STUDENT NAME:

MATHEMATICAL METHODS (CBE) UNIT 3
SAC 1 – PART 1

Reading time: 15 minutes
Writing time: 120 minutes

QUESTION AND ANSWER BOOK

Number of questions	Number of questions to be answered	Number of marks
5	5	70
	Total	70

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one laptop with Mathematica and any number of Mathematica files.
- Students are NOT permitted to bring into the examination: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 18 pages.
- Formula sheet

Instructions
- Write your name in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Note: Sections between these symbols are story only and are not needed to answer the questions

In the land of Median-Earth

Much that once was is lost, for none now live who remember it.

It began with the deriving of the Great Trigonometric Identities. Three were given to the Elves, immortal, wisest and purest of all mathematicians. Seven to the Dwarf-Lords, great engineers and applied mathematicians of the mountain halls. And nine, nine Trigs were gifted to the race of Men, who above all else desire indices. For within these Trigs was bound the strength and the will to govern each race. But they were all of them deceived, for another Trig was made. Deep in the land of Mathdor, in the Fires of Mount Domain, the Dark Lord SaurJohn derived a master trigonometric identity, and into this Trig he poured his cruelty, his malice and his will to dominate all maths.

One Trig to rule them all.

One by one, the free lands of Median-Earth fell to the power of the Trig, but there were some who resisted. A last alliance of men and elves marched against the armies of Mathdor, and on the very slopes of Mount Domain, they fought for the freedom of Median-Earth. Victory was near, but the power of the Trig could not be undone. It was in this moment, when all hope had faded, that Bohni-ldur, son of the king, took up his father's sword.

SaurJohn, enemy of the free peoples of Median-Earth, was defeated. The Trig passed to Bohni-ldur, who had this one chance to destroy evil forever, but the hearts of men are easily corrupted. And the Trig of power has a will of its own. It betrayed Bohni-ldur, to his death.

And some identities that should not have been forgotten were lost. History became legend. Legend became myth. And for two and a half thousand years, the Trig passed out of all knowledge. Until, when chance came, it ensnared another bearer.

It came to the creature Corkllum, who took it deep into the tunnels of the Misty Mountains. And there it consumed him. The Trig gave to Corkllum unnatural long life. For five hundred years it poisoned his mind, and in the gloom of Corkllum's cave, it waited. Darkness crept back into the forests of the world. Rumor grew of a shadow in the East, whispers of a nameless function, and the Trig of Power perceived its time had come. It abandoned Corkllum, but then something happened that the Trig did not intend. It was picked up by the most unlikely creature imaginable: a hobbit, BilBui Baggins, of the Shire.

For the time will soon come when hobbits will shape the fortunes of all.

The Function of the Trig

In the Shire, in the village of JMSS-ton.
The entire village is celebrating the eleventy-first birthday (11+1=111) of BilBui and the coming of age of her fresh-faced cousin and heir, Frodo-lambous. The evening entertainment has been supplied by the wizard GanDoug the Gray.

Question 1 (17 marks)

GanDoug launches a particular sky rocket whose path can be modelled by the equation

$h(t) = 19 - 17\cos(\frac{\pi t}{8})$, $0 \le t \le 60$ where h is the height in metres of the sky rocket above the ground t seconds after it is launched.

    a. State the period and amplitude of the function.   2 marks

Period: In[2]:= $\dfrac{2\pi}{\frac{\pi}{8}}$ Out[2]= 16 16 seconds Amplitude = 17 metres Units not required	A1 A1

    b. Find the maximum height reached by the rocket.   1 mark

19 + 17 =36 metres Units required	A1

c. Sketch the graph of $h(t)$ for the entire domain labelling endpoints and turning points with their coordinates.   3 marks

	A1 Shape
	A1 end points labelled correctly A1 Turning points all labelled correctly
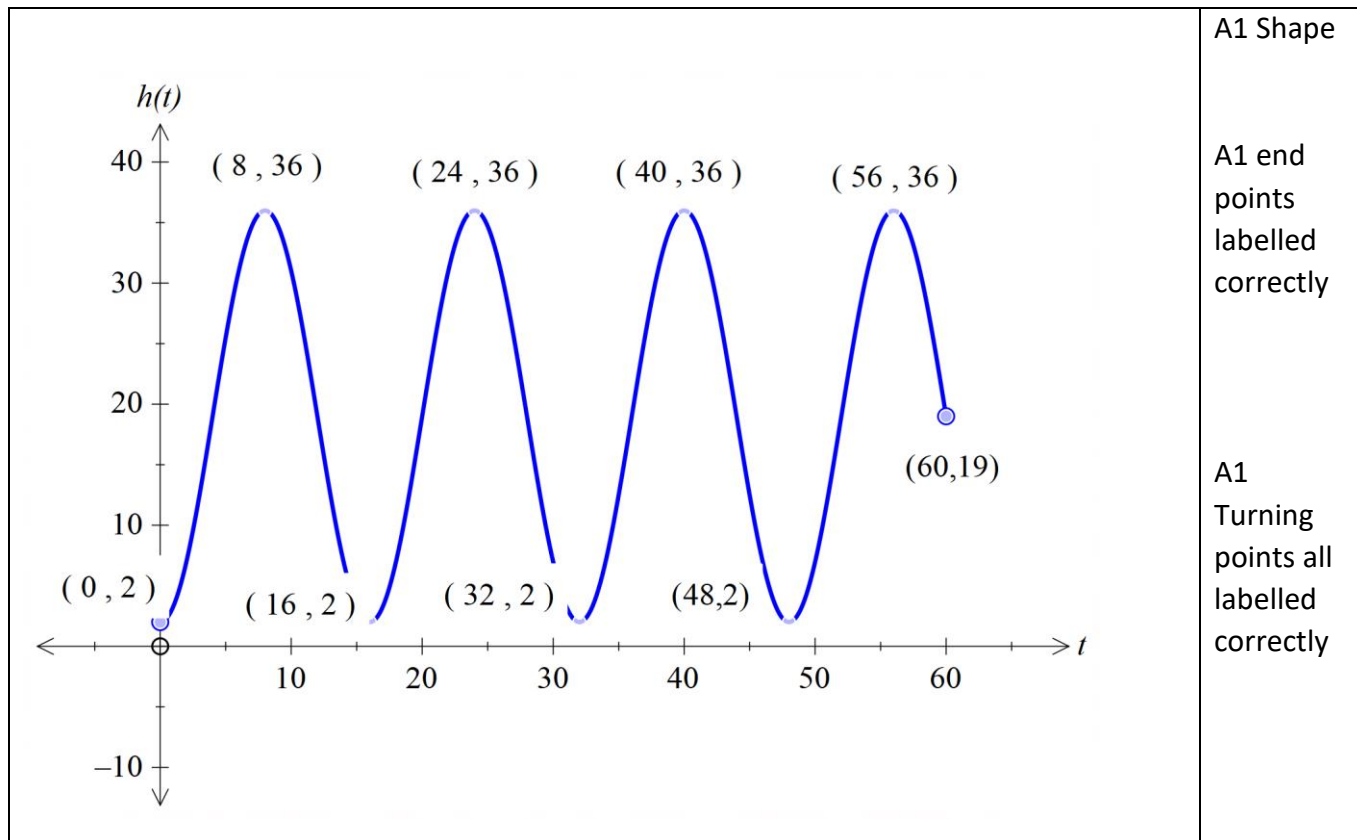
The sky rocket is visible to everyone in the village when its height is equal to or greater than 20 metres above the ground.

