JOHN MONASH
SCIENCE SCHOOL

VICTORIAN CERTIFICATE OF EDUCATION
2019

STUDENT NAME:

MATHEMATICAL METHODS

SAC 1 – PART 2

Reading time: 15 minutes
Writing time: 120 minutes

QUESTION AND ANSWER BOOK

Number of questions	*Number of questions to be answered*	*Number of marks*
4	4	63
	Total	63

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one laptop with Mathematica and any number of Mathematica files.
- Students are NOT permitted to bring into the examination: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 18 pages.
- Formula sheet

Instructions

- Write your name in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

General marking comments:

- Giving decimals rather than exact values – 1 mark across paper
- Rounding errors – 1 mark across paper
- Units – 1 mark across paper
- No penalty for Mathematica syntax unless it doesn't show the required information

Instructions

Answer **all** questions in the spaces provided
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Note: Sections between these symbols are story only and are not needed to answer the questions

Previously in:

The Two Transformations

While Mayagorn, Legolainger and Gregli are chasing them, the Uruk-i are ambushed by the Riders of Rangehan and Merry and Pippin escape into the forest. There they meet a talking tree, Chis-beard, Assistant Principal of the Ents. They try to convince him to help in the war, but first Chis-beard needs to carefully follow all the policies and procedures before making a decision.

Upon reaching the Uruk-i, they find them all slain and the hobbits missing.
Mayagorn, Legolainger and Gregli instead find Gandoug, now Gandoug the White and are taken to the kingdom of Rangehan. There they find Toan-oden, king of Rangehan, under the control of SaruMann. Gandoug frees him and the people of Rangehan head to the mountain fortress, Helms Deep. GanDoug leaves to find reinforcements. "Look to my coming on the first light of the fifth day. Look to the east." He tells Mayagorn before galloping off.

Meanwhile..

Frodo-lambous and Schmidt-wise, after walking around in circles for half a book, capture Corkllum. They convince him to guide them to Mathdor and nothing could possibly go wrong.

Frodo-lambous and Schmidt-wise follow Corkllum to a secret path into Mathdor and everything goes wrong.

Question 1 (14 marks)
While travelling through the secret cave into Mathdor, Frodo-lambous is bitten by the evil giant spider Shelob. Shelob injects poison into his blood stream. His natural Hobbit immunity means that the amount of poison in his blood stream, after reaching a maximum level begins to decrease.

The amount of poison in blood stream can be modelled by $P(t) = ate^{\frac{12-kt}{5}}$, $0 \le t \le 5$, where P is the number of units of poison in the blood stream t hours after being bitten.

After 30 minutes there is $\frac{9e^2}{2}$ units of poison in the blood stream and after 3 hours there is 27 units of poison in the blood stream.

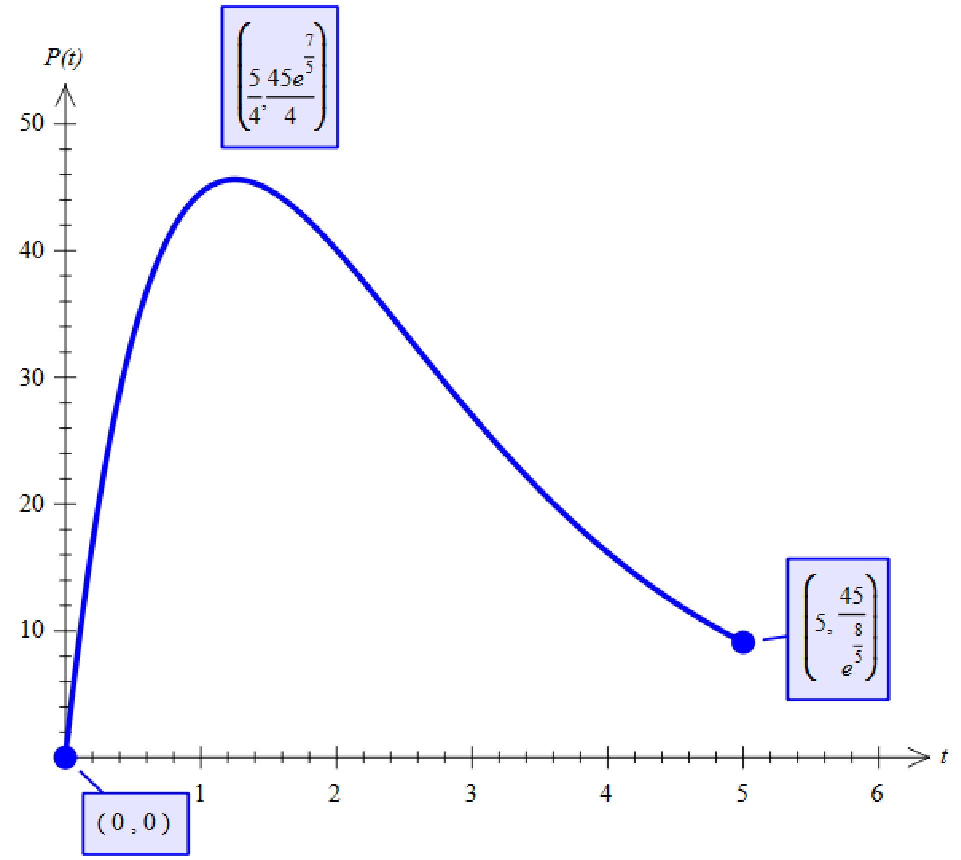
a. Use algebra to show that $a = 9$ and $k = 4$ 2 marks

