space ship. (Assume that the 20 Outriders taken in the second random sample are representative of the entire Outrider population).

2 marks

On another space ship that Proxima Midpoint investigates, there are a total of 34 Outriders. After she has taken a sample of 20 Outriders from this space ship, she marks them with a green dye and returns them to the space ship.

Let Y represent the random variable ‘the number of marked Outriders in the sample’.

Proxima Midpoint takes a second sample of 20 Outriders from this space ship.

- b. i.** State the population proportion, p , of marked Outriders in the space ship. 1 mark

- ii.** State the smallest and largest possible values of Y . 2 marks

- c.** Find $\Pr(Y = 10)$ correct to 4 decimal places. 3 marks

After surveying all of the space ships in the Outrider Army, Proxima Midpoint is able to determine that the population proportion of Four-Armed Outriders is 70%.

Assume that the population of Outriders is very large.

- d.** Find the probability, correct to 4 decimal places, that from a random sample of 18 Outriders, 13 or more are Four-Armed Outriders 2 marks

- e.** Find the probability, correct to 4 decimal places, that from a random sample of 18 Outriders, no more than 10 are Four-Armed Outriders given that at least 3 are Four-Armed Outriders. 2 marks

- f.** Find the minimum number of Outriders Proxima Midpoint must collect in order to be at least 99% certain of having a minimum of 10 Four-Armed Outriders in her sample. 2 marks



Despite the best efforts of the Mathengers, John “Thanos” Kermond arrives on Earth and successfully manages to retrieve the final Infinity Stone by taking it from The Vision’s head.

He then proceeds to click his finger and, one by one, Vivienne Luu, along with half of all of the other students and teachers at JMSS disintegrate into nothingness.

Question 6 (8 marks)

Each of the Superheroes at JMSS can be categorised into 3 distinct phases: Phase 1, Phase 2 and Phase 3. There are 24 superheroes at JMSS in total. Of these, eight are from Phase 1, eleven are from Phase 2 and five are from Phase 3.

- a. When John “Thanos” Kermond clicked his finger, six superheroes disintegrated one after another.
- i. Calculate the probability, correct to 4 decimal places, that the first four superheroes to disintegrate were from Phase 2, followed by one superhero from phase 1 and then another superhero from phase 3. 2 marks

- ii. Calculate the probability, correct to 4 decimal places, that exactly two superheroes from each of the three Phases (Phase 1, Phase 2 and Phase 3) were disintegrated. 3 marks

- iii. Calculate the probability, correct to 4 decimal places, that at least five of the six superheroes that were disintegrated came from Phase 2. 3 marks

Question 7 (13 marks)

The probability of an individual superhero from Phase 2 disintegrating follows a discrete distribution based on how many Marvel movies they have been in.

Let X be the random variable ‘the number of Marvel movies a superhero has been in’. The probability that a character appearing in x movies will disintegrate when John “Thanos” Kermond clicks his finger is shown in the following table.

x	1	2	3	4	5
$\Pr(X = x)$	0.3	0.2	$-2b$	0.1	b^2

- a. Find the value of b for this distribution. 3 marks

- b. i. Find the mean of the random variable X , correct to two decimal places. 1 mark

- ii.** Find the median of the random variable X . 1 mark

- iii.** Find the standard deviation of the random variable X , correct to two decimal places. 2 marks

- c. i.** Find the probability that a superhero will disintegrate if they have been in 3 or more Marvel movies. 2 marks

- ii.** Find the probability that a superhero who has been in exactly 4 Marvel movies and a superhero who has been in exactly 1 Marvel movie will both disintegrate when John “Thanos” Kermond clicks his finger. 2 marks

- iii. Find the probability that two superheroes who have, in combination, appeared in 8 Marvel movies, will both disintegrate when John “Thanos” Kermond clicks his finger. Give your answer correct to 4 decimal places.

2 marks



With his objective complete and a ‘balance’ restored to JMSS, John “Thanos” Kermond retires to a remote world where he can write Specialist Maths SACs in peace.

Meanwhile, the remaining heroes of JMSS are at a loss as to what to do next. Can they rally and find a way to defeat the Mad Titan? Is this all part of the timeline that Doctor Strange saw when he looked into the future?

Maybe next year, we shall find out.

END OF SAC 3