d. For how long, to the nearest second, during its flight is the sky rocket visible to everyone in the village?   3 marks

	M1 Attempt to solve equation or inequation
`Reduce[{19 - 17 Cos[πt/8] ≥ 20, 0 ≤ t ≤ 60}, t] // N` `4.14988 ≤ t ≤ 11.8501 || 20.1499 ≤ t ≤ 27.8501 || 36.1499 ≤ t ≤ 43.8501 || 52.1499 ≤ t ≤ 59.8501` `11.850120591847382` – 4.149879408152618`` `7.70024` `7.700241183694764` * 4`  `30.801` show all digits scientific form integer part digits more... 31 seconds Must have units	M1 Find duration of one interval A1

GanDoug, who has always been a fan of matrices and transformations, launches a second sky rocket. The equation of the second sky rocket can be found by transforming $h(t) = 19 - 17\cos(\frac{\pi t}{8})$, $0 \le t \le 60$ using

the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2, T\begin{bmatrix} t \\ h \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} t \\ h \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

e. Show that the equation describing the motion of the second sky rocket is given by

$k(t) = 37 - 34\cos(\frac{\pi t}{12})$. **3 marks**

$\text{Solve}\left[\left(\begin{smallmatrix} tp \\ hp \end{smallmatrix}\right) == \left(\begin{smallmatrix} \frac{3}{2} & 0 \\ 0 & 2 \end{smallmatrix}\right) \cdot \left(\begin{smallmatrix} t \\ h \end{smallmatrix}\right) + \left(\begin{smallmatrix} 0 \\ -1 \end{smallmatrix}\right), \{t, h\}\right]$ $\{\{t \to \frac{2\,tp}{3}, h \to \frac{1 + hp}{2}\}\}$ $\text{Solve}\left[\frac{1 + hp}{2} == 19 - 17\,\text{Cos}\left[\frac{\pi}{8} * \frac{2\,tp}{3}\right], hp\right]$ $\{\{hp \to 37 - 34\,\text{Cos}\left[\frac{\pi\,tp}{12}\right]\}\}$ Assuming a list of rules | Use as _a two-dimensional array_ instead $k(t) = 37 - 34\cos(\frac{\pi t}{12})$ Must be $k(t)$	M1 Attempt to find t in terms of t' and h in terms of h' M1 Substitution into original equation A1

**f.** List a series of transformations that would transform $h(t)$ into $k(t)$. **2 marks**

• Dilation of factor $\frac{3}{2}$ from the h axis • Dilation of factor 2 from the t axis • Translation of 1 unit in the negative h direction Translation must be after h axis dilation Must have correct axes titles Or. . . . • Translation of $\frac{1}{2}$ in the negative h direction • Dilation of factor $\frac{3}{2}$ from the h axis • Dilation of factor 2 from the t axis	A1 for dilations A1 for translation

The path of a third sky rocket can be modelled by the function $m(t) = j(t) + k(t) + 8$ where the graphs of $j(t)$ and $k(t)$ are shown below.