$P\left(\frac{1}{2}\right) = \frac{9e^2}{2}$, $P(3) = 27$ $\frac{9e^2}{2} = \frac{a}{2}e^{\frac{12-\frac{1}{2}k}{5}}$ eqn 1 $27 = 3ae^{\frac{12-3k}{5}}$ eqn 2 divide equation 2 by equation 1 $\frac{27\times 2}{9e^2} = 6e^{\frac{12-3k}{5}-\frac{12-\frac{1}{2}k}{5}}$ $e^{-2} = e^{\frac{-3k+\frac{1}{2}k}{5}}$ $-2 = \frac{-\frac{5}{2}k}{5}$ $k = 4$ Sub into eqn 2 (or 1) $27 = 3ae^0$ $a = \frac{27}{3}$ as required $a = 9$	M1– Set up and attempt to solve simultaneous equations M2– correct algebra to $a = 9$ and $k = 4$

b. Find the maximum level of poison in the bloodstream to two decimal places and the time, to the nearest minute, that it occurs. 2 marks

`{t, P[t]} /. Solve[P'[t] == 0, t] // N` `{{1.25, 45.621}}` Max level is 45.62 units at 1 hour and 15 minutes after being bitten.	M1 – Setting derivative to 0 A1– max level and time.

c. Sketch the graph of $P(t) = 9te^{\frac{12-4t}{5}}$, $0 \le t \le 5$, labelling end points and turning points with their **exact coordinates**. 3 marks



Shape – 1 mark Endpoints – 1 mark Maximum point – 1 mark

It is known that the poison will be deadly if it continuously maintains a level of 40 units for more than 2 hours.

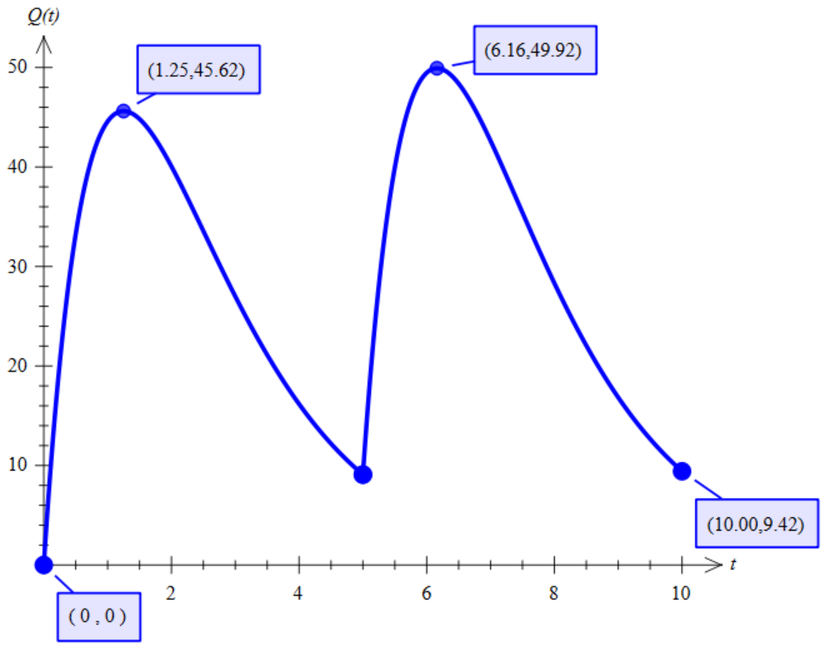
d. Show that the poison is not deadly according to this model. 2 marks

`Reduce[P[t] > 40 && 0 ≤ t ≤ 5, t] // N` `0.713546 < t < 2.00496` `2.0049640952980927` - 0.713546083137432`` `1.29142` Above 40 units for 1.29 hours, therefore it is not deadly	M1 – Inequality A1 – 1.29 hours and statement