g. Use addition of ordinates to sketch the graph of $m(t)$ on the axes below. 3 marks
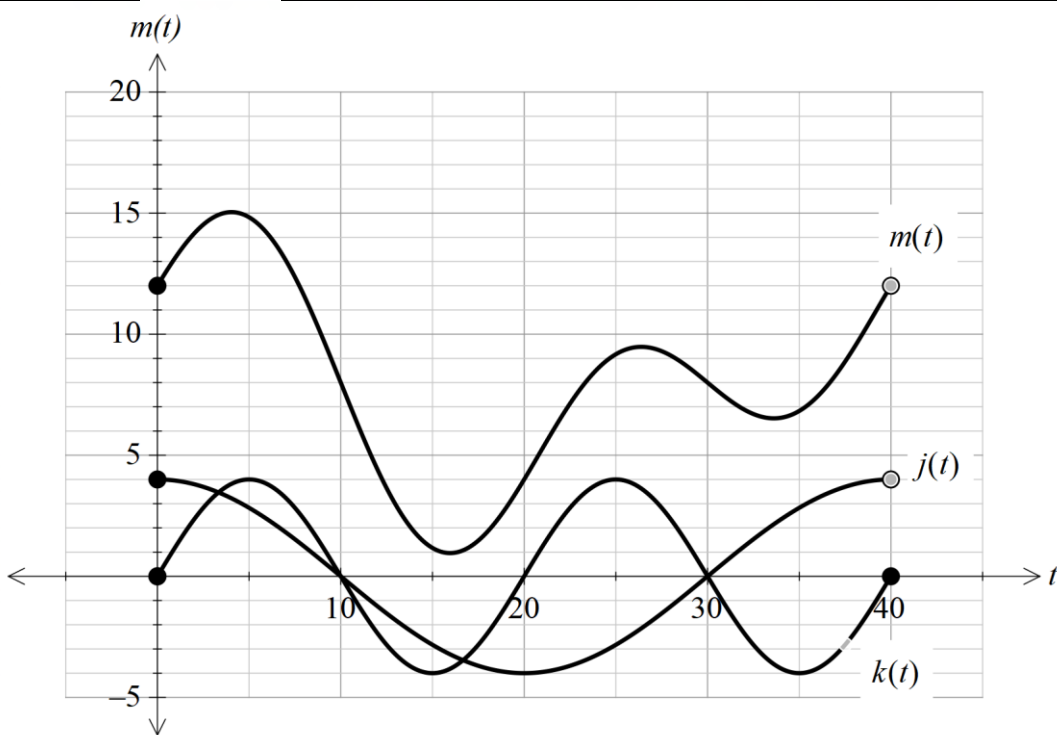
	1A Correct endpoints (0,12),(40,12)
	1A Correct points (10,8),(20,4), (30,8)
	1A Shape

If +8 is missed, pay only the shape mark
Points do not need to be labelled, just in correct position.

Yearning for adventure BilBui uses the power of the Trig to disappear from the party. Packing her belongings, she tells GanDoug "I want to see Maths again GanDoug, Maths beyond the hills of the Study Design."
"You are leaving the Trig with Frodo-lambous as we agreed, correct?"
*With a struggle, BilBui drops the Trig on the floor and leaves the Shire forever.*
Sometime later Frodo-lambous arrives to find BilBui gone. GanDoug, suspecting that the Trig is no regular identity, places it in the fireplace revealing the differential equations of Mathdor.
"It is the One Trig, forged by SaurJohn deep within Mathdor. We must leave, I will meet you in the village of VCEBree, I must tell the leader of my order, SaruMann, what I have learned."

Frodo-lambous, Schmidt-wise Gamgee, Merry Bandy-Xu and Pippin Toronyi leave the Shire for the human village of VCBree.

Question 2 (17 marks)

The hobbits, on their way to the village of VCBree find themselves point *A* on the south side of a rectangular forest that is 2km wide, and need to get to point to the village VCBree located at point *C*, 20km to the east and on the other side of the forest. Point *B* is directly to the north of point *A* and on the edge of the forest as shown. There is a track running along the far side *BC* of the forest. They need to get from *A* to *C*. They can travel at a speed of 24km/hr on the track but only at a speed of 12km/hr through the forest. Note: The Diagram is not to scale.
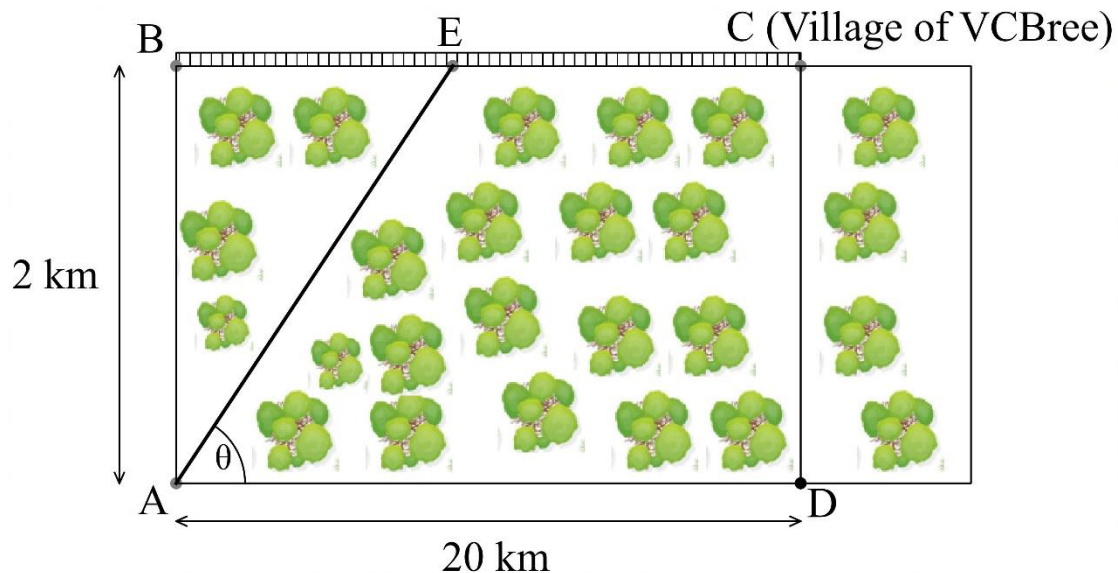
a. Calculate the time taken to travel directly from *A* through the forest to *B* and then from *B* to *C* using the track. 1 mark

Time = distance/Speed

$$\frac{d_{AB}}{12} + \frac{d_{BC}}{24}$$

```
:=
    2     20
   ── +  ──
   12     24
4]=
    1
form ◌ prime? perfect number? Roman numerals more... ◌
```

=1 hour

Must have units   1A

b. Calculate the time taken, to the nearest minute, to travel directly from A to C through the forest.

2 marks

$d_{AC} = \sqrt{2^2 + 20^2}$ $\quad = \sqrt{404}$ Time = distance/Speed $\dfrac{\sqrt{20^2 + 2^2}}{12}$ // N 1.67498 1.674979270186815` * 60 100.499 scientific form integer part digits more... =100 minutes or 1hour and 40 minutes Must have units. BEWARE of rounding. Must be correct	1A Distance 1H

As they reach the forest the hobbits realise that they are being pursued by the menacing dark riders and need to reach the safety of VCBree as quickly as possible.
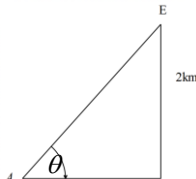
Schmidt-wise Gangee, being a keen mathematician realises that there would be an optimal path from A to C which would involve traveling in a straight line from A through the forest to a point E on BC then along the path to point the village as shown. The angle that the line AE makes with AD is $\theta°$.

c. Explain why the possible values of θ, to the nearest degree, are $6° \le \theta \le 90°$. 2 marks

Angle for direct from A to C $N\left[\text{ArcTan}\left[\dfrac{2}{20}\right] / \text{Degree}\right]$ 5.71059 Angle for direct through forest then along path is 90° So $6° \le \theta \le 90°$ to the nearest degree.	1A 1A

d. Find in terms of θ:

    i. the distance AE 1 mark

  E 2km θ A $\sin\theta = \dfrac{2}{AE}$ $AE = \dfrac{2}{\sin\theta}$	1A

    ii.   the distance *EC*   2 marks

$BE+EC = 20$ $EC = 20 - BE$ $\tan\theta = \dfrac{2}{BE}$ $BE = \dfrac{2}{\tan\theta}$ $EC = 20 - \dfrac{2}{\tan\theta}$ Consequential if EC = 20 – (Their *BE*)	1M Find *BE* 1A/H

Note: Some of the following identities may be needed for part e.

$$\cot(x) = \frac{1}{\tan(x)} = \frac{\cos(x)}{\sin(x)}, \ \ \csc(x) = \frac{1}{\sin(x)}, \ \ \sec(x) = \frac{1}{\cos(x)}$$
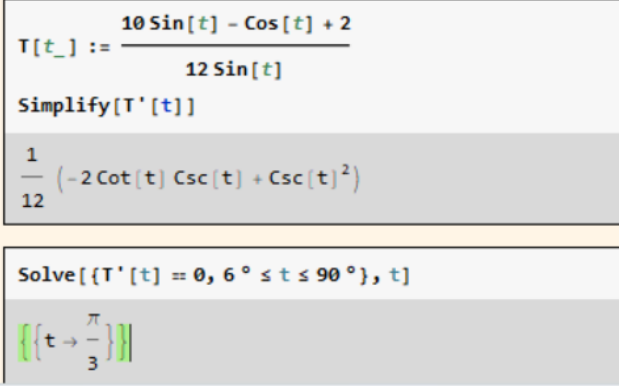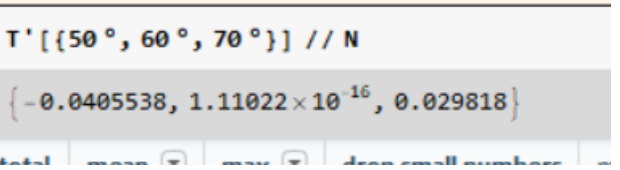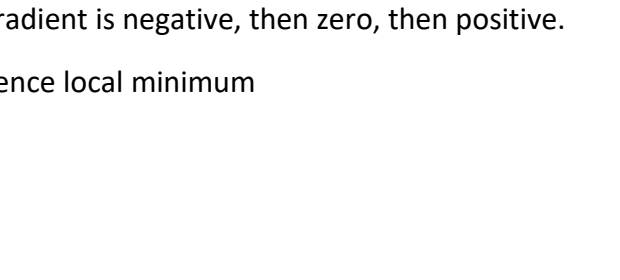
e. Hence, use algebra to show that the time taken, *T* hours, to travel from *A* to *C* via this route is given by:

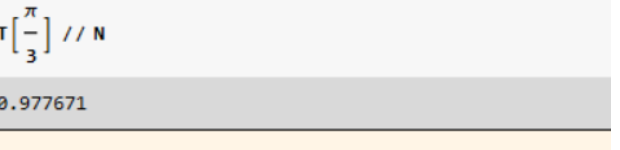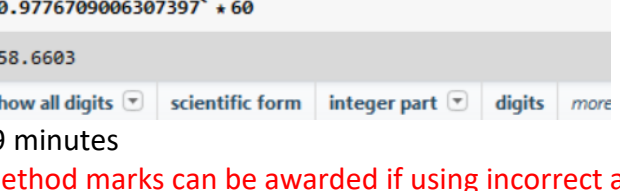$$T(\theta) = \frac{10\sin(\theta) - \cos(\theta) + 2}{12\sin(\theta)}$$
    3 marks

Time = distance/Speed $T = \dfrac{d_{AE}}{12} + \dfrac{d_{EC}}{24}$ $= \dfrac{\dfrac{2}{\sin\theta}}{12} + \dfrac{20 - \tan\theta}{24}$ $= \dfrac{\dfrac{2}{\sin\theta}}{12} + \dfrac{20 - \dfrac{2\cos\theta}{\sin\theta}}{24}$ $= \dfrac{4 + 20\sin\theta - 2\cos\theta}{24\sin\theta}$ $= \dfrac{10\sin\theta - \cos\theta + 2}{12\sin\theta}$ Need to see answers from part d used Need to see some form of algebraic simplification First two method marks can be awarded if using incorrect answers from part d	1M Equation for total time 1M Replace tan with sin/cos 1A

f. Write the equation for $\dfrac{dT}{d\theta}$ and hence find the angle $\theta°$ required to minimise the time taken to travel from A to C. You must verify that this is a minimum. **4 marks**

`T[t_] :=` $\dfrac{10\,\text{Sin}[t] - \text{Cos}[t] + 2}{12\,\text{Sin}[t]}$ `Simplify[T'[t]]` $\dfrac{1}{12}\left(-2\,\text{Cot}[t]\,\text{Csc}[t] + \text{Csc}[t]^2\right)$	**1A** Statement of T'(t)
`Solve[{T'[t] == 0, 6° ≤ t ≤ 90°}, t]` $\left\{\left\{t \to \dfrac{\pi}{3}\right\}\right\}$ $t = 60°$	**1M** Solving T'(t)=0 **1A**
`T'[{50°, 60°, 70°}] // N` $\{-0.0405538,\ 1.11022\times10^{-16},\ 0.029818\}$ Gradient is negative, then zero, then positive. Hence local minimum	**1A** Sign test or other valid method such as drawing graph or second derivative.