Five hours after being bitten, he is bitten again. The amount of poison in the bloodstream can now be modelled by the piecewise function

$$Q(t)=\begin{cases} P(t), & 0\le t\le 5 \\ P(t)+P(t-5), & 5<t\le 10 \end{cases}$$

e. Sketch the graph of $Q(t)$ on the axes below labelling end points and turning points with their coordinates correct to 2 decimal places. 3 marks



Shape – 1 mark
Endpoints – 1 mark accept (10 ,9.42)
Maximum points – 1 mark

f. Is this second bite enough to cause death? Show working to justify your answer. 2 marks

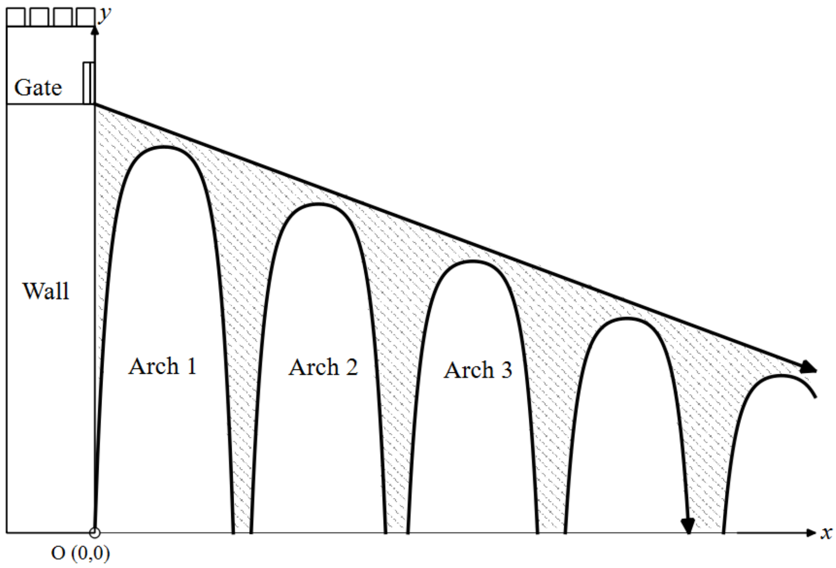
`Reduce[P[t] + P[t - 5] > 40 && 5 ≤ t ≤ 10, t] // N` `5.5014 < t < 7.18623` `7.186233673564575` - 5.501404730479687`` `1.68483` Q(t) is greater than 40 for the same interval as in part d (not deadly). It is greater than 40 for 1.68 hours from $5.50<t<7.19$ and this is also not deadly.	M1 – inequality for second region A1 – interpretation of second region being less than 2 hours

Schmidt-wise believes Frodo-lambous to be dead and takes the One Trig, hoping to destroy it himself. Frodo-lambous is discovered and captured by orcs, Schmidt-wise hears this and realises that perhaps he was too hasty.

Over at Helms Deep, the Rangehan army is filing up the bridge into the fortress.

Question 2 (25 marks)

As the army enters the fortress of Helms Deep, Legolainger and Gregli consider the shape of the bridge leading into up to the gates. The section of the bridge leading to the wall is shown below. The arches and the walkway continue until the walkway reaches the x axis.



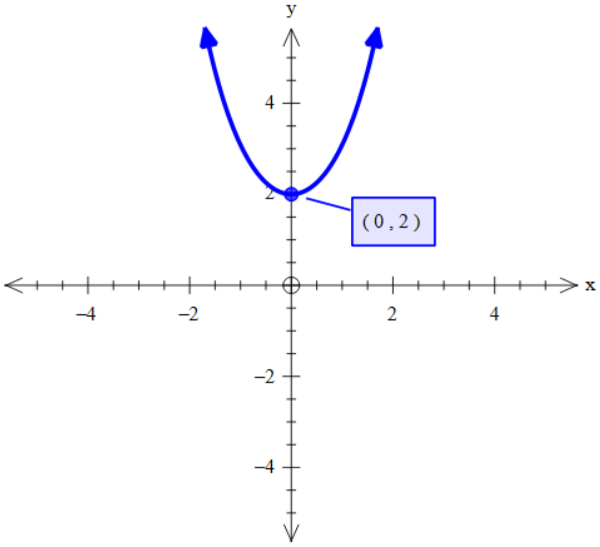
They take the base of the wall to be the Origin, O and the ground in front of the wall to be the x axis. The wall and gate form the y axis of their coordinate system. **Distances along the x and y axis are in metres.**

The bridge leading to Helms deep has a number of arches. These arches have the shape of a catenary.