g. Calculate the minimum time taken to travel from A to C to the nearest minute. **2 marks**

$T\left[\dfrac{\pi}{3}\right]$ `// N`
 	
`0.977671`
 	
`0.9776709006307397` `* 60`
 	
`58.6603`
 	
show all digits ▼ scientific form integer part ▼ digits more
	M1
	Substitution into T[x]

59 minutes Method marks can be awarded if using incorrect answers from part h	**A1**

In VCBree the hobbits meet a mysterious man, Strider, who helps them escape from the dark riders. "Those are the Nazgul, the Invigilators of SaurJohn." He tells the hobbits. "Quick we must head to Derivendell, the Assistant Principal will know what to do."
Along the way, the travellers are ambushed by the Nazgul and Frodo-lambous is stabbed in the shoulder with a cursed blade. Strider knows they must get to Derivendell so the Elves can heal him. Luckily they meet ArweNadine, Princess of Derivendell, who takes Frodo-lambous on her horse and gallops for safety.

Question 3 (15 marks)

ArweNadine and Frodo-lambous are weaving through the woods along a path defined by:
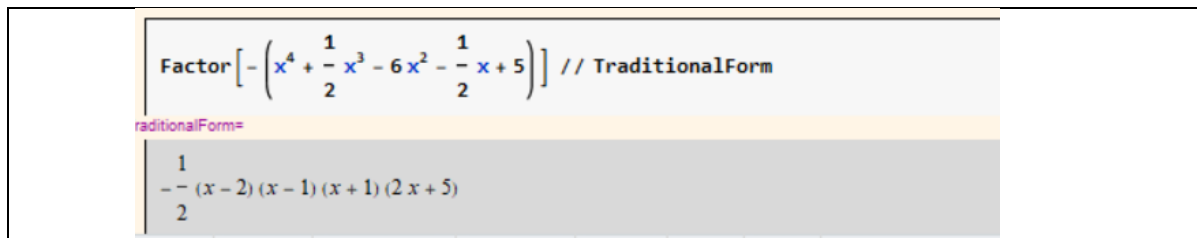$$f:[-\pi, \pi] \to \mathbb{R}, f(x) = a\,Sin(x), \ a \in \mathbb{R}^+$$

The Nazgul attack, coming in on a different path given defined by:
$$g:\left[-\frac{5}{2}, 2\right] \to \mathbb{R}, g(x) = -\frac{a}{3}\left(x^4 + \frac{1}{2}x^3 - 6x^2 - \frac{1}{2}x + 5\right), \ a \in \mathbb{R}^+$$

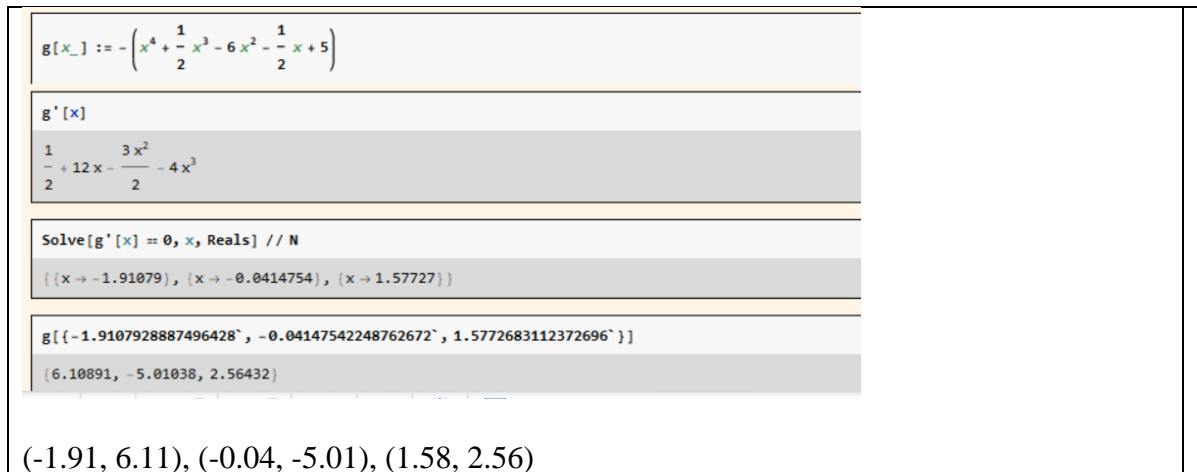Given that $a = 3$ (use for parts a, b, c, d and e).

a. Express $g(x)$ in factorised form. — 1 mark

	1A

b. State $g'(x)$ and find the cords of stationary points of $g(x)$ correct to 2 decimal places. — 3 marks