A catenary without translations has the form $y=\frac{a}{2}\left(e^{\frac{x}{a}}+e^{-\frac{x}{a}}\right)$

NOTE: In the unlikely event that Mathematica outputs Cosh[ ] or Sinh[ ], use TrigToExp [ ] to convert the expression to an exponential function.

a. Sketch the graph of $y = e^x + e^{-x}$ for $x \in \mathbb{R}$ on the axes below. Label any axial intercepts with their coordinates. 2 marks



Shape – 1 mark y intercept – 1 mark No scale needed on *x* axis

b. Consider the functions $g(x)=\frac{a}{2}\left(e^{\frac{x}{a}}+e^{-\frac{x}{a}}\right)$ where $a=1$, $h(x)=\frac{a}{2}\left(e^{\frac{x}{a}}+e^{-\frac{x}{a}}\right)$ where $a=-1$ and $k(x)=\frac{a}{2}\left(e^{\frac{x}{a}}+e^{-\frac{x}{a}}\right)$ where $a=2$.

State the transformations applied to $g(x)$ to obtain the graphs of :

i. $h(x)$ — 1 mark

Reflection in the x axis (Reflection in the y axis -optional as it doesn't change the shape)	A1

ii. $k(x)$ — 2 marks

Dilation by a factor of 2 from the *x* axis. Dilation by a factor of 2 from the *y* axis (or k(*x*) axis)	A1 A2

c. Explain why *a* must be less than zero to model the arches under the bridge? — 1 mark

All arches are inverted from the base graph (g(x)), therefore they must be reflected in the x axis. This means $a<0$	A1

The arches under the bridge have the shape such that $a = -\frac{1}{4}$

That is, they have the general form $j(x) = -\frac{1}{8}\left(e^{4x} + e^{-4x}\right)$

d. State the maximum value of $j(x)$ 1 mark

`{x, j[x]} /. Solve[j'[x] == 0, x, Reals]` $\{\{0, -\frac{1}{4}\}\}$ Maximum value is $-\frac{1}{4}$	A1 (not just coords)

Each arch has an equation given by $f_n(x)$ where $n \in \mathbb{Z}^+$ is the arch number, starting with $n = 1$

The first catenary has the form $f_1(x) = j(x-h)+k$

e. Given that the local maximum of the first arch is 27m above the ground, Show that $k = \frac{109}{4}$

2 marks

j(x) must be translated up such that its maximum value is 27. That is: $-\frac{1}{4}+k = 27$ $\therefore k = 27+\frac{1}{4}$ $\Rightarrow k = \frac{109}{4}$	M1- Recognition of vertical translation M2- Statement of $27+\frac{1}{4}$ giving $\frac{109}{4}$

f. The first arch has x intercepts at $(0,0)$ and $(m,0)$ where m is a positive real number.

Show that the equation of the first arch is $f_1(x) = -\frac{1}{8}\left(e^{4(x-h)} + e^{-4(x-h)}\right) + \frac{109}{4}$

where $h = \frac{1}{4}\log_e(109+6\sqrt{330})$ Note: Technology may be used. 2 marks

$j(x)+\frac{109}{4} = 0$ must be translated in the positive x direction such that its x intercept is at (0,0) `Solve[j[x] + 109/4 == 0, x, Reals]` $\{\{x \to \frac{1}{4}\text{Log}[109 - 6\sqrt{330}]\}, \{x \to \frac{1}{4}\text{Log}[109 + 6\sqrt{330}]\}\}$ $f_1(x) = j(x-h)+k$ must be translated in the positive x direction so therefore h must be positive. Taking the positive solution: $h = \frac{1}{4}\log_e\left(109+6\sqrt{330}\right)$ as required	M1– Solving for $j(x)+\frac{109}{4} = 0$ M2– explanation/reasoning for choosing positive value as h.

g. What are the coordinates of the points where $f_1(x)=0$? Give your answers to 2 decimal places. 2 marks

``f1[x_] := -1/8 (e^(4 (x - 1/4 Log[109+6 √330])) + e^(-4 (x - 1/4 Log[109+6 √330]))) + 109/4``` ```Solve[f1[x] == 0, x, Reals] // N``` ```{{x → 0.}, {x → 2.69224}}```	$f_1(x)=0$ at $(0,0)$ and $(2.69,0)$	A1 and A2 Correct coordinates.