	1A
	1M G'[x]=0 and substitute
(-1.91, 6.11), (-0.04, -5.01), (1.58, 2.56)	1A

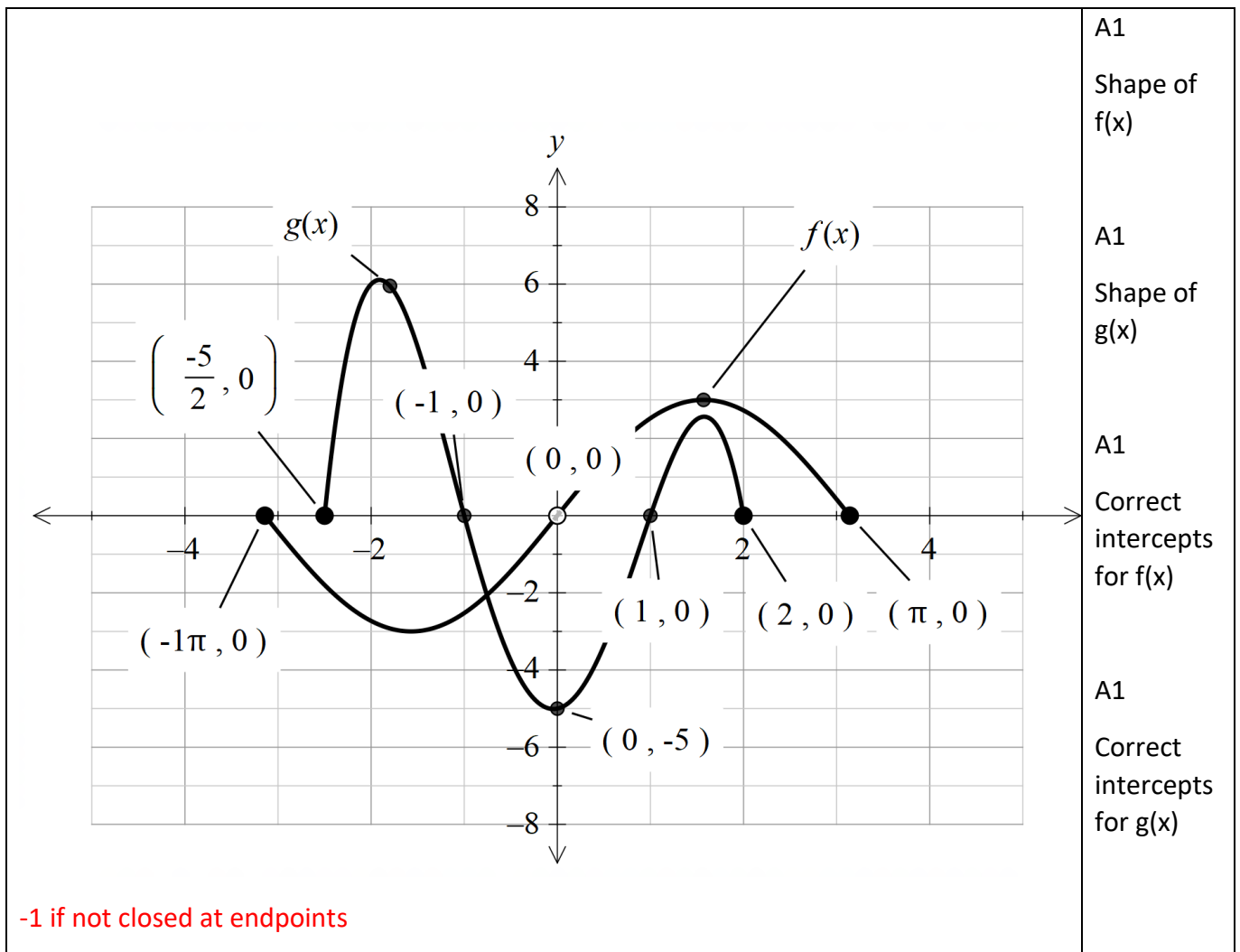**c.** Find the point of intersection of *f* and *g* to 2 decimal places. — 2 marks

	1M
(-0.76, -2.06) Other value is outside domain!	1A

Need this level of accuracy. If -0.76 is substituted y coord is -2.07

d. Sketch the graph of *f* and *g* on the same axes. Label endpoints and *x* and *y* intercepts.   4 marks

A1

Shape of f(x)

A1

Shape of g(x)

A1

Correct intercepts for f(x)

A1

Correct intercepts for g(x)

Graph labels: $g(x)$, $f(x)$, $\left(\dfrac{-5}{2}, 0\right)$, $(-1, 0)$, $(0, 0)$, $(-1\pi, 0)$, $(1, 0)$, $(2, 0)$, $(\pi, 0)$, $(0, -5)$

-1 if not closed at endpoints

e. State with reasons which, if any, of the composite functions $f(g(x))$ and $g(f(x))$ exist.

2 marks

	Domain	Range
f	$[-\pi, \pi]$	$[-3, 3]$
g	$[-\dfrac{5}{2}, 2]$	$[-5.01, 6.11]$

For $f(g(x))$ to exist, range $g(x) \subseteq$ domain $f(x)$

$[-5.01, 6.11] \not\subset [-\pi, \pi]$, so $f(g(x))$ does NOT exist

A1

For $g(f(x))$ to exist, range $f(x) \subseteq$ domain $g(x)$

$[-3, 3] \not\subset [-\dfrac{5}{2}, 2]$, so $g(f(x))$ does NOT exist

Domains and ranges must be shown.
If both correct with correct reasoning, but now values shown, then 1 mark only

A1

f. For what values of a are both $f(g(x))$ and $g(f(x))$ are defined? Give your answer to 3 decimal places. 2 marks

	Domain	Range
f	$[-\pi, \pi]$	$[-a, a]$
g	$[-\frac{5}{2}, 2]$	$[-1.670a, 2.036a]$

Need $[-a, a] \subseteq [-\frac{5}{2}, 2]$

AND $[-1.670a, 2.036a] \subseteq [-\pi, \pi]$

		$a \leq 2$
$-1.670a \geq -\pi \Rightarrow$	$1.670a \leq \pi \Rightarrow$	$a \leq 1.881$
$2.036a \leq \pi$	$\Rightarrow$	$a \leq 1.543$

Intersection of these intervals, so $a \in (0, 1.543]$

1M
Setting up subsets or similar

1A

ArweNadine decides on the **maximum** value of a for which $g(f(x))$ is defined and makes for the river at the border of Derivendell. The river at the boundary of Derivendell lies on the line $x = 2$.