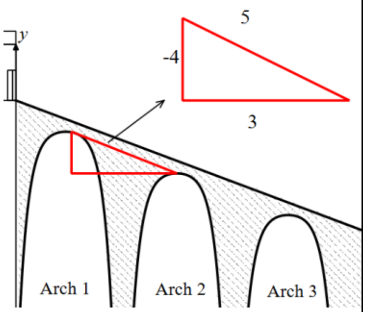
The walkway of the bridge has the equation $y=30-\dfrac{4x}{3}$.

h. Find the vertical distance between the top of the first arch and the walkway. Give your answer to 3 decimal places 2 marks

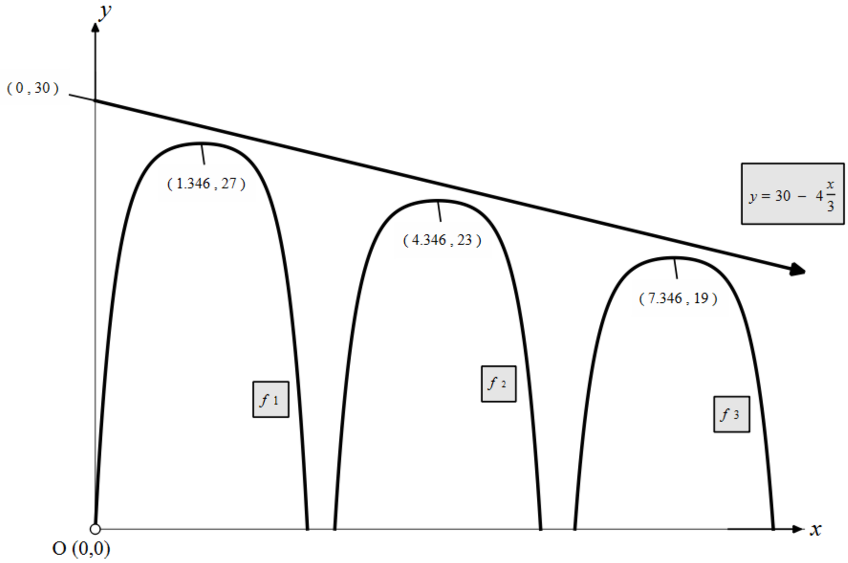
``{x, f1[x]} /. Solve[f1'[x] == 0, x, Reals] // N``` ```{{1.34612, 27.}}``` ```(30 - 4/3 * 1.3461185050312694`) - 27``` ```1.20518``` Vertical distance is 1.205 m	M1- x coordinate of the turning point A1- vertical distance

The maximum points of each arch have a **straight-line** distance of 5m between them.

i. Given that all arches have the same vertical distance from their local maximum to the walkway, explain why $f_2(x) = f_1(x-3)-4$ and $f_3(x) = f_1(x-6)-8$ 3 marks

As the top of each arch has the same vertical distance to the walkway, the line joining the tops must be parallel to the walkway. Therefore the gradient of the line joining the top of each arch must be $-\frac{4}{3}$. This forms a 3,4,5 right angled triangle as shown to the right.  Therefore the 2nd arch must be translated 3 units right and 4 units down and the 3rd arch must be translated 6 units right and 8 units down giving equations of $f_2(x) = f_1(x-3)-4$ and $f_3(x) = f_1(x-6)-8$	M1- recognition that the line joining the maximums is parallel to the walkway. (or giving gradient between maximums as $-\frac{4}{3}$) M2- recognition of the 3,4,5 triangle M3- explanation of translations Can still receive M2 and M3 without M1

j. Sketch the graph of f_1, f_2, f_3 **and the walkway** on the same axes over an appropriate domain.
Label the coordinates of any turning points to 3 decimal places and the coordinates of any y intercepts. You do not need to label any x intercepts. 4 marks



y intercepts– 1 mark Arches and walkway shape. – 2 marks. 1 mark if one mistake. Maximums – 1 mark

k. Let P_n be the coordinate of the local maximum of the nth arch. Express the coordinates of P_n in terms of n, where $n \in \mathbb{Z}^+$. Constants may be expressed to 3 decimal places. 2 marks

x coordinates shift 3 units right for each arch starting at 1.346 First arch is n=1, therefore, the x coordinate is $1.346+3(n-1)$	1A – x coord
y coordinates shift by 4 units down for each arch starting at 27 First arch is n=1, therefore, the y coordinate is $27-4(n-1)$	2A – y coord
Coordinates are therefore: $\left(1.346+3(n-1),\ 27-4(n-1)\right)$ OR $\left(-1.654+3n,\ 31-4n\right)$	(Alternate: 1 method mark available for reasoning/setting up)

l. Given that the arches continue until the walkway reaches the ground ($y=0$), state the number of arches in the bridge. 1 mark

Let $27-4(n-1)=0$ $n=7.75$ Therefore there are 7 arches (8th would be below ground)	1A

After a long and protracted battle sequence in which many orcs are slain…

Battle weary, Mayagorn, Legolainger and Gregli see the rising sun and GanDoug leads the charge with the riders of Rangehan into the flank of the orc army. The battle is won.
Meanwhile, Chis-beard finishes his checklist and concludes that SaruMann is a very bad wizard. The Ents, along with Merry and Pippin attack Sarumann's tower, routing his armies.
They are joined by Gandoug, Mayagorn, Legolainger, Gregli and the armies of Rangehan.
GanDoug strips SaruMann of his powers and imprisons him in his own tower.

The story continues in…

The Return of the Graphing

SaurJohn is raising an army to attack Gondilation. After discovering this Mayagorn and the Human armies travel the Paths of the Dead, hoping to recruit the Dead Men of Dun-h(ρ(x)) and save Gondilation.

Question 3 (8 marks)

The Dead Men of Dun-h(ρ(x)) only fight for supreme mathematicians. They set the following problem for our heroes to solve.

Consider the function below, defined on its maximal domain:

$$k(x)=\sqrt{\frac{a(x+1)^2}{3}-b}\text{ , where } a\in R^+,\ b\in R^+.$$

a. **Use algebra** to find the implied domain of $k(x)$ 2 marks

We require $\frac{a(x+1)^2}{3}-b\geq 0$ let $\frac{a(x+1)^2}{3}-b=0$ $(x+1)^2=\frac{3b}{a}$ $x=-1\pm\sqrt{\frac{3b}{a}}$ $\therefore\ x\in\left(-\infty,-1-\sqrt{\frac{3b}{a}}\right]\cup\left[-1+\sqrt{\frac{3b}{a}},\infty\right)$	1M – recognition that $\frac{a(x+1)^2}{3}-b\geq 0$ 1A – Correct domain (must use algebra) Watch out for $x\geq -1\pm\sqrt{\frac{3b}{a}}$ (incorrect)

b. Find the values of a and b for which the tangent to the graph of $k(x)$ at (0, 1) makes an angle of 60^{o} with the positive direction of the x-axis. 2 marks

$k(x)=\sqrt{\frac{a(x+1)^2}{3}-b}$ $m=\tan(60°)=\sqrt{3}$ $k(0)=1$ and $k'(0)=\sqrt{3}$ $\therefore a=3\sqrt{3}$ $b=-1+\sqrt{3}$	1M - $k'(0)=\sqrt{3}$ 1A – Correct values of a and b

c. Show that the graph of $k(x)$ does not have any stationary points. 2 marks

$k'(x)=\frac{a(x+1)}{\sqrt{3a(x+1)^2}}$ let $k'(x)=0$ $\therefore a(x+1)=0$ $x=-1$ However this is outside of the domain, $\therefore\ x\in\left(-\infty,-1-\sqrt{\frac{3b}{a}}\right]\cup\left[1+\sqrt{\frac{3b}{a}},\infty\right)$ Therefore there are no stationary points.	1M- calculation of $x=-1$ 2M – statement that it is out of the domain.

d. Find the relationship between a and b for which the distance between the two x-intercepts of $k(x)$ is 1 unit. 2 marks

When $k(x)=0$, $x=-1\pm\sqrt{\frac{3b}{a}}$ Require the distance between the x intercepts to be 1 $\therefore\left(-1+\sqrt{\frac{3b}{a}}\right)-\left(-1-\sqrt{\frac{3b}{a}}\right)=1$ $2\sqrt{\frac{3b}{a}}=1$ $\frac{3b}{a}=\frac{1}{4}$ $3b=\frac{a}{4}$ $12b=a$ or $\mathrm{b}=\frac{a}{12}$	1M- subtraction of one x intercept from the other A1 – simplified relationship between a and b Pay A1 as long as a and b are on opposite sides of the equation

The Dead men of Dun-h(ρ(x)) are impressed and promise to fight.
Together they march to Gondilation and save the city. There is much rejoicing. Knowing that Frodo-lambous will be sneaking into Mathdor they gather their forces and march to distract SaurJohn and allow the hobbits a chance to destroy the Trig.
At the gates the forces of good and evil compete in a mathematical competition for the ages.

Meanwhile…

Schmidt-wise sneaks into Mathdor and saves Frodo-lambous. Together they head to Mount Domain to destroy the One Trig once and for all. Finally, Frodo-lambous is at the place where the one Trig was derived. He just has to drop it into the lava, unfortunately the power of the Trig does not give up so easily and Frodo-lambous turns away.
Just as he does, Corkllum suddenly appears and attacks Frodo-lambous. In the struggle Corkllum bites off Frodo-lambous' finger with the Trig and they both fall into the lava. Schmidt-wise grabs Frodo-lambous and pulls him to safety.