Let b be the point at which f and g **first** intersect.

g. ArweNadine and Frodo-lambous travel along $f(x)$ until $x = b$. They are then chased along $g(f(x))$ until they reach the river at $x = 2$. State the point at which they cross the river to 2 decimal places. 1 mark

1A


```
In[11]:=   f[x_] := 1.543 Sin[x]

In[12]:=   g[x_] := - 1.543/3 (x^4 + 1/2 x^3 - 6 x^2 - 1/2 x + 5)

In[13]:=   g[f[2]]

Out[13]=   1.16066
```
show all digits ▾   integer part ▾   digits   root approximant   more...

(2.00, 1.16)

As ArweNadine and Frodo-lambous cross the river the mathemagical elven protections cause a local maximum in the water levels and wash away the Nazgul. Frodo-lambous is rushed into the Derivendell hospital for treatment.

After recovering in Derivendell, Frodo-lambous attends the council of Sally Chea-rond, Assistant Principal of Derivendell. A gathering of representatives from each race are in attendance to determine what must be done with the One Trig. Strider is revealed to be Mayagorn, Son of Mayathorn, heir of Bohni-ldur and rightful king of Gondilation. Legolainger of the Elves, Gregli of the Dwarves, Boro-Mclean, Steward of Gondilation and GanDoug the Gray argue about who should take the Trig.
"I will take it." Frodo-lambous interjects. And thus the Function of the Trig was formed. Frodo-lambous, Schmidt-wise, Merry, Pippin, Legolainger, Gregli ,Boro-Mclean, Mayagorn and GanDoug set out for Mathdor to destroy the One Trig forever. Along the way GanDoug tells the hobbits of how SaruMann has allied with SaurJohn and of how he escaped from his prison on a giant eagle.

After much walking….

The group attempt to cross the mountain pass but are blocked by a blizzard sent by SaruMann. They resolve to pass through the Mines of Mode-ia.
Inside they discover that the dwarves that once lived there are dead and goblins have taken over. Chased through the mines, they hear drums, drums in the deep.
The heroes run across the last bridge, only to be pursued by the fiery figure of Alexandrog.

Gandoug challenges Alexandrog to a mathematical battle yelling "You shall not pass (my question)!"
Alexandrog flashes a fiery glare and attacks the problem immediately.

Question 4 (2 marks)
Consider the equations

$$y = \frac{x}{k-1} + k \qquad \text{where } k \in \mathbb{R} \setminus \{1\}$$
$$y = (k-1)x^2 + 2x + k$$

Explain why there is always at least one solution to this set of simultaneous equations.

$\text{Solve}\left[\dfrac{x}{k-1} + k == (k-1)\,x^2 + 2x + k,\ x\right]$ $\left\{\{x \to 0\},\ \left\{x \to \dfrac{3-2k}{(-1+k)^2}\right\}\right\}$ Assuming a list of rules | Use as *a two–dimensional array* instead $x = 0$ is always a solution, so there is always at least one solution. If stated that x=0 is always a solution this is sufficient. $x = \dfrac{3-2k}{(-1+k)^2}$ is also a solution (=0 when $k = \dfrac{3}{2}$) OR $\Delta = (-3+2k)^2$ $\Delta \geq 0$ So always at least one solution	1A Solving equations 1A explain that there is at least one solution.

As GanDoug and the Alexandrog are completing the mathematical battle, the bridge collapses. "Fly you fools, Fly!" GanDoug yells before they both fall into the darkness
The group escapes the mine and pass through the Elven forest of Lothlorien. Galadri-ovska, ruler of Lothlorien gives the company mathematical gifts and sends them on their way.
After sailing to the end of the river Boro-Mclean, poisoned by the power of the One Trig attempts to steal it from Frodo-lambous. Just as he does this the company is attacked by the Uruk-i and Boro-Mclean falls, trying to protect the hobbits. Merry Brandy-Xu and Pippin Toronyi are captured and Frodo-lambous and Schmidt-wise leave to go to Mathdor alone.
Behind, the ominous figure of Corkllum follows…

The story continues in…

The Two Transformations

The Fellowship is now broken. With Merry and Pippin captured by the Uruk-i, Mayagorn, Legolainger and Gregli pursue, desperate to save their friends before they are delivered to SaruMann.

Question 5 (12 marks)

Mayagorn and Legolainger use their tracking skills to determine that the Uruk-i are travelling on a path with the equation $U(x) = k - (x - h)^3$ where $h \in \mathbb{R}$ and $k \in \mathbb{R}$

They don't know the exact values of h and k but decide to investigate and chase along the path of $U^{-1}(x)$.

   a. Mayagorn and Legolainger first consider the case when $h = 0$ and $k = 0$.
   Use algebra to show that $U^{-1}(x) = -\sqrt[3]{x}$ 1 mark

$U(x) = -x^3$ Let $U^{-1}(x) = y$ $x = -y^3$ $y = -x^{\frac{1}{3}}$ $U^{-1}(x) = -\sqrt[3]{x}$ Must use algebra	1A

b. While Gregli is trying to keep up he decide to consider the case when $h = 4$ and $k = -1$.

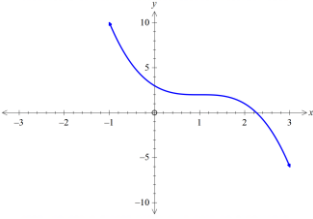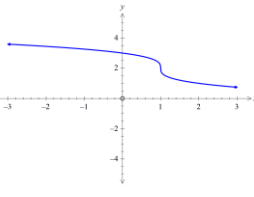Find the coordinates of any point(s) of intersection between $U(x)$ and $U^{-1}(x)$ to 3 decimal places