Question 4 (16 marks)

The One Trig falls into the lava in the depths of Mt Domain. **Initially the Trig has a temperature of 30 degrees Celcius**. It falls into the lava and its **change** in temperature, ΔT (in 100's of degrees Celsius), at time, t mins, after falling into the lava is given by the function:

$$\Delta T(t) = \frac{3}{e}\left(e^{\frac{t}{5}} - 1\right) \text{ where } a \in \mathbb{R}^+ \text{ and } b \in \mathbb{R}^+$$

The Trig is imbued with mathematical magic such that it gains protection equal to the inverse of the danger (the increase in temperature).

a. Given that $M(t)$ is the inverse function of $\Delta T(t)$, show using algebra that $M(t)$ has the rule:

$M(t) = 5log_e\left(\frac{e}{3}t + 1\right)$ 2 marks

let $t = \frac{3}{e}\left(e^{\frac{y}{5}} - 1\right)$ where $y = M(t)$ $\frac{e}{3}t = e^{\frac{y}{5}} - 1$ $e^{\frac{y}{5}} = \frac{e}{3}t + 1$ $\frac{y}{5} = \log_e\left(\frac{e}{3}t + 1\right)$ $y = 5\log_e\left(\frac{e}{3}t + 1\right)$ As required	1M – swap t and T (with correct notation) 2M – Correct algebra to required answer

The One Trig will melt when the change in temperature, $\Delta T(t)$, becomes greater than the magical protection, $M(t)$.

b. Find the time, to 2 decimal places, at which the One Trig melts. 1 mark

require $\Delta T(t) > M(t)$ let $\Delta T(t) = M(t)$ $\frac{3}{e}\left(e^{\frac{t}{5}} - 1\right) = 5\log_e\left(\frac{e}{3}t + 1\right)$ $t = 12.59$ minutes	1A

c. At what **temperature** does the Trig melt? Give your answer to the nearest degree Celcius.
1 mark

$T_f = \Delta T(12.5925) + 30$ $T_f = 1259 + 30$ $T_f = 1289°C$	1A

While they are waiting for the Trig to melt they decide to investigate $\Delta T(t)$ and $M(t)$ further:

d. At what time(s) is the rate of increase of both $M(t)$ and $\Delta T(t)$ the same? Give your answer to the **nearest second**.
1 mark

$\mathrm{D}\left[\frac{3}{E}\left(E^{t/5}-1\right), t\right]$ $\frac{3}{5}e^{-1+\frac{t}{5}}$ $\mathrm{D}\left[5\,\mathrm{Log}\left[\frac{E}{3}t+1\right], t\right]$ $\frac{5e}{3\left(1+\frac{et}{3}\right)}$ $\mathrm{Solve}\left[\frac{5e}{3\left(1+\frac{et}{3}\right)} == \frac{3}{5}e^{-1+\frac{t}{5}}, t\right] \mathbin{//} \mathrm{N}$ $\{\{t \to 5.88202\}\}$ $t = 353\mathrm{sec}$	1A – Correct Answer

e. At some time, t, the rate of increase of the change in temperature, $\Delta T'(t)$ is equal to 1.
Find the exact time at which this occurs. Let this point on $\Delta T(t)$ be P.
1 mark

$\mathrm{Solve}\left[\frac{3}{5}e^{-1+\frac{t}{5}} == 1, t, \mathrm{Reals}\right] \mathbin{//} \mathrm{Simplify}$ $\left\{\left\{t \to 5 + \mathrm{Log}\left[\frac{3125}{243}\right]\right\}\right\}$ $t = 5 + \log_e\left(\frac{3125}{243}\right)$ minutes	1A

f. Show that the equation of the normal to the graph of $\Delta T(t)$ at point P, y_N has the equation

$$y_n = -t + 10 + \log_e\left(\frac{3125}{243}\right) - \frac{3}{e}$$ 2 marks

`3/E (E^(t/5) - 1) /. t → 5 + Log[3125/243] // Simplify` `5 - 3/e` $m_n = \frac{-1}{1} = -1$ $y_n - \left(5 - \frac{3}{e}\right) = -1\left(t - \left[5 + \log_e\left(\frac{3125}{243}\right)\right]\right)$ $y_n = -t + 5 + \log_e\left(\frac{3125}{243}\right) + 5 - \frac{3}{e}$ $y_n = -t + 10 + \log_e\left(\frac{3125}{243}\right) - \frac{3}{e}$	1M(H) – Correct gradient and y value (consequential on their value from e) 1A – correct equation (with sufficient working) (no consequential on answer)

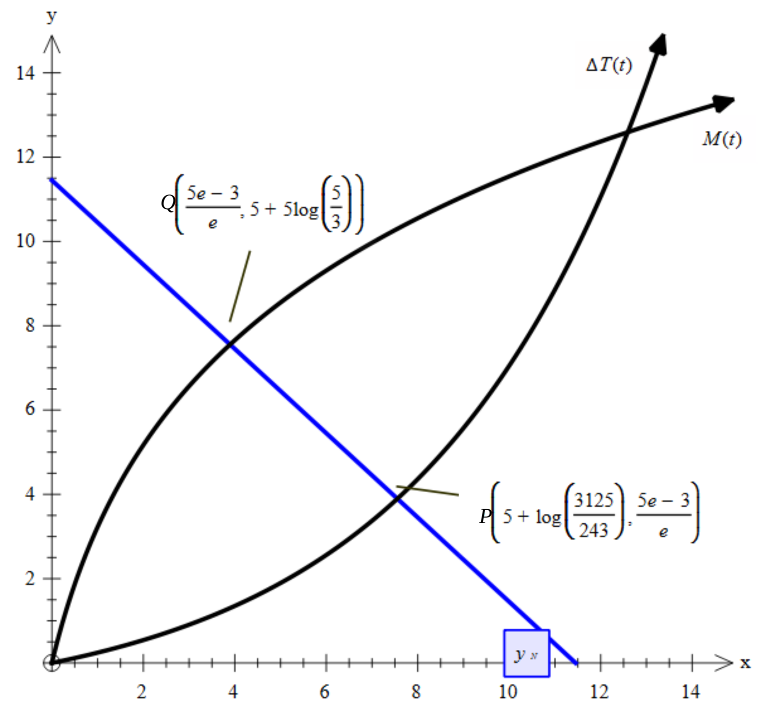
g. At what time is the rate of increase of $M(t)$ equal to 1? Let this point on $M(t)$ be Q. 1 mark

`Solve[D[5 Log[E/3 t + 1], t] == 1, t]` `{{t → (-3 + 5 e)/e}}` $t = \frac{5e-3}{e}$ minutes	1A

h. Show that Q lies on the line y_N. 1 mark

When $t = \frac{5e-3}{e}$ `5 Log[E/3 t + 1] /. t → (-3 + 5 E)/E // FullSimplify` `5 (1 + Log[5/3])` `-t + 10 + Log[3125/243] - 3/E /. t → (-3 + 5 e)/e // FullSimplify` `5 + Log[3125/243]` `(5/3)^5` `3125/243` $M\left(\frac{5e-3}{e}\right) = 5 + 5\log_e\left(\frac{5}{3}\right)$ $y_n = 5 + \log_e\left(\frac{3125}{243}\right)$ $\left(\frac{5}{3}\right)^5 = \frac{3125}{243}$ Therefore Q lies on y_N as their y values match at $t = \frac{5e-3}{e}$	1A – Show that their y values are the same

i. Sketch $\Delta T(t)$, $M(t)$ and the normal to P on the axes below.
Label the exact coordinates of the points P and Q.
Label each function with its name (you do not need its equation) 4 marks



$M(t)$ and $\Delta T(t)$ -1 mark each
y_N - 1 mark
P and Q – 1 mark

j. The area bounded by $\Delta T(t)$ and $M(t)$ is divided by the normal to point P creating two distinct areas. Find the ratio of the two areas. Explain your answer without the use of calculus. 2 marks

The ratio of the areas is 1:1

As the tangents have a gradient of 1, they are parallel to the line $y = x$, which joins the intersections of $\Delta T(t)$ and $M(t)$ which are themselves reflected across the line $y = x$.

Points P and Q are reflections along the line $y = x$ as they both lie on the normal, which is perpendicular to both tangents and $y = x$

This means that P and Q must lie halfway along the arc of each function and the line $\overrightarrow{PQ}$ must bisect the bounded are.
NOTE: This is incorrect as the curvature of the functions is not symmetric along the interval. It would work for something like a semi-circle as in this case the point with gradient 1 would be halfway along the arclength of the interval and the normal would bisect the bounded area. The actual ratio for this case is approximately 0.87 : 1 but this requires calculus and setting up of 4 integrals to find the areas between curves. This is shown over the page.

```
T[t_] := 3/E (E^(t/5) - 1)

M[t_] := 5 Log[E/3 t + 1]

y[t_] := -t + 10 + Log[3125 / 243] - 3 / E

Integrate[M[t] - T[t], {t, 0, (5 E - 3)/E}] +
  Integrate[y[t] - T[t], {t, (5 E - 3)/E, 5 + Log[3125 / 243]}] // N

28.0857

Integrate[M[t] - y[t], {t, (5 E - 3)/E, 5 + Log[3125 / 243]}] +
  Integrate[M[t] - T[t], {t, 5 + Log[3125 / 243], 12.59}] // N

32.3557

28.08565778282653` / 32.35574543021633`

0.868027
```

The One Trig is destroyed and with it, SaurJohn and his forces. The people of Median-Earth are finally free.
The surviving members of the Function of the Trig return to their homes to spend their days happily working on mathematics that won't destroy the world.

END OF SAC 1 – PART 2