2 marks

```U[x_] := -1 - (x - 4)³```
 	
```InverseFunction[U][x]```
 	
```4 - (1 + x)^(1/3)```
 	
```Solve[U[x] == 4 - Surd[1 + x, 3], x] // N```
 	
```{{x → 2.48402}}```
 	
```U[2.48401977230717`]```
 	
```2.48402```
	M1
	Attempt to solve
	$U(x) = U^{-1}(x)$
	or
	$U(x) = x$

(2.484, 2.484)	A1

Consider the general case $U(x) = k - (x - h)^3$

c. Show that U and U^{-1} are strictly decreasing.   2 marks

$U(x) = k - (x-h)^3$ $U'(x) = -3(x-h)^2$ $U'(x) \leq 0 \ \text{for } x \in \mathbb{R}$ Sketch of $U'(x)$ or If $b > a$, Then $f(b) < f(a)$ So strictly decreasing.	$U^{-1}(x) = h - \sqrt[3]{x-k}$ $(U')^{-1}(x) = -\dfrac{1}{3[(x-k)^{\frac{1}{3}}]^2}$ $(U')^{-1}(x) \leq 0 \ \text{for } x \in \mathbb{R}$ Sketch of $(U')^{-1}(x)$ or If $b > a$, Then $f(b) < f(a)$ So strictly decreasing.	A1 reasonable attempt at explanation A1 Full and correct explanation

d. State the number of x and y intercepts of $U(x)$ and $U^{-1}(x)$. Explain your answer.   2 marks
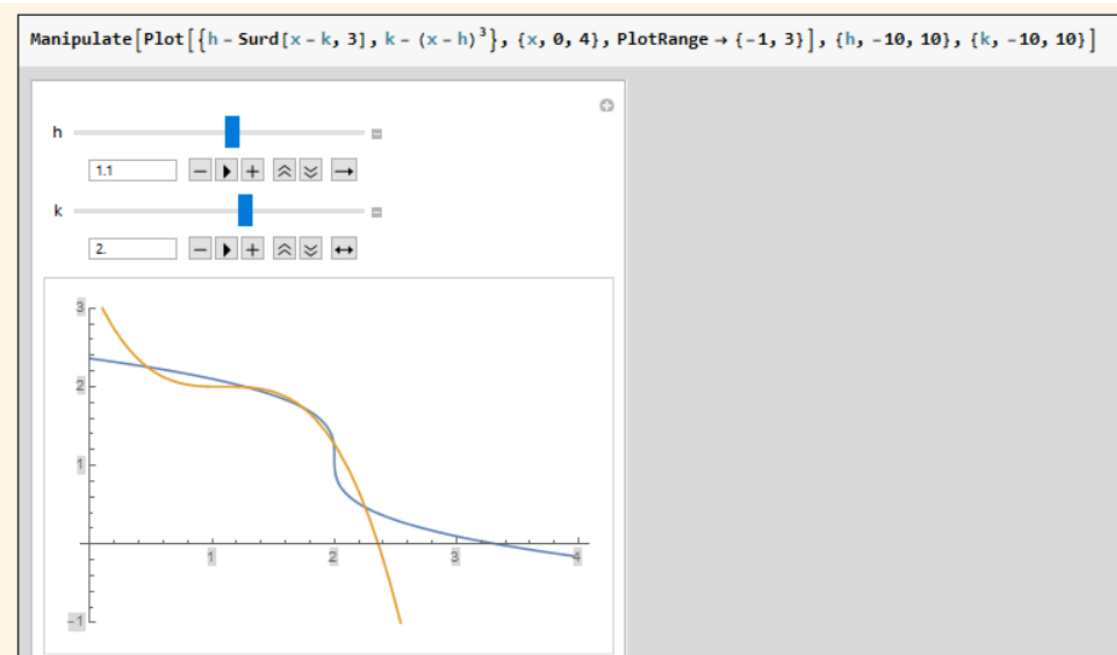
As the functions are both strictly decreasing they will have only one x-intercept.	A1
Since x=0 is included in the domain they have one y-intercept. (or similar)	A1
Must have explanations.	

e. Mayagorn and Legolainger stop to track the Uruk-i every time their paths cross. Recall that the Uruk-*i* travel along $U(x)$ and the heroes travel along $U^{-1}(x)$.

State the possible number of crossings for their paths. 2 marks

1, 3 or 5 Manipulate[Plot[{h - Surd[x - k, 3], k - (x - h)³}, {x, 0, 4}, PlotRange → {-1, 3}], {h, -10, 10}, {k, -10, 10}] Have seen examples of 1 and 3 points of intersection. Here is 5	A1 for answer of 1,3. MUST HAVE BOTH A1

f. If $h = k$, **use algebra** to **show** that there must be three solutions. 3 marks

$U(x) = U^{-1}(x)$ $h - (x-h)^3 = h - (x-h)^{\frac{1}{3}}$ $(x-h)^3 = (x-h)^{\frac{1}{3}}$ $(x-h)^9 = (x-h)$ $(x-h)^9 - (x-h) = 0$ $(x-h)[(x-h)^8 - 1] = 0$ $x-h = 0 \quad or \quad (x-h)^8 = 1$ $x-h = 0 \quad or \quad (x-h) = \pm 1$ $x = h, \quad h \pm 1$ Thus three distinct solution	M1 Equate expressions M1 Set = 0 A1 three solutions

NOTE: Some may solve $a^9 = a$ Intuitively with a = 0, 1, -1. This would be valid and gain full marks if clearly demonstrated/explained.

While Mayagorn, Legolainger and Gregli are chasing them, the Uruk-i are ambushed by the Riders of Rangehan and Merry and Pippin escape into the forest. There they meet a talking tree, Chis-beard, Assistant Principal of the Ents. They try to convince him to help in the war, but first Chis-beard needs to carefully follow all the policies and procedures before making a decision.

Upon reaching the Uruk-i, they find them all slain and the hobbits missing.

Mayagorn, Legolainger and Gregli instead find Gandoug who tells the story of how fought Alexandrog and smote him down on top a mountain. There he lay and was reborn as Gandoug the White.
They travel to the kingdom of Rangehan. There they find Toan-oden, king of Rangehan, under the control of SaruMann. Gandoug frees him and the people of Rangehan head to the mountain fortress, Helms Deep. GanDoug leaves to find reinforcements. "Look to my coming on the first light of the fifth day. Look to the east." He tells Mayagorn before galloping off.

END OF SAC 1 – PART